23 Computer aided prediction of the vibration and rebound velocity characteristics of tennis rackets with various physical properties

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1 Introduction

The performance of the tennis racket in terms of the coefficient of restitution is closely related to impact phenomena. Nevertheless, there are a number of unclarified points regarding the impact phenomenon and the optimum design of tennis rackets (*Groppel*, 1987).

In this paper, the vibration of a racket-frame and the rebound velocity of a ball when it hits the strings of rackets with various physical properties, such as the frame stiffness, mass distribution and the string tension, are predicted using a simple impact model. The model is based on the idea that the contact duration, which has a strong influence on the racket-frame vibration and is not much affected by the frame stiffness, is determined by the natural period of a whole system composed of the mass of a ball, the non-linear stiffness of a ball and strings, and the reduced mass of racket at the impact point on the string face. The rebound velocity of the ball can be derived by considering the main sources of energy loss during impact, such as the instantaneous deformation of the ball and strings, rotation of the racket and vibration of racket-frame (*Kawazoe*, 1989, 1992, $a-e$).

2 Approximate Non-linear Impact Analysis and the Prediction of a Ball Rebound Velocity

The coefficient of restitution e_{BG}, when the racket head is firmly clamped in the collision between a ball and strings, is closely related to the energy loss due to the instantaneouse deformation of the ball and strings. If the vibration of the racket frame is neglected, the impulse could be approximately described as Eq.(1) using the mass m_B of a ball, the reduced mass M_r of a racket at the hitting point on the string face, and the ball velocity V_{Bo} before impact (*Kawazoe*, 1992a, e).

$$\int F(t)dt = V_{Bo}(1 + e_{BG})m_B/(1 + m_B/M_r) \ . \tag{1}$$

Assuming that the contact duration, which is not much affected by the frame stiffness accoding to the experiment, is determined by the natural period of a whole system composed of the mass of a ball, the nonlinear

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stiffness K_{GB} of a ball and strings, and the reduced mass of racket, the contact duration between the ball and the racket might be

$$T_c = \pi {m_B}^{1/2}/(K_{GB}(1 + m_B/M_r))^{1/2} . \tag{2}$$

The stiffness K_{GB}, however, has a strong non-linearity and its value changes during impact also depending on the impact velocity ($Kawazoe, 1992a$). In order to make the analysis simpler, the equivalent force F_{MEAN} can be introduced during impact time T_C ($\int F(t)dt = F_{MEAN}T_C$). Accordingly, the relationship between F_{MEAN} and corresponding K_{GB} is represented as Eq.(3) from Eq.(1) and Eq.(2).

$$F_{mean} = V_{Bo}(1 + e_{BG}){m_B}^{1/2}{K_{GB}}^{1/2}/(\pi(1 + m_B/M_r)^{1/2}) \tag{3}$$

On the other hand, each curve of restoring forces F_B, F_G and F_{GB} vs. the deformations of a ball, strings and the compound system of ball and strings can be determined respectively. This is so as to satisfy a number of experimental data using the least square method assuming that a ball with concentrated mass deforms only at the side in contact with the strings. Also, the curves of the corresponding stiffness K_G, K_B, and K_{GB} can be derived by differentiation of the restoring forces with respect to deformation ($Kawazoe, 1992a$). Thus, the relationship between F_{GB} and K_{GB} can be derived using the least square method as Eq.(4) by eliminating the deformation graphically from the above two relations.

$$F_{mean} = func.(K_{GB}) \tag{4}$$

From Eq.(3) and Eq.(4), the parameters K_{GB} and F_{MEAN} against the impact velocity V_{Bo} (when the pre-impact velocity of racket $V_{Ro} = 0$) can be derived. Also, the contact duration T_C can be determined against the impact velocity V_{Bo} from Eq.(2) ($Kawazoe, 1992a$).

The force-time curve of impact is approximated as a half-sine pulse shown on the left in Fig.11. Its mathematical expession is

$$F(t) = F_{max} sin(\pi t/T_c)(0 \leq t \leq T_c) \tag{5}$$

where

$$\int F(t)dt = F_{MEAN}T_C, \qquad F_{max} = \pi F_{mean}/2 \tag{6}$$

The fourier transform of Eq.(5) is shown on the right in Fig.11 and represented as Eq.(7), where f is the frequency ($Kawazoe, 1992a$).

$$S(f) = 2F_{max}T_c \mid cos(\pi f T_c) \mid /[\pi \mid 1 - (2fT_c)^2 \mid] \tag{7}$$

The amplitude of racket vibration due to impact can be simulated by applying the impact force to the experimentally identified racket vibration model ($Kawazoe, 1989, 1992c$). When the impact force $S_j(\omega_k)$ applies to the arbitrary point j on the string face, the amplitudes X_{ijk} at the arbitrary point i of the racket are expressed as

$$X_{ijk} = r_{ijk}S_j(\omega_k) \tag{8}$$

where $\omega_k = 2\pi f_k$, r_{ijk} denotes the residue of k-th mode, and $S_j(\omega_k)$ is the impact force component of k-th frequency ω_k in the frequency region (*Kawazoe*, 1992*a*). When the hitting position and the pre-impact velocity are given, the vibration of the racket can be simulated by using Eq.(7) and Eq.(8). The energy loss due to the racket frame vibration can be derived from the amplitude distribution of the velocity and the mass distribution along a racket frame when a ball collides with a hand-held tennis racket (*Kawazoe*, 1992*c*, *e*).

The rebound velocity of a ball when a ball strikes the string face can be derived, considering the main sources of energy loss during impact (*Kawazoe*, 1992*e*). If the longitudinal mass distribution of racket frame is assumed to be uniform, the energy loss ΔE_1 due to racket-frame vibrations can be calculated. Another energy loss ΔE_2 due to the collision between the ball and strings with the reduced mass of racket at the hitting point could also be derived using the measured coefficient of restitution e_{BG}. Accordingly, the coefficient of restitution e_r with respect to the relative velocities corresponding to the total energy loss $\Delta E(=\Delta E_1 + \Delta E_2)$ is given by

$$e_r = (1 - 2\Delta E(m_B + M_r)/(m_B M_r V_{Bo}^2))^{1/2} \tag{9}$$

The coefficient e of the post-impact velocity relative to the pre-impact velocity of a ball is given by

$$e = (1 - 2\Delta E(m_B + M_r)/(m_B M_r V_{Bo}^2))^{1/2} - m_B(1 + e_{BG})/(m_B + M_r) \tag{10}$$

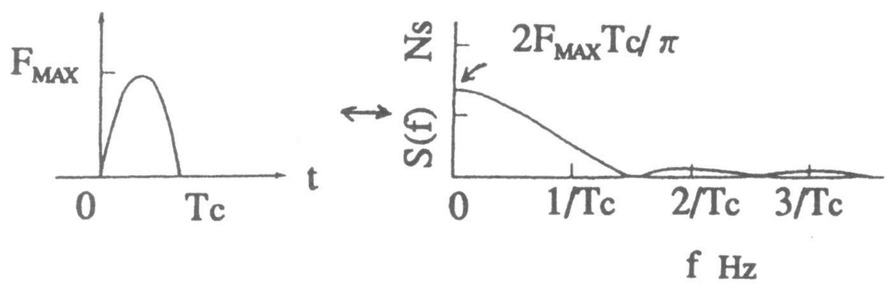


Fig.1 Shock shape and its spectrum during ball/racket impact.

3 Results and Discussion

The calculated contact time agrees well with the measured one during actual forehand strokes. Also, the measured distribution of the coefficient e on the string face (impact velocity: 26.4 m/s) and the calculated one (30.0 m/s) showed good agreement. It tends to be maximized along the longitudinal axis of the racket and peaks close to the throat due to the mass distribution of a racket (*Kawazoe*, 1992*e*).

Figure 2 shows the predicted post-impact vibrations of rackets with various physical properties when a ball hits the top, the centre and the

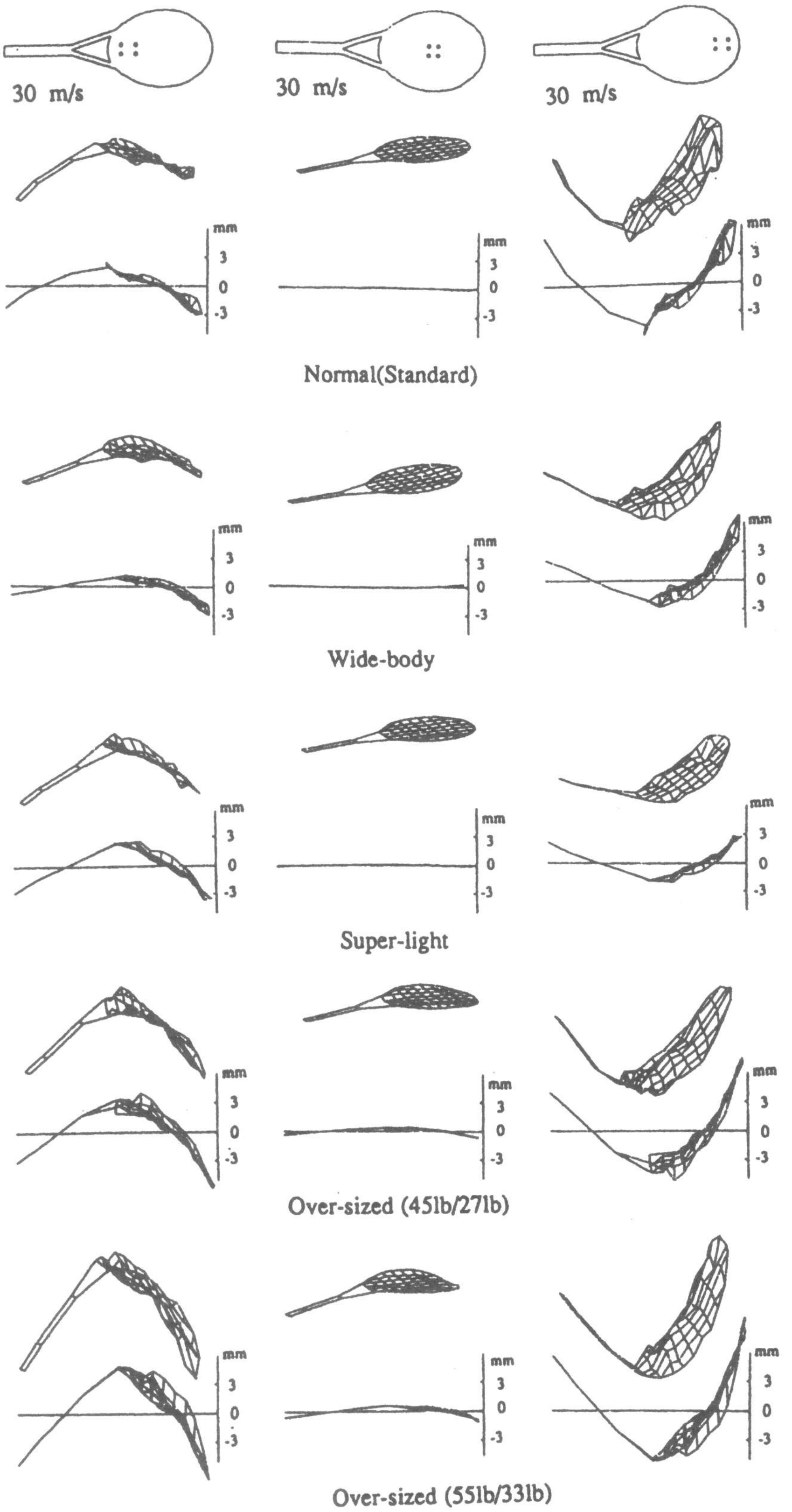
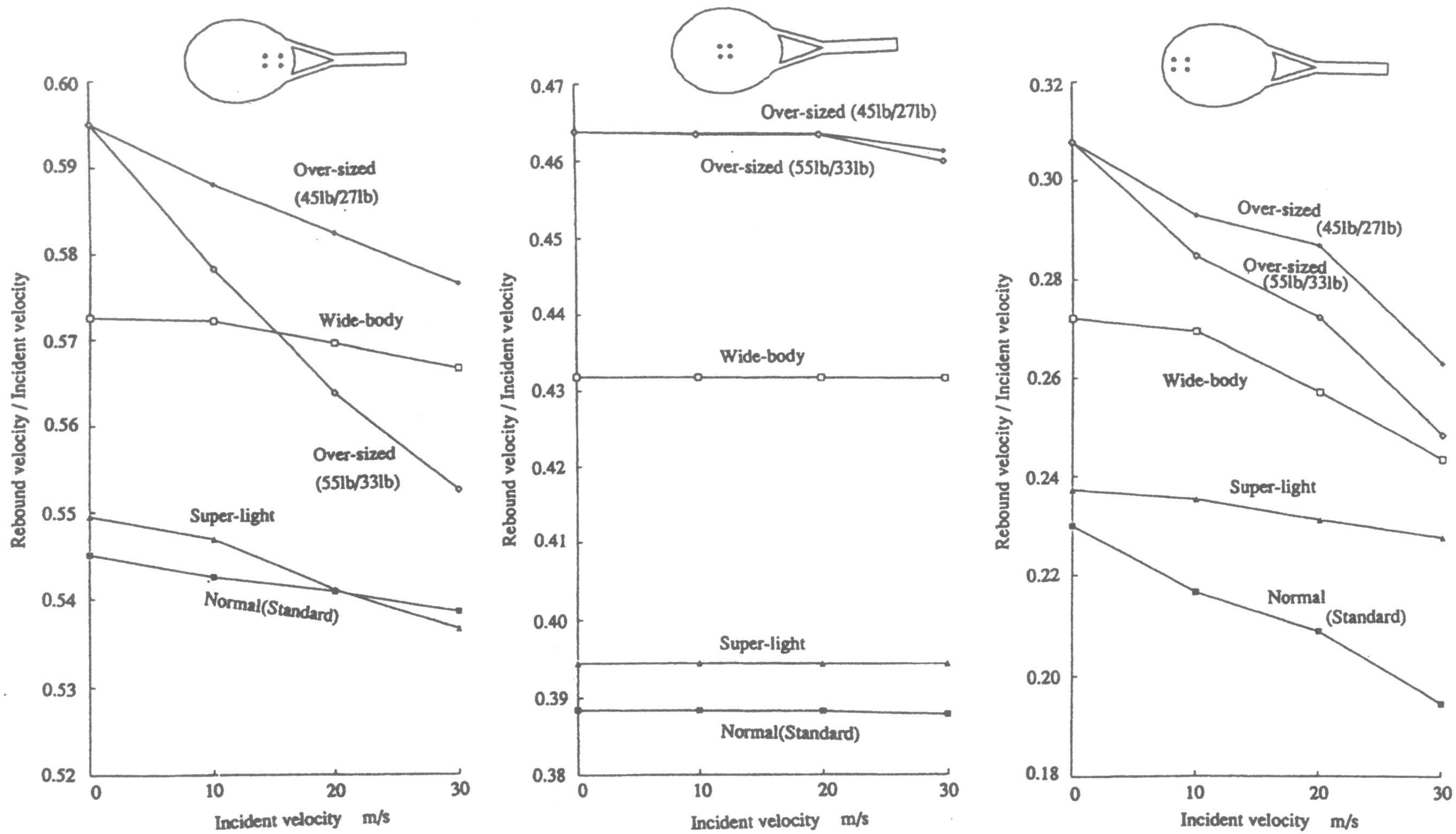


Fig.2 Predicted amplitude of the racket frame vibration immediately after impact when a ball hits the racket (near, center, top) at a velocity of 30 ms^{-1}.



(a) Impact at the near on the string face. (b) Impact at the centre on the string face. (c) Impact at the top on the string face.

Fig.3 Predicted rebound velocity against incident velocity when a ball strikes rackets with different physical properties at the near, centre, and top on the string face.

near on the string face at a velocity of 30 m/s respectively. The rackets tested are Yamaha's EX-II (360 g,length 680 mm,area 100 in^2, centre of gravity from the grip end: 308 mm, 1st vibration mode: 122 Hz), a PROTO-02 (370 g, centre of gravity: 317 mm, 215 Hz), a PROTO-EX110 (366 g,685 mm,110 in^2, centre of gravity:325 mm,132 Hz) strung loosely, a PROTO-EX110 strung tightly, and a EOS100 (290 g, 680 mm, centre of gravity: 350 mm, 171 Hz). It is seen that the amplitude of racket vibration is very small when a ball hits the centre, whereas it is large when a ball hits the off-centre (the top or the place close to the throat) on the string face. The amplitude of vibrations with the wide body racket is rather small compared with the normal one. The amplitude of the super-light and top-heavy racket is small at the top, whereas the amplitude with the over-sized racket is large at the near on the string face. If strung tightly, the amplitude of vibration increases remarkably in the off-centre impact.

Figure 3 shows the predicted coefficient e against the impact velocities when the ball strikes the racket with different physical properties. It is seen that each racket has its own characteristics with respect to the rebound velocity. The rebound coefficient with PROTO-02 (wide body) and EOS100 (super light and top-heavy) are not affected by an increase of impact velocity at the top and higher than that with the normal one. The super-light and top-heavy racket decreases gradually in rebound coefficient with an increase of impact velocity at the near on the string face. Although the rebound coefficient with PROTO-EX110 (over-sized) strung loosely is the highest among them, it decreases with an increase of impact velocity. If strung tightly, it decreases remarkably in the off-centre impact due to the energy loss caused by the vibration of racket-frame.

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