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Chaos-Entropy Analysis and Acquisition of Individuality and Proficiency of Human Operator's Skill Using a Neural Controller*

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Abstract

The emergence of intelligence in an autonomous robot exists in the dexterity of humans or creatures as complex systems and research and development procedures along this approach seems necessary for realization of an intellectual robot. However, although strict judgment is required during stabilizing control of an unstable system, such as an inverted pendulum on a cart by human operators, it is assumed that human operators exhibit complex behavior intermittently. A previous paper investigated the skill of a human operator and investigated the formation of a complex system in the learning process of human operators with objects difficult to control. It also considered the mechanism of robustness of human operators against such a disturbance. The current paper shows that the neural network controller identified from time series data of each trial of several operators exhibits the human-generated decision-making characteristics with the chaos and a large amount of disorder. It also confirms that the estimated degrees of freedom of motion increases and the estimated amount of disorder decreases with an increase of proficiency. In addision, this paper shows that the agreement between the neural control simulation and the experimental results of neural control for the degrees of freedom of motion and the entropy ratio is particularly good when the simulated wave form and the measured wave form are similar in appearance.

Key words: Human's Dexterity, Skill, Proficiency, Identification, Complex System, Chaos, Time Series Data, Neural Controller, Inverted Pendulum, Degrees of Freedom of Motion, Amount of Disorder

1. Introduction

Motion is a phenomenon that clearly indicates the living nature of a creature. Motion can be observed as a change in physical status. However, it is difficult to clarify how motion is acquired. There is an infinite variety of motions ranging from our daily activities to the exceptional movement of an athlete or a musician(1).

Based on his extensive observation of child growth, Gesell (1945) stated some empirical rules. In particular, he noted that the development of motion progresses from a generally integrated state to an individualized state in which individual sections have specialized functions. He also noted that the number of degrees of freedom of the motion increase with development, and that periods of unstabilization and stabilization are repeated to advance development by taking advantage of such fluctuations well. Finally, he observed

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that chaos plays a very important role in motion[1].

The human process of learning motion can also be studied by focusing on the degrees of freedom. When a person who normally writes with his or her right hand (i.e., their dominant hand) is asked to write with the left hand (i.e., their non dominant hand), the number of degrees of freedom of each joint is initially fixed; but, after training, each joint moves according to a peculiar phase relationship after training (Newell & Van Emmerik, 1989). This implies that we are rigid when we attempt a new motion, but become more relaxed after getting accustomed to it. The term "relax" in sports refers to a freeing of the degrees of freedom. To achieve a new motion, it is considered necessary to initially stiffen the body or to freeze the degrees of freedom[1].

Machinery and human beings are absolutely of a different nature at the present stage, but most research work on man-machine systems has dealt with the linear characteristics of human behavior[2]. As an example, many studies on control systems for stabilizing the inverted pendulum as an inherently unstable system have been presented. These studies focus on the linear characteristics of human behavior. There seem to be few studies and a number of unknowns regarding both the nonlinear characteristics of human behavior in an inherently unstable man-machine system as well as the learning process of the human operator with difficult control objects[3]-[5].

In order to stabilize an unstable system such as the inverted pendulum, strict judgment of the situation is required. Accordingly, it can be expected that the human operators exhibit a complex behavior intermittently. In the author's previous papers[6]-[14], it was found that there are various nonlinear features in the stabilizing behavior of a human operator. In this study, the definition of nonlinear stability means that an inverted pendulum does not fall for 60 consecutive seconds.

The behavior during stabilizing control of an inverted pendulum by a human operator exhibits a random-like or limit-cycle like fluctuation, and the stabilizing control by the human operator is robust against the disturbance. This may be because the limit-cycle-like fluctuation with a digital computer control, which means lineally unstable, is more robust against the disturbance than the lineally stable fluctuation investigated in previous experiments[6], [14]. Furthermore the limit cycle is very stable in the sense of nonlinearity, which means it is very strong against the disturbance, because the trajectories near the limit cycle spiral onto it from its inside and outside[15]-[17].

This paper shows that the estimated degrees of freedom of motion increases and the estimated amount of disorder decreases with an increase of proficiency. It also shows that the agreement between simulated and experimental values for the degrees of freedom of motion and the entropy ratio is particularly good when the simulated wave form and the measured wave form are similar in appearance.

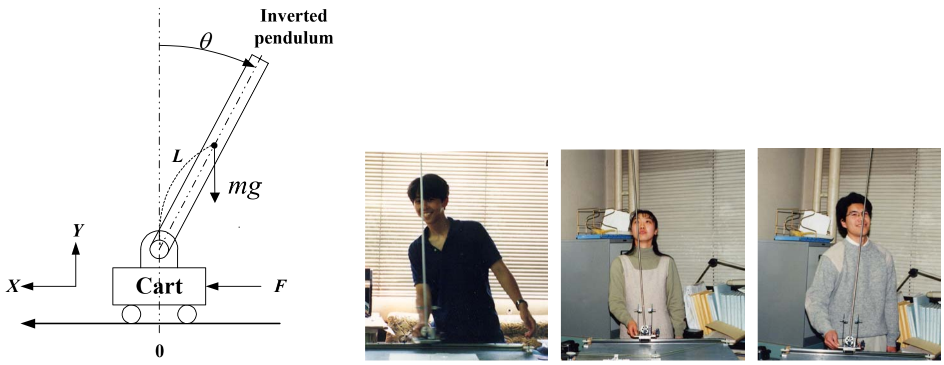
The entropy is estimated from the time series data as a measure of the amount of disorder in a system, and the degrees of freedom of the motion are estimated by the dimensions when curves of the largest Lyapunov exponents are saturated against the embedding dimensions for quantifying the proficiency.

2. Nonlinear Behavior of a Human Operator during Stabilizing Control of an Inverted Pendulum on a Cart

Figure 1 shows the experimental situation. The inverted pendulum is mounted on a cart which can move along the line of a sliding rail of limited length while it is hinged to the cart so that it rotates in one plane. A human operator manipulates the cart directly by hand. Although it takes some time and is needed intensive training for a human operator to succeed in stabilizing the pendulum for 60 seconds, it becomes less difficult after the first success of stabilizing.

Human operators in an experiment were trained so that they were skilled to some extent in stabilizing the pendulum by training, and the data of ten trials per person were successively taken for an analysis. The angle that the pendulum makes with the vertical axis and the displacement of the cart were measured, from which the derivatives and the force that moves the cart can be derived.

Figure 2 shows phase plane representations of the behavior of the inverted pendulum on a cart during stabilizing control by a human operator, identified as NK during the first trial after training.



(a) (b)

Fig.1 Stabilizing control of an inverted pendulum.

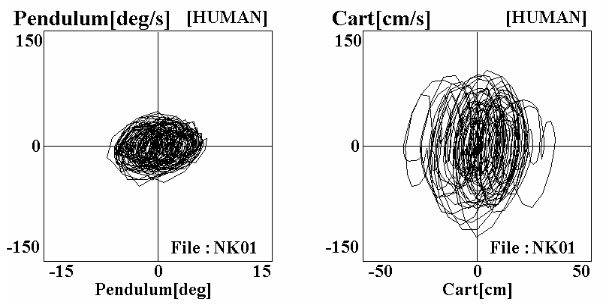


Fig.2 Behavior of an inverted pendulum in a phase plane.

3. Chaos-Entropy Analysis of Time Series Data during Stabilizing Control of an Inverted Pendulum by a Human Operator

In order to stabilize an unstable system such as the inverted pendulum, strict judgment of the situation is required. Accordingly, it can be expected that a human operator exhibits a complex behavior intermittently.

Consider a hypothetical statistical system for which the outcome of a certain measurement must be located on a unit interval. If a line is subdivided into N subintervals, we can associate a probability p_i with the i- th subinterval containing a particular range of possible outcomes. The entropy of the system is then defined as

$$S = -\sum_{i=1}^{Nc} p_i \log p_i \qquad (1)$$

This quantity may be interpreted as a measure of the amount of disorder in the system or as the information necessary to specify the state of the system. If the subintervals are equally probable so that $p_i = 1/N$ for all i, then the entropy reduces to $S = \log_e N$, which can be shown to be the maximum value. Conversely, if the outcome is known to be in a particular subinterval, then $S = 0$ is the minimum value. When $S = \log_e N$, the amount of further information needed to specify the result of a measurement is at a maximum, and when $S = 0$ no further information is required[(18), (19)]. We applied this formulation to the time series data by establishing N bins or subintervals of the unit interval into which the value of time series data may fall. We define S as the net entropy calculated with Eq.(1) and $S/(\log_e N)$ as the entropy ratio.

It is necessary to analyze time series data for detecting chaotic dynamics and characterizing it quantitatively when the model of a whole system is unknown. Methods for dynamical analysis of time series data are still developing, but a common method is a two-step process: (1) reconstruction of the strange attracter of the unknown dynamical system from the time series, and (2) determination of certain invariant quantities of the system from the reconstructed attracter. It is possible to obtain the dynamics from a single time series without reference to other physical variables(18). This concept was given a rigorous mathematical basis by Takens(20) and Mane (21).

Since the attracter dimension is unknown for time series data and the required embedding dimension M is unknown, it is important that the reconstruction be embedded in a space of sufficiently large dimension to represent the dynamics completely. Thus, the dimension of the embedding space is increased one by one; the attractor is reconstructed and its largest Lyapunov exponent is calculated. The process is continued until the largest Lyapunov exponent saturates against the embedding dimensions and a dimension, that is, the degrees of freedom of the system behavior, is estimated. The largest Lyapunov exponent can be obtained from a time series data using an algorithm given by Wolf *et al.*(22). The Lyapunov exponent can be used to obtain the measure of the sensitivity upon initial conditions. This measure of sensitivity is characteristic of chaotic behavior. If the Lyapunov exponent is positive, nearby trajectories diverge, and so the evolution is sensitive to initial conditions and therefore chaotic.

Consider the time series data $x(t_1)$, $x(t_2)$, - - - - -. Successive points in the phase space formed from time-delay coordinates can be written as vectors $\boldsymbol{X_i}$:

$$\begin{aligned}
\mathbf{X}_1 &= (x(t_1), x(t_{1+\tau}), \cdots, x(t_{1+(m-1)\tau})) \\
\mathbf{X}_2 &= (x(t_2), x(t_{2+\tau}), \cdots, x(t_{2+(m-1)\tau})) \\
&\vdots \\
\mathbf{X}_i &= (x(t_i), x(t_{i+\tau}), \cdots, x(t_{i+(\mathrm{m}-1)\tau}))
\end{aligned} \tag{2}$$

where the symbol τ denotes the time delay and the symbol m denotes the embedding dimension.

The choice of an appropriate delay τ is important to the success of the reconstruction. If τ is too short then the coordinates are almost equal to each other, and the reconstruction is useless. If τ is too large then the coordinates are so far apart as to be uncorrelated. If the system has some rough periodicity, then a value comparable to but somewhat less than that period is typically chosen. Because there is no simple rule for choosing τ in all cases, some times τ is adjusted until the results seem satisfactory. The time τ is typically some multiple of the spacing between the time series points(18). We chose seven times the spacing between the time series points, that is, 7 x 0.0293 [s], as the value of τ because the calculated largest Lyapunov exponents were not too sensitive to τ, and because the curves of the largest Lyapunov exponents against embedding dimensions were smooth within a reasonable range τ, whereas the dominant period of the experimental time series data was 0.5 ～1.0 [s].

Because the time series is presumed (by hypothesis) to be the results of a deterministic process, each x_{n+1} is the result of a mapping. That is

$$x_{n+1} = f(x_n) \tag{3}$$

The differentiation of the above equation is approximated as

$$\frac{df(x_j)}{dx_j} = \frac{dx_{j+1}}{dx_j} = \frac{x_{j+1} - x_j}{x_j - x_{j-1}} = f'(x_j) \quad (4)$$

Thus, the general expression of the Jacobian matrices and the orthogonal vectors $\boldsymbol{b}_{ij}(i = 1,2,\cdots,m)$ can be obtained(13). The Lyapunov exponents λ_i against each embedding dimension i are then obtained as

$$\lambda_i = \frac{1}{t_n - t_0}\sum_{j=1}^{n-1}\log_e \boldsymbol{b}_{ij} \quad (i = 1,2,3,\cdots,m) \quad (5)$$

Figure 3 shows the convergence of the largest Lyapunov exponent calculated from the experimental time series.

4. Generation of a Neural Controller from Time Series Data during Stabilizing Control of an Inverted Pendulum by a Human Operator

Figure 4 shows the multilayer feed-forward neural network for identification of individual skill from the time series data of stabilizing control of an inverted pendulum by a human operator. There, the circles represent the neurons (weights, bias, and activation function) and the lines represent the connections between the inputs and neurons, and between the neurons in one layer and those in the next layer. This is a three-layer perceptron with three stages of neural processing between the inputs and outputs. More layers can be added by concatenating additional hidden layers of neurons.

For the pendulum, we assigned angle θ_t, angular velocity θ'_t, angular acceleration θ''_t, and for the cart, we assigned displacement X_t, velocity X'_t, acceleration X''_t and force F_{t-1} that moves the cart as input variables, and the force F_{t+1} that moves the cart as output of the neural controller, in order to identify the nonlinear characteristics of the human operator from the experimental time series data. We choose the sigmoid function as an activation function and gave various values to the parameters affecting the slope of the sigmoid function.

Figure 5 shows the neural network used as an identification system for a human operator using a neural network. A very important part of neural net work models is the algorithms that allow the weight matrix W to change. The leaning algorithms determine the weight W in such a way that the network produces certain output values when certain input values are given.

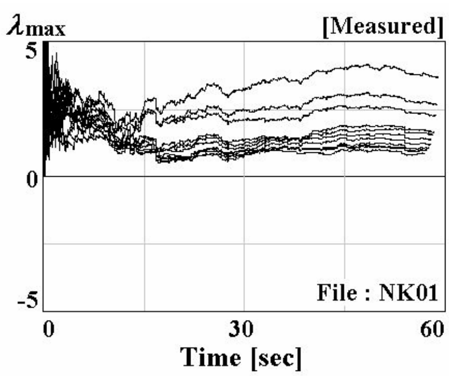


Fig.3 Convergence of the largest Lyapunov exponent calculated from the experimental time series.

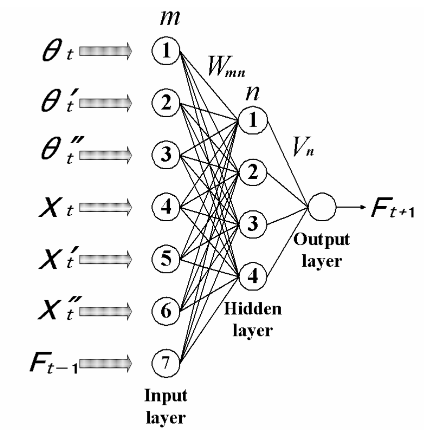


Fig.4 Multilayer feedforward neural network for identification

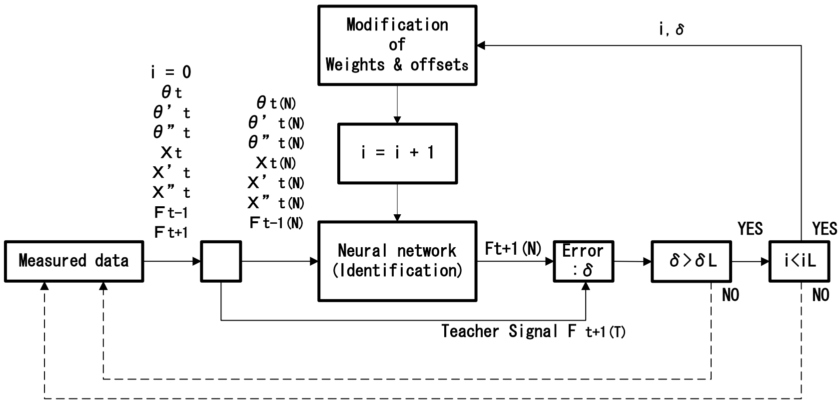


Fig.5 Identification system using a neural network for a human operator using a neural network.

The network should also react in a reasonable way, when new, unknown input values are presented. To achieve this goal the provided known input patterns are propagated through the network. The determined outputs are then compared to the target output patterns. Then the weight matrix W is changed in a way that the network comes closer to the target output, when the same input pattern is propagated again. To express whether two inputs or outputs are similar or close to each other we define a similarity or error measure. Here, learning means finding acceptable values for the weights by an iterative procedure which processes the measured time series data. The procedure is guided by an error measure that rates the current degree of performance. We used backpropagation, which indicates the backward propagation of an error signal through the network. After propagating a pattern through the network, the output pattern is compared with a given target pattern and the error of the output unit is calculated. This error is propagated backwards, that is, in the direction of the input layer through the net work. By using the error signal the hidden units are able to determine their own error. Finally, the errors of the units are used to modify the weights. The determination of error values for hidden units is an important achievement of this learning procedure.

Figure 6 shows a block diagram of the stabilizing control simulation with a neural controller identified from the stabilizing behavior of a human operator. The sampling time for control is 0.06 [s] and the initial pendulum angle is 3.0 [deg].

Figure 7 shows the simulated waveform using the acquired neural controller compared to the measured waveform of human operator NK's third trial (NK03) as an example.

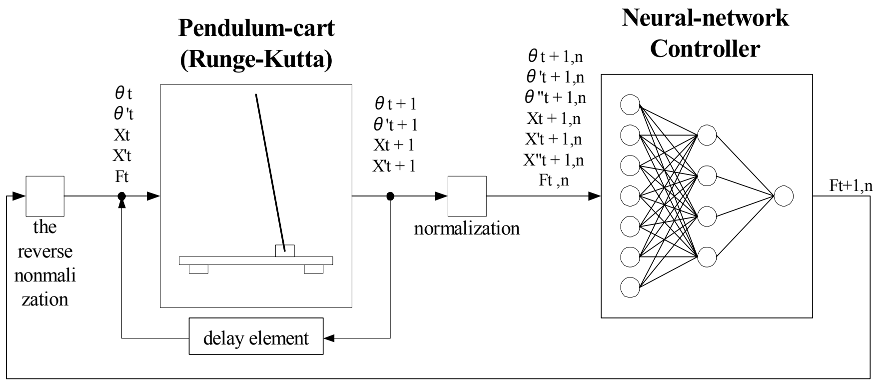
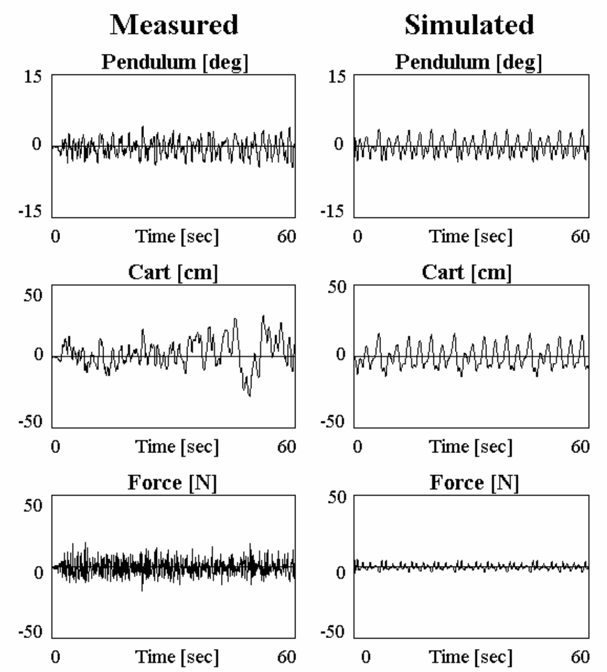


Fig.6 Block diagram of stabilizing control simulation with neural controller identified from human operator.



(a) Measured (b) Simulated

Fig.7 Simulated waveform using acquired neural controller vs. measured waveform of human operator NK's third trial (NK03), where u_0=2.0, α=12.0, β=3.0.

5. Diagnosis of Proficiency of Stabilizing Control of an Inverted Pendulum by a Human Operator using Chaos-Entropy Analysis

Figure 8 shows an example of the estimated ratio of entropy to maximum entropy, saturated as the number of partitioned cells increases. The entropy can be interpreted as a measure of the amount of disorder in the system and the maximum entropy as a random process with a uniform probability. Figure 9 shows the entropy ratios vs. trial number of human operator NK. It is recognized that the simulated time series data have a large amount of disorder according to the result of the estimated entropy ratio.

Figure 10 shows the maximum Lyapunov exponents against the embedded dimension, converging as the dimension increases. Figure 11 shows the estimated dimension (degrees of freedom) of motions vs. the trials of operator NK. The estimated degrees of freedom of motion increases with the increase of proficiency.

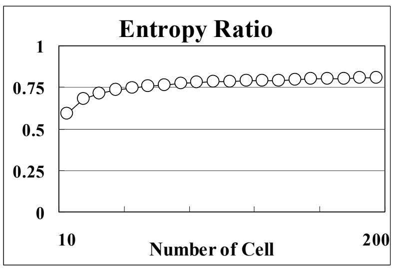


Fig.8 Example of estimated Ratio of Entropy to maximum entropy, saturated as the number of cells increases.

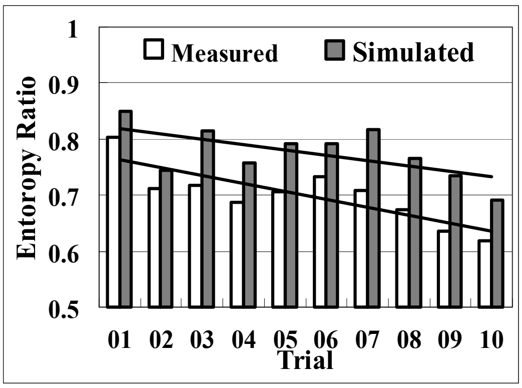


Fig.9 Entropy Ratios vs. trials of human operator NK.

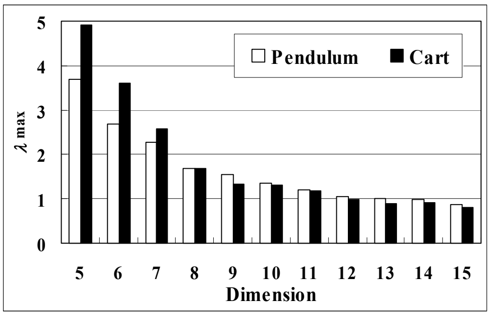


Fig.10 Maximum Lyapunov Exponents against embedded dimension, converging as dimension increases.

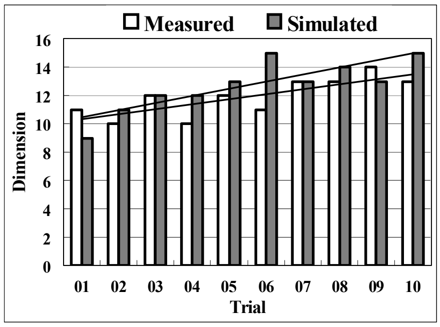


Fig.11 Estimated dimension (degrees of freedom) of motions vs. trial numbers of operator NK.

6. Remarks on Identification of Proficiency Using Neural Controller

Figure 12 shows the waveforms of inverted pendulum angles during stabilizing control by human operator FT from this subject's first trial (FT01) to tenth trial (FT10) (Measured), indicating the process of proficiency. Figures 13 and 14 show the estimated entropy ratios and the estimated dimension (degrees of freedom) of motion against the trial numbers of human operator FT, respectively. Each stabilizing behavior of human operator FT in Fig.12 seems to exhibit the proficiency except the first trial. The neural control simulation of the first trial failed to stabilize the inverted pendulum for 60 seconds as shown in Figs.13 and 14. Furthermore, the entropy ratio is rather large and the degrees of freedom are rather small in the first trial. We can see that the neural controller identifies the skilled up behavior better than the behavior in the initial stage of dexterity, contrary to the fuzzy controller which identifies the initial stage better than the skilled up behavior. A comparison between the neural identification and the fuzzy identification will be separately reported in the near future.

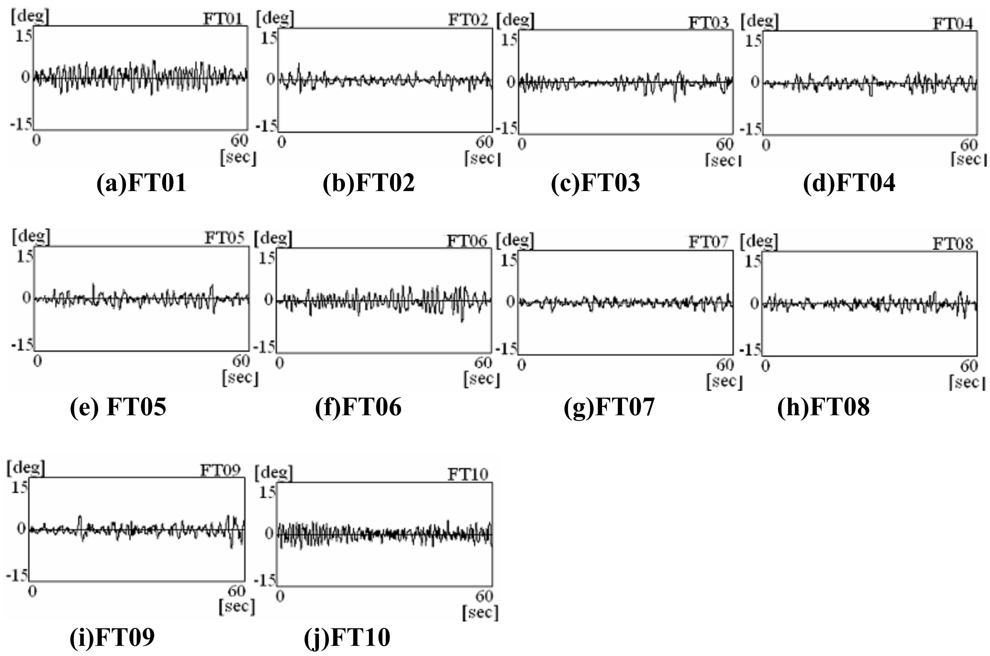


Fig.12 Proficiency of operator FT from first (FT01) to tenth trial (FT10) (Measured).

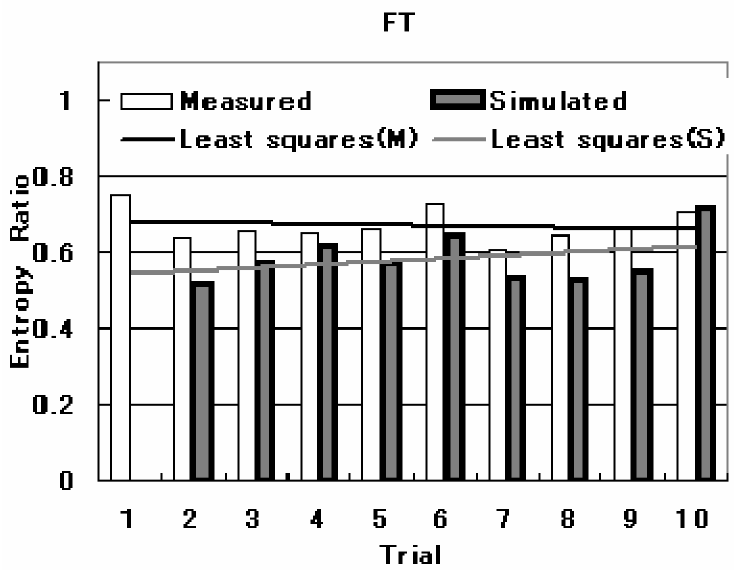


Fig.13 Entropy ratios vs. trial numbers of operator FT.

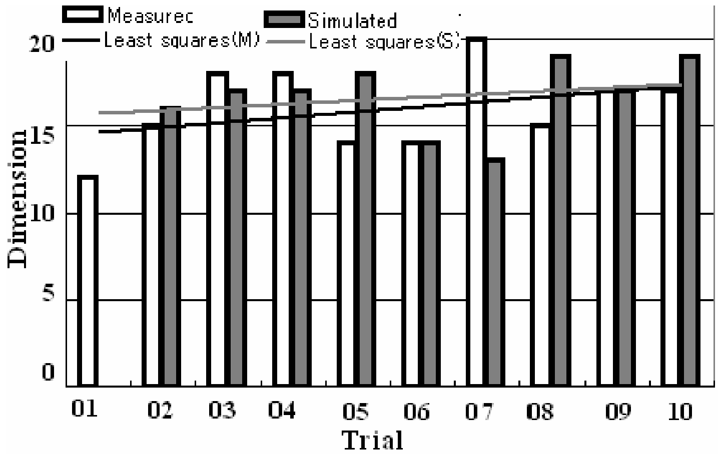


Fig.14 Estimated dimension (degree of freedom) of motions vs. trial numbers of operator FT.

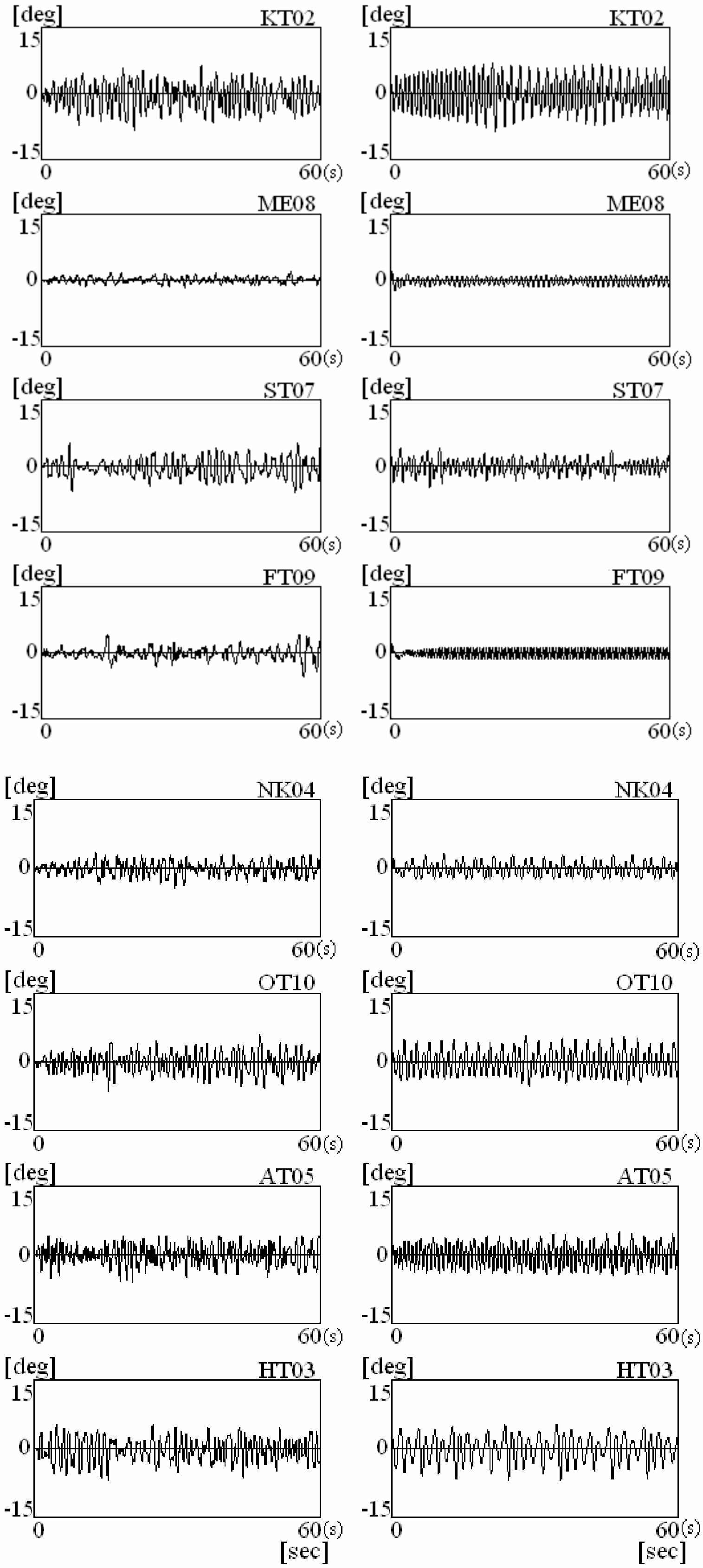
7. Acquisition of Amount of Disorder and Degrees of Freedom of Motion during Human Operator's Trial by Using the Neural Controller

Figure 15 shows an example of the simulated wave forms and the measured wave forms of pendulum angles resembling each other in appearance during stabilizing control by eight human operators.

Figure 16 shows a comparison of entropy ratios during stabilizing control by the eight human operators between the experiments and the neural control simulations resembling each other in waveform appearance of wave forms.

Figure 17 shows a comparison of the estimated dimension (degrees of freedom) during stabilizing control by the eight human operators between the experiments and the neural control simulations resembling each other in waveform appearance .

It is seen that the agreement between the simulated and experimental values for the degrees of freedom of motion and the entropy ratios is particularly good when the simulated wave form and the measured wave form resemble each other in appearance.



(a) Measured (b) Simulated

Fig.15 Simulated and measured wave forms of pendulum angles similar in appearance during stabilizing control by eight human operators.

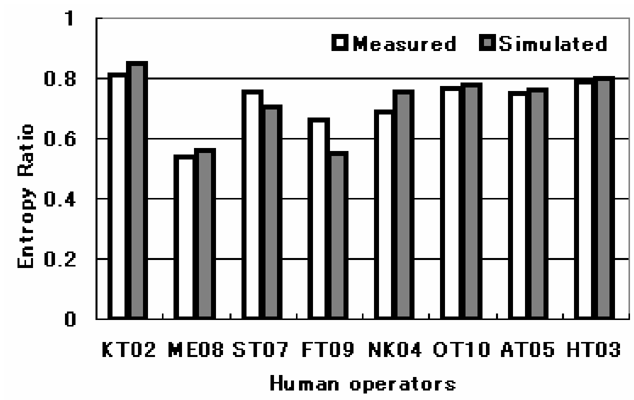


Fig.16 Comparison of entropy ratios during stabilizing control by eight human operators between the experiments and the neural control simulations for waveforms similar in appearance.

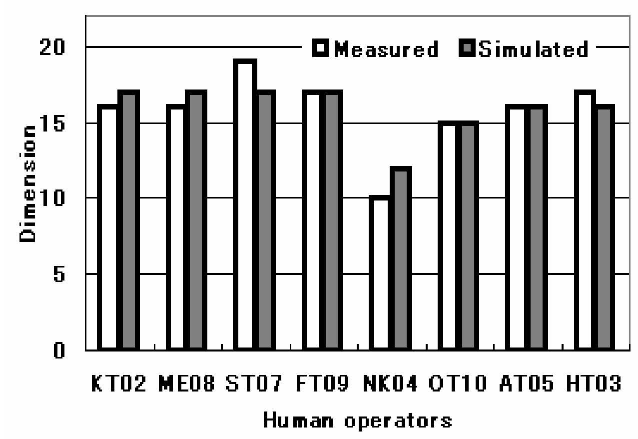


Fig.17 Comparison of estimated dimension (degrees of freedom) during stabilizing control by eight human operators between the experiments and the neural control simulations for waveforms similar in appearance.

8. Consideration of the Dexterity and the Degrees of Freedom

The enormously excessive degrees of freedom seem to give us considerable advantages. In many cases a more flexible instrument, which is certainly much more challenging to work with, has undeniable advantages for giving better results. An experienced master will always prefers an instrument with more degrees of freedom than an instrument that is easier to use but also constrains the worker. For example, a bicycle is harder to control than a tricycle, but anyone who has mastered a bicycle will probably never want to ride a tricycle again. The bicycle is preferred, because, in the hands of an experienced rider, it is more flexible and maneuverable, and, becomes more stable than the tricycle. Likewise, light weight children's skates with their wide blades are less flexible and maneuverable than sharp-bladed speed skates. Nature follows the same route, avoiding frets and props in the movement apparatus and generously scattering degrees of freedom. The practical problem of gaining dexterity is at the early stages. This fascinating and extremely important area can move us closer to the deepest, concealed caches of knowledge about the human brain and its functioning(23).

It seems that the emergence of intelligence in an autonomous robot exists in the dexterity of human or creatures as complex systems and research and development along this approach are necessary for realization of an intellectual robot.

9. Conclusions

The behavior during stabilizing control of an inverted pendulum by a human operators exhibited the random-like or limit-cycle like fluctuation and the the stabilizing control was robust against the disturbance. The limit-cycle like fluctuation with digital computer control, which is lineally unstable, was more robust against the disturbance than was the lineally stable fluctuation in the experiments performed in this study. In addition, the limit cycle was very stable nonlinearly, which means it was very robust against the disturbance, because trajectories near the limit cycle spiraled onto it from its inside and outside.

This paper shows that the neural network controller identified from time series data of each trial of each human operator clearly exhibited well the human-generated decision-making characteristics with chaos and a large amount of disorder. It also shows that the estimated degrees of freedom of motion increases and the estimated amount of disorder decreases with an increase of proficiency. In addition, this paper shows that the agreement between the simulation and experimental results for degrees of freedom and entropy ratios of motion is particularly good when the simulated waveform and the measured waveform are similar in appearance. It was clarified that a simple neural controller can be very useful for identifying the individuality and proficiency of skills of a human operator when stabilizing an unstable system.

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