

**Dynamics and Equivalent Damping of the Fuel Control  
System of the Pneumatically Governed  
Compression-ignition Engine\***

by **Yoshihiko KAWAZOE\*\***

An equation describing the dynamic behavior of a fuel control system of the pneumatically governed engine is derived, and its equivalent mass, damping, and effective diaphragm area were revealed. Moreover, it is shown that a simplified equation using equivalent damping coefficient can be used at idling where low speed hunting occurs. From the results of experiments and calculations, the effects of a liaison pipe and a chamber of reduced pressure on damping were made clear. The equivalent damping coefficient of the fuel pump with a chamber 50cm<sup>3</sup> in volume equipped with a liaison pipe 8mm in diameter and 40cm long is twice that of the pump without chamber and pipe.

**Key Words :** Vibration, Compression-Ignition Engine, Fuel Control System, Pneumatic Governor, Damping

### 1. Introduction

A Bosch-type individual fuel injection pump with its control rack locked delivers an increased quantity of fuel in each injection with an increased engine speed except in high speed running; thus the engine torque also increases with an increasing engine speed, so that the equilibrium of idling speed is statically unstable. To get rid of this drawback, a governor is additionally provided. However, in a pneumatically governed compression-ignition engine the idling engine speed is hardly kept constant, and consequently a low frequency noise of its own is generated. This fluctuation of the engine speed is called low speed hunting (1)~(5).

Many studies on the governing of engines have been carried out for a long time. As for hunting in actual systems, however, there have been few studies; further, these studies do not give a sufficient explanation for these phenomena. Welbourn et al. (6) investigated the behaviors of closed loops on the basis of the frequency characteristics of both mechanical governor and hydraulic governor. A remarkable feature of their work is consideration of phase lag of each element, and they point out that there exists a phase lag between the control rack displacement and the developed torque. However the calculated results do not agree with the experimental ones of the actual system, due to a non-viscous damping of the governor according to their reasoning. Fujihira (4) and Ishimaru (5) have measured characteristic values of a 6-cylinder engine, a fuel pump and a pneumatic governor, particularly the value

of damping of the governor and they have found out the hunting speed range on the basis of the stability criterion of a small oscillation. Nevertheless, there yet remain questions about the measuring method of damping coefficient of the pneumatic governor system in their works. In addition, the conventional linear theory of the engine-governor loop gives a hunting frequency estimation different from that of the actual system. They measured the damping from the damped oscillatory motion of the fuel control rack when the rack was set free at a manually displaced position, while the diaphragm chamber was given a reduced pressure with a vacuum pump. But the motion of the fuel control rack of an actual system at hunting comprises a component of the hunting frequency and a component of a higher frequency caused by the suction process of each piston. This higher frequency is usually much higher than the natural frequency of the governor system. The damping characteristic when the control rack fluctuates with a higher frequency is supposed to be different from that without it, accordingly the true value cannot be measured near the natural frequency. The same method of measurement as above has been used for mechanical governors (6), or an attempt has been made to obtain the time constant of the first-order from the initial response test because of the difficulty of measuring the damping coefficient (7). Low speed hunting is a problem of control of a closed loop composed of the crank shaft system, governor, fuel injection pump and combustion torque. In order to investigate the hunting phenomena, it is necessary to assess the dynamic characteristics of each element of the loop. The elements of equation of motion of the fuel control system of the pneumatically governed engine, particularly the damping, equivalent mass and effective diaphragm area

\* Received 30th September, 1983.

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should be estimated.

In the present report, the author measures the damping of the fuel control system with its fuel control rack given an excitation by the reduced pressure caused by the suction stroke of each piston as well as an excitation by a shaker, and he shows that the results from the above two methods are in good agreement. Further, measurements of the damping coefficient without a liaison pipe and a reduced pressure chamber have been made using the frequency response test with a shaker, and this value is put into the equation of motion of the fuel control system, and the results are compared with the experiments. A Bosch-type individual fuel injection pump having plungers 6.5mm in diameter, cam lift 8mm high, and a diaphragm 60mm in its outer diameter is used.

## 2. The Fuel Control System of the Pneumatically Governed Engine

As illustrated in Fig.1, a pneumatically governed engine controls the fuel delivery by displacing the fuel control rack with a reduced pressure taken at a narrow passage called subventuri beside a throttle valve, whose opening determines the mean engine speed. Reduced pressure caused by an increased engine speed is applied to displace the control rack in the direction of decreasing fuel delivery through a diaphragm combined with a spring. The motion of the fuel control rack when hunting occurs comprises a component of the hunting frequency and a component of a higher frequency caused by the suction process of each piston, and fuel is sprayed during the injection period determined by the cam angle of the fuel injection pump. A series of connected elements: the subventuri pressure, the liaison pipe, the reduced pressure chamber and the displacement of the fuel

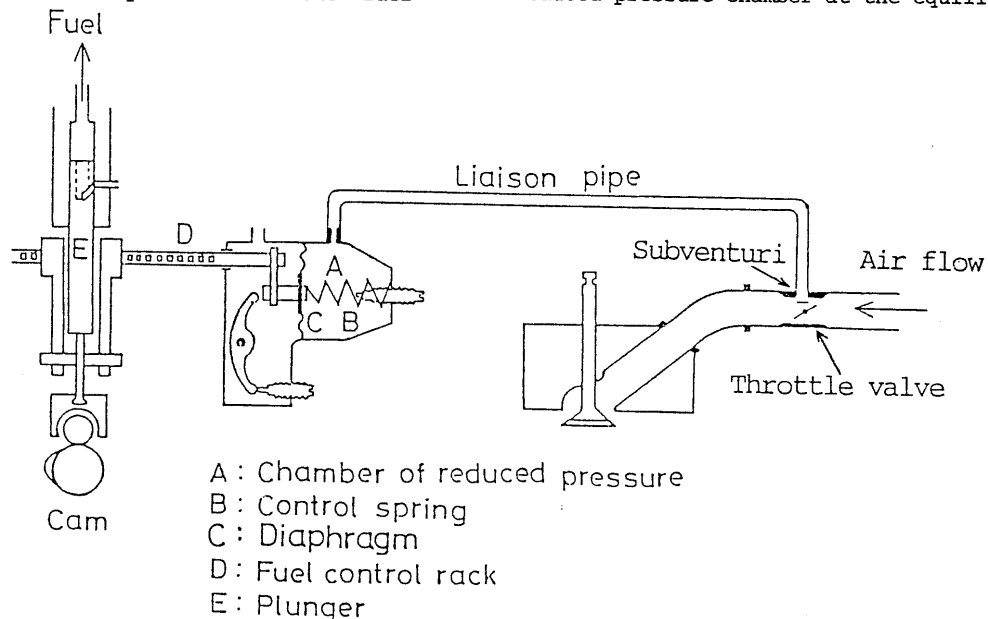


Fig.1 Fuel control mechanism of a pneumatically governed engine.

control rack is called a fuel control system.

## 3. Equation of Motion of the Fuel Control System (Theoretical Analysis)

Consider a one-dimensional flow in a liaison pipe. Let  $t$ ,  $x$  and  $A$ , denote the time, the coordinate in the flow direction, and the constant cross sectional area of the liaison pipe respectively;  $\rho_0$  and  $u_0$  denote the density and velocity of air at an equilibrium state respectively, and  $\rho$  and  $u$  denote the variations from  $\rho_0$  and  $u_0$ . Then the equation of continuity is given by the expression

$$\frac{\partial \rho(x, t)}{\partial t} + \rho_0 \frac{\partial u(x, t)}{\partial x} = 0 \quad (1)$$

on the assumption that  $u_0 = 0$ , where the higher terms above the second can be omitted.

The equation of motion for a liaison pipe flow can be expressed as

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = - \frac{\partial p}{\partial x} - Ru \quad (2)$$

where  $p$  and  $R$  are the pressure and the pressure drop per unit length and unit velocity respectively. For an incompressible fluid the integration of Eq.(2) can be written as

$$\rho_0 \cdot l_e \frac{du}{dt} + R \cdot l_e \cdot u = p_v - p_d \quad (3)$$

where  $p_v$  is the subventuri pressure,  $p_d$  the pressure in the diaphragm chamber, and  $l_e$  the length of the liaison pipe.

Let  $V_0$  and  $V$  denote the volume of the reduced pressure chamber at the equilibri-

um state and at any time, and the higher terms above the second order can be omitted. Then the boundary condition with respect to the connection between a liaison pipe and a diaphragm chamber is given by the expression

$$\rho_0 \frac{dV}{dt} + V_0 \frac{d\rho}{dt} = C_s \cdot A_p \cdot \rho_0 \cdot u \quad (4)$$

where  $C_s$  is the flow coefficient of the pipe joint. If the velocity in the diaphragm chamber can be neglected and the pressure changes adiabatically, Eq.(4) becomes

$$\rho_0 \cdot A_d \frac{dx_d}{dt} + V_0 \frac{\rho_0}{n P_0} \frac{dp_d}{dt} = C_s \cdot A_p \cdot \rho_0 \cdot u \quad (5)$$

where  $P_0$ ,  $n$ ,  $A_d$  and  $x_d$  are the equilibrium pressure, the specific-heat ratio, the effective area of a diaphragm, and the control rack displacement in the direction of increasing fuel delivery.

The equation of motion of the fuel control rack can be expressed as

$$m_e \frac{d^2 x_d}{dt^2} + C_{e0} \frac{dx_d}{dt} + k \cdot x_d = A_d \cdot p_d \quad (6)$$

where  $m_e$  is the equivalent mass of the fuel control rack system,  $k$  the control spring stiffness,  $C_{e0}$  the viscous damping coefficient without a liaison pipe and a diaphragm chamber (measured with a diaphragm chamber opened to atmosphere). Thus, the motion of the fuel control rack system can be expressed from Eqs.(1),(2), (5), and (6). If the motion of the fluid in the liaison pipe is considered incompressible, Eqs.(3),(5), and (6) reduce to the equations

$$\begin{aligned} & \frac{l_e V_0 \rho_0 m_e}{C_s A_p n P_0} \frac{d^4 x_d}{dt^4} + \frac{l_e V_0 \rho_0 C_{e0}}{C_s A_p n P_0} \frac{d^3 x_d}{dt^3} + \left( \frac{l_e A_d^2 \rho_0}{C_s A_p} + \frac{l_e V_0 \rho_0 k}{C_s A_p n P_0} + \frac{l_e V_0 R C_{e0}}{C_s A_p n P_0} + m_e \right) \frac{d^2 x_d}{dt^2} \\ & + \left( \frac{l_e A_d^2 R}{C_s A_p} + \frac{l_e V_0 R k}{C_s A_p n P_0} + C_{e0} \right) \frac{dx_d}{dt} + k x_d = A_d p_v \end{aligned} \quad (7)$$

From Eq.(7) the dimensionless amplitude and the phase lag of the control rack displacement relative to the sinusoidal fluctuation of the subventuri pressure with an angular frequency  $\omega$  are given by the expression

$$\left| \frac{x_d}{p_v} \right| \frac{k}{A_d} = 1 / \left[ \sqrt{ \left\{ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \right\}^2 + \left\{ 2 \frac{C_{e0} \omega}{C_c \omega_0} \right\}^2 } \sqrt{H_1^2 + H_2^2} \right] \quad (8)$$

$$\phi = \angle \left( \frac{x_d}{p_v} \right) = \tan^{-1} \left[ 2 \frac{C_{e0} \omega}{C_c \omega_0} \left\{ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \right\}^{-1} \right] + \tan^{-1} (H_2 / H_1) \quad (9)$$

where,

$$\begin{aligned} H_1 &= 1 + \frac{l_e}{C_s A_p} \left[ \left\{ \frac{n P_0 A_d^2}{k} \frac{1 - (\omega/\omega_0)^2}{\{1 - (\omega/\omega_0)^2\}^2 + \{2 C_{e0} \omega / (C_c \omega_0)\}^2} + V_0 \right\} \left\{ - \frac{\rho_0 \omega^2}{n P_0} \right\} \right. \\ & \quad \left. + \frac{2 C_{e0} \omega / (C_c \omega_0)}{\{1 - (\omega/\omega_0)^2\}^2 + \{2 C_{e0} \omega / (C_c \omega_0)\}^2} \frac{A_d^2 R \omega}{k} \right] \\ H_2 &= \frac{l_e}{C_s A_p} \left[ \frac{A_d^2 \rho_0 \omega^2}{k} \frac{2 C_{e0} \omega / (C_c \omega_0)}{\{1 - (\omega/\omega_0)^2\}^2 + \{2 C_{e0} \omega / (C_c \omega_0)\}^2} \right. \\ & \quad \left. + \frac{R \omega}{n P_0} \left\{ \frac{n P_0 A_d^2}{k} \frac{1 - (\omega/\omega_0)^2}{\{1 - (\omega/\omega_0)^2\}^2 + \{2 C_{e0} \omega / (C_c \omega_0)\}^2} + V_0 \right\} \right] \end{aligned}$$

$$\omega_0 = \sqrt{k/m_e}, \quad C_c = 2\sqrt{m_e k}$$

Furthermore, provided  $X$  and  $P$  denote the control rack displacement from the position for no fuel delivery and its corresponding subventuri pressure, the equation of motion of the control rack response to the subventuri pressure fluctuation becomes

$$m_e \frac{d^2 X}{dt^2} + C_e \frac{dX}{dt} + k(X + L_0 - L) = A_d P \quad (10)$$

where  $m_e$  is the equivalent mass of the moving parts of the fuel control rack system,  $C$ , the equivalent viscous damping coefficient,  $k$  the stiffness of the rack spring,  $L_0$  the length of the rack spring at  $X=0$ ,  $L$  the free length of the rack spring, and  $A_d$  is the effective diaphragm area.

#### 4. Equivalent Mass and Effective Diaphragm Area

The equivalent mass  $m_e$  of the fuel control rack system is composed of an equivalent mass of the moments of inertia of plungers, pinions and sleeves,  $I_{PL}$ ,  $I_{PN}$ , and  $I_S$  respectively and also of the mass  $m_R$  of control rack itself and  $m_D$  of moving parts of the diaphragm. Let  $r_p$  be the radius of pinion's pitch circle geared to the control rack,  $\theta$  the angular rotations of the pinion,  $x$  the displacement of control rack. Then the total amount of the kinetic energy of the plunger, pinion and sleeve is given as

$$1/2(I_{PL} + I_{PN} + I_S)\dot{\theta}^2 = 1/2m_{er}\dot{x}^2 \quad (11)$$

where  $m_{er}$  is the equivalent mass and  $\dot{x} = r_p \dot{\theta}$ . Hence the equivalent mass  $m_{er}$  is

$$m_{er} = (I_{PL} + I_{PN} + I_S) / r_p^2 \quad (12)$$

Thus, the equivalent mass of the fuel control rack system of a 4-cylinder fuel injection pump is given as

$$m_e = m_R + m_D + m_C + 4(I_{PL} + I_{PN} + I_S) / r_p^2 \quad (13)$$

where  $m_C$  is the mass of the core of the differential transformer for the measuring the rack displacement. The values of each mass and moment of inertia of the actual system are  $m_R = 117$  g,  $m_D = 48.0$  g,  $m_C = 2$  g,  $I_{PL} = 1.31$  g·cm<sup>2</sup>,  $I_{PN} = 12.32$  g·cm<sup>2</sup>, and  $I_S = 12.25$  g·cm<sup>2</sup>; thus, the equivalent mass  $m_e$  becomes 0.273 kg.

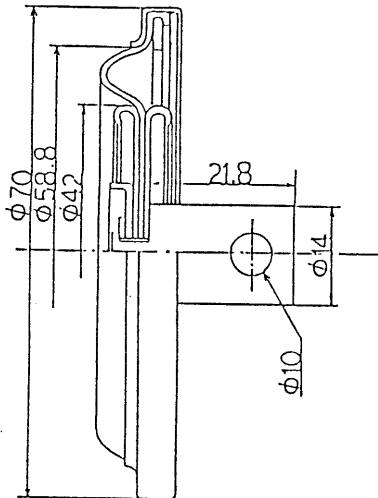


Fig.2 Diaphragm.

The value of  $A_d$  of the effective diaphragm area was obtained by an equilibrium between a dead weight force given to the end of the diaphragm assembly with its central axis vertically held, and the air force in the diaphragm chamber reduced by a vacuum pump; the diaphragm chamber pressure was measured with manometers by controlling the opening of a valve between the vacuum pump and the tank. The effective diaphragm area somewhat increases with an increasing fuel delivery, i.e. with an increasing diaphragm chamber volume. For the standard diaphragm shown in Fig.2, the value of  $A_d$  is from 20.2 to 21.4 cm<sup>2</sup>. The area of a circle having a diameter amounting to the mean value of 58.8 and 42 mm, which are outer and inner diameters of the diaphragm respectively, is 20.0 cm<sup>2</sup>; This value well agrees with the experimental result.

#### 5. Damping Coefficient

##### 5.1 Damping coefficient of the fuel control rack system without a liaison pipe and a reduced pressure chamber

A control spring retainer opened to atmosphere (Fig.3) has been prepared and attached to the fuel pump instead of the reduced pressure chamber to investigate the damping of the fuel control rack system without the liaison pipe and the chamber. The relationship between the force applied to the fuel control rack and its displacement has been measured with the control rack given a harmonic excitation by a shaker through the ring spring provided for measurement of the exciting force, where two strain gauges are attached to the ring spring, as illustrated in Fig.4. The ring springs' diameter and width are 110 mm and 30 mm respectively, and the first spring made of a plate 0.6 mm in thickness gives a measured stiffness amounting to 25.28 N/cm, while the second one of a plate 0.7 mm in thickness gives 46.65 N/cm, their natural frequencies being 57 and 67 Hz respectively. A

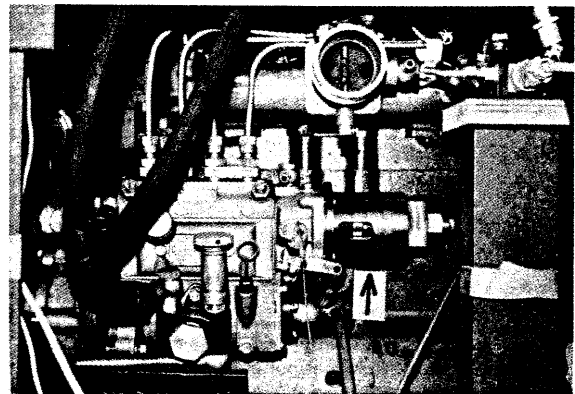


Fig.3 Control spring retainer attached to the fuel injection pump.

differential transformer was used to measure the rack displacement. The set force of the rack spring in the retainer was adjusted to be equal to that of the actual system, and the equilibrium position as well as the amplitude of the rack displacement was set almost the same as those in the engine idling.

In Fig.5 is shown a result of the frequency response tests, where  $\Delta F$  and  $\Delta X$  are the amplitudes of the external force and the rack displacement. The measured values scatter in the lower range of frequencies under 10 Hz corresponding to engine speed of 300 rpm; the damping characteristics are very complicated, and these cannot be explained even if the combined Coulomb and viscous friction is assumed. But, in the higher range of frequencies above 10 Hz which is the range of engine running, the value of the viscous damping coefficient  $C_{e0}$  of the fuel control rack system without both liaison pipe and reduced pressure chamber is close to 12.4 N·s/m, provided that the equation of this system is written as

$$m_e \frac{d^2 x_d}{dt^2} + C_{e0} \frac{dx_d}{dt} + kx_d = \Delta F \sin \omega t \quad (14)$$

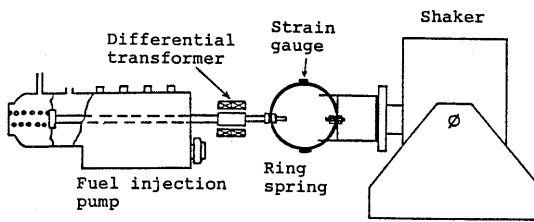


Fig.4 Schematic apparatus for measuring frequency response under shaker excitation.

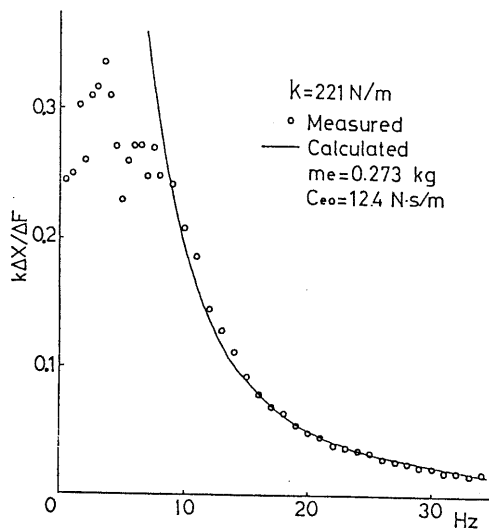


Fig.5 Frequency response of the fuel control system without liaison pipe and chamber of reduced pressure.

Investigators except the author tried to measure the damping characteristics in lower frequency range near the natural frequency, getting incorrect damping values as already stated in Section 1. With the engine tested, the hunting occurs in the range of mean engine speeds from 650 to 850 rpm and the hunting frequency is about 2 Hz. The above measured value may be used for  $C_{e0}$ , because the motion of the control rack comprises a component of higher frequency caused by the suction process of each piston.

### 5.2 Equivalent viscous damping

In order to investigate the frequency characteristics of the fuel control rack system, frequency response tests excited by the subventuri pressure on a motored engine have been carried out. In Fig.6 is

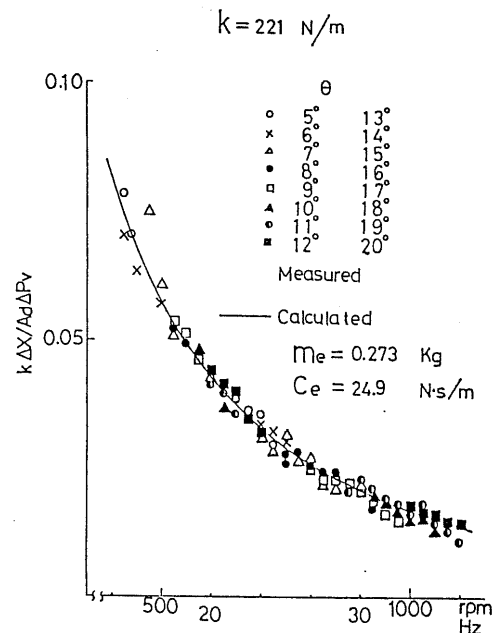


Fig.6 Amplitude of the control rack displacement relative to that of the subventuri pressure.

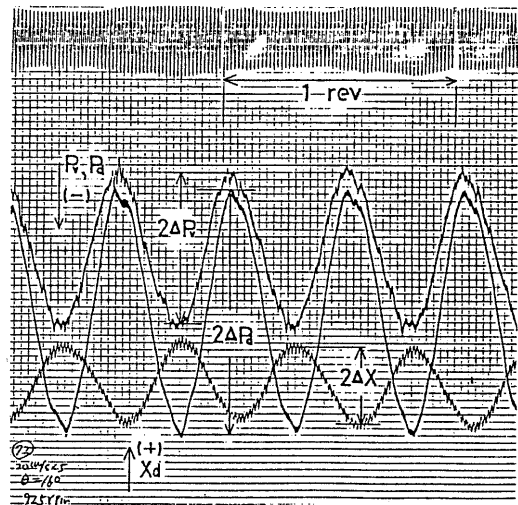


Fig.7 An example of recorded wave forms of frequency response tests excited by the subventuri pressure on a motored engine.

shown the amplitude  $\Delta X$  of the rack displacement relative to the subventuri pressure amplitude  $\Delta P_v$ . Throttle valve opening has no effects on the results. If the system behaves as the following equation consisting of equivalent mass  $m_e$ , equivalent damping coefficient  $C_e$  and rack spring stiffness  $k$ , then the value of  $C_e$  derived is close to 24.9 N.s/m in the case of  $k = 2.21$  N/cm when equipped with a liaison pipe 40 cm long and 0.8cm in diameter.

$$m_e \frac{d^2 x_d}{dt^2} + C_e \frac{dx_d}{dt} + kx_d = A_d \Delta P_v \sin \omega t \quad (15)$$

The ratio of  $C_e$  to  $C_{e0}$  is 2, and this discrepancy must be due to the damping of the system consisting of a liaison pipe and a chamber of reduced pressure. In Fig.7 is shown a recorded example of the frequency response test, where  $P_v$ ,  $P_d$  and  $X_d$  are subventuri pressure, pressure of diaphragm chamber and control rack displacement respectively. The phase of the rack displacement is opposite to that of the reduced pressure.

On the other hand, frequency response tests have been carried out with a shaker using the same method as that of Section 5.1, where the liaison pipe is detached from the pressure tap of subventuri. Measurements have been made both on a motored engine and a stopped engine to examine the effects of engine running on the damping of the control rack. As shown in Fig.8, engine revolution has no effect on the result in higher range of frequencies above 10 Hz; the equivalent damping coefficient  $C_e$  is close to 24.9 N.s/m, which is the same as that in the case of excitation by subventuri pressure.

The natural frequency of the fuel control rack system with a standard con-

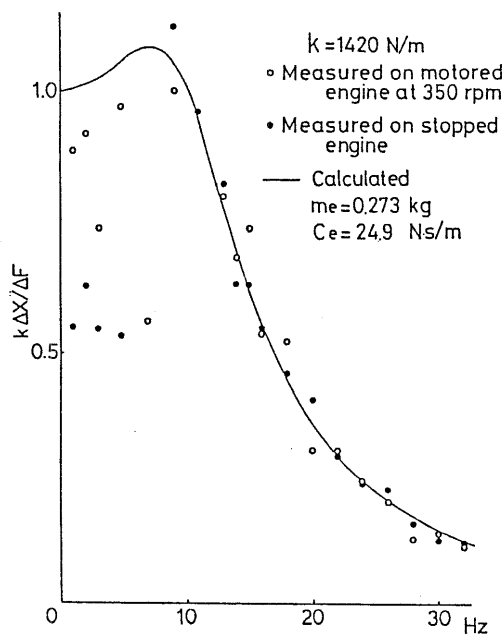


Fig.8 Frequency response of the fuel control system excited by a shaker.

trol spring ( $k = 221$  N/m) is lower than 10 Hz, and the recorded wave forms of exciting force and rack displacement at low frequency are distorted by large friction. It is therefore difficult to estimate the natural frequency accurately. However it seems to be close to the calculated value of 4.5 Hz judged from the phase variation with the frequency.

### 6. Calculated Results and Considerations

Figure 9 shows the amplitude  $\Delta X$  of rack displacement relative to  $\Delta P_v$  of subventuri pressure as a numerical result of Eq.(8), in the cases of liaison pipe 40 cm and 110cm long respectively, where the pressure drop  $R = 8\pi\mu/A_p$  ( $\mu$  is the coefficient of viscosity of air and  $A_p$  the cross sectional area of liaison pipe) and the following values are used,

$$m_e = 0.273 \text{ kg}, k = 221 \text{ N/m}, C_{e0} = 12.4 \text{ N}\cdot\text{s/m}, \\ A_p = 0.503 \text{ cm}^2, A_d = 21.0 \text{ cm}^2, V_0 = 50 \text{ cm}^3, \mu = 19 \\ \mu\text{Pa}\cdot\text{s}(40^\circ\text{C}), P_0 = 99.3 \text{ kPa}, \rho_0 = 1.11 \text{ kg/m}^3(40^\circ\text{C}), \\ C_s = 0.8, n = 1.40.$$

Both the calculated results (solid lines) agree well with the experimental ones. In lower range of engine speeds under 960 rpm (33 Hz) the amplitude ratio for  $l_e = 110$  cm is smaller i.e. damping is larger than that for  $l_e = 40$  cm, while the superiority is reversed above 960 rpm.

The natural frequencies of the system consisting of a liaison pipe and a diaphragm chamber were experimentally investigated in the cases of  $l_e = 110$  cm and 40 cm. The diaphragm and the chamber of reduced pressure were removed from the fuel injection pump, and the central portion of the diaphragm was excited directly by the shaker, as illustrated in

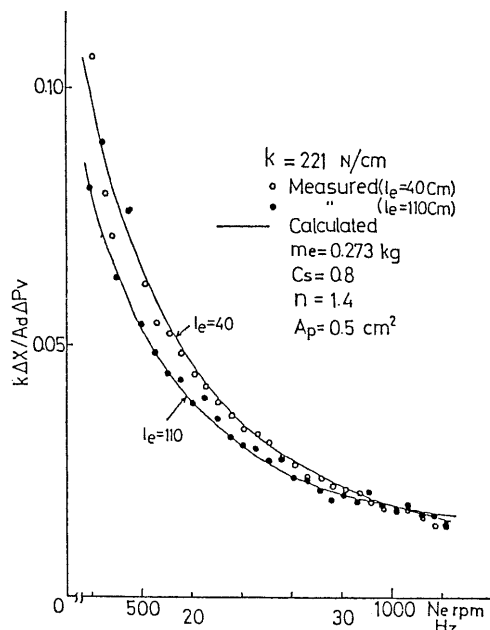


Fig.9 Amplitude of the control rack displacement relative to that of the subventuri pressure.

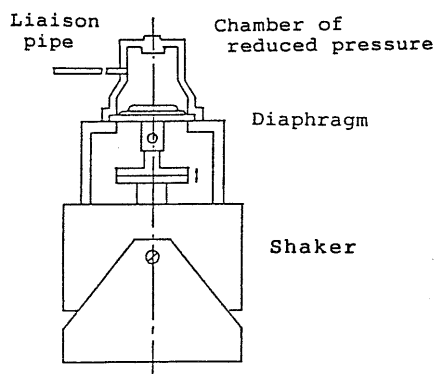


Fig.10 Apparatus for measuring natural frequencies of the system consisting of a liaison pipe and a chamber of reduced pressure.

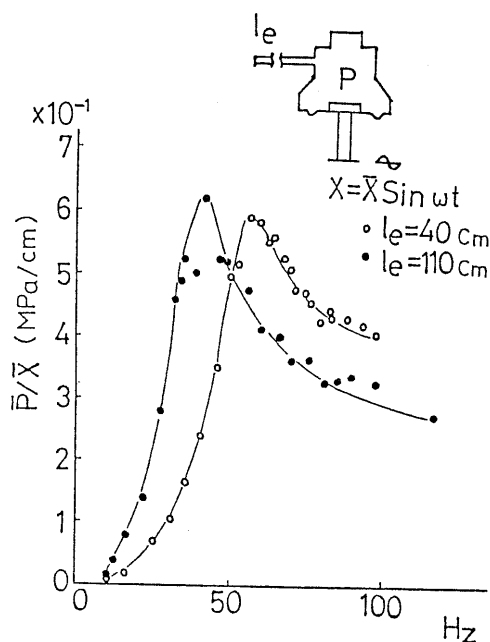


Fig.11 The amplitudes of the diaphragm chamber pressure relative to the sinusoidal variation of diaphragm displacement (measured).

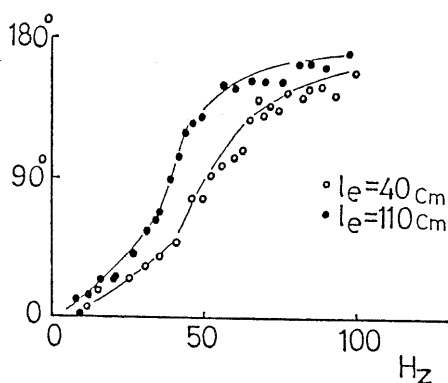


Fig.12 Phase lags of the diaphragm chamber pressure relative to the sinusoidal variation of diaphragm displacement (measured).

Fig.10. The amplitude  $\bar{P}$  of the pressure in the chamber relative to  $\bar{X}$  of the displacement of the diaphragm center and the phase variation are shown in Fig.11 and Fig.12 respectively. From these figures it is seen that the natural frequencies in the cases of  $l_e = 110$  cm and  $l_e = 40$  cm are 42 Hz and 55 Hz respectively.

At idling where the low speed hunting occurs, a simplified equation using the equivalent damping coefficient can be used. The damping increases with a decreasing cross sectional area  $A$ , and with an increasing length  $l_e$  of liaison pipe over lower range of engine speeds.

## 7. Conclusions

An equation describing the dynamic behaviors of the fuel control system of the pneumatically governed diesel engine was derived, and its equivalent mass, damping, and effective diaphragm area were revealed. Moreover, it was shown that a simplified equation using the equivalent damping coefficient can be used at idling where low speed hunting occurs. From the results of experiments and calculations, the effects of a liaison pipe and a chamber of reduced pressure on the damping of the fuel control system were made clear. For the fuel pump tested with a chamber  $50 \text{ cm}^3$  in volume equipped with a liaison pipe 8 mm in diameter and 40 cm long, the equivalent damping coefficient was twice that for a pump without chamber and pipe. In addition it was confirmed that the vibration of the fuel injection pump itself due to engine revolutions has no effects on damping.

## Acknowledgements

The author is grateful to Professor K.Tsuda of the Faculty of Engineering, Yokohama National University, Professor H.Sakai, Messers N.Koizumi, Y.Ohtake and T.Someya of the Faculty of Engineering, University of Tokyo for their suggestions and encouragements throughout this study.

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