Prediction of Rebound Power and Comfort of Tennis Racket

Analysis of Factors Associated with Impact between the Racket and the Ball in the Forehand Drive

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ABSTRACT: **At the current stage, the performance of tennis racket is based on the feeling of an experienced tester or a player. However, the optimum racket depends on the physical and technical levels of each user. Accordingly, there are a number of unclear points regarding the relationship between the performance estimated by a player and the physical properties of a tennis racket. This paper investigates the racket performance in terms of rebound power and the mechanism of forces transmitted to the wrist joint from a racket and predicts the shock vibrations of the wrist joint in the impact of tennis in order to give physical explanation for the recorded waveform of measured acceleration of the wrist joint. It is based on the experimental identification of the dynamics of racket-arm system and the approximate nonlinear impact analysis. The predicted result of impact shock vibrations of the player's wrist joint agrees fairly well with the measured one. This study enables us to predict quantitatively the factors associated with impact between a various types of racket with various strings and various types of ball.**

1. INTRODUCTION

Advanced engineering technology has enabled manufacturers to discover and synthesize new materials and new design for sport equipment. There are rackets of all compositions, sizes, weights, shapes and string tension. At the current stage, very specific designs are targeted to match the physical and technical levels of each user. However, ball and racket impact in tennis is an instantaneous phenomenon creating vibrations and large deformations of ball/strings in the racket. The problem is further complicated by the involvement of humans. Many unknown factors are involved in the mechanisms that explain how the specifications and physical properties of the racket and the ball influence the racket capabilities.

This work investigated the mechanism of racket performance in terms of the rebound power of tennis racket and the feel or comfort of the arm or hand during a tennis impact. Based on an impact analysis between the ball and the racket, it clarifies the mechanism of forces transmitted to the wrist joint from a racket and predicts the shock vibrations of the wrist joint in order to give physical explanation for the recorded waveform of measured acceleration of the wrist joint. It deals with the shock vibrations of the wrist joint caused by the impact when a male tournament player hits flat forehand drive. It presents a new approach and a method for the prediction of the impact shock vibrations of a racket grip and a player's wrist joint in order to give the physical explanation for the measured waveforms. It is based on the identification of the racket characteristics, the damping of the racket-arm system, the equivalent mass of the player's arm system and an approximate nonlinear analysis of the

impact in tennis.

This study enables us to predict quantitatively the factors associated with impact between a various types of racket with various strings and various types of ball.

2. DERIVATION OF THE IMPACT FORCE AND THE CONTACT TIME AT THE INSTANT WHEN THE PLAYER HITS THE BALL WITH HIS RACKET

2.1 MAIN FACTORS ASSOCIATED WITH THE ENERGY LOSS AND COEFFICIENT OF RESTITUTION DURING IMPACT

2.1.1 Nonlinear restoring force characteristics of a ball and strings and a composed ball/strings system

Figure 1 shows the test for obtaining the applied force-deformation curves schematically, where the ball was deformed between two flat surfaces as shown in (a) and the ball plus strings were deformed with a racket head clamped as shown in (b). The results for the ball and racket strung at a tension of 246 N (55 lbs) are shown in Fig.2. According to the pictures of a racket being struck by a ball, it seems that the ball deforms only at the side, which contact to the strings.

Assuming that a ball with concentrated mass deforms only at the side in contact with the strings (Kawazoe, 1994), the curves of restoring force F_B vs. ball deformation, restoring force F_G vs. strings deformation, and the restoring force F_{GB} vs. deformation of the composed ball/strings system are obtained from Fig.2 as shown in Fig.3. These restoring characteristics are determined so as to satisfy a number of experimental data using the least square method. The curves of the corresponding stiffness K_B, K_G and K_{GB} are derived as shown in Fig.4 by differentiation of the equations of restoring force with respect to deformation, respectively.
The stiffness K_B of a ball, K_G of strings and K_{GB} of a composed ball/strings system exhibit the strong nonlinearity.

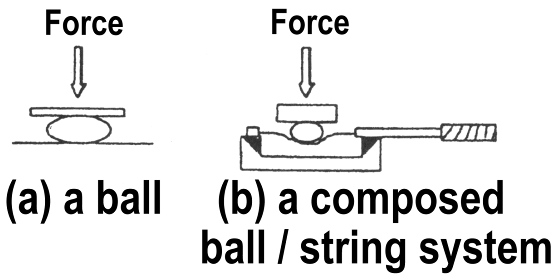

Fig.1 Illustrated applied force-Deformation test

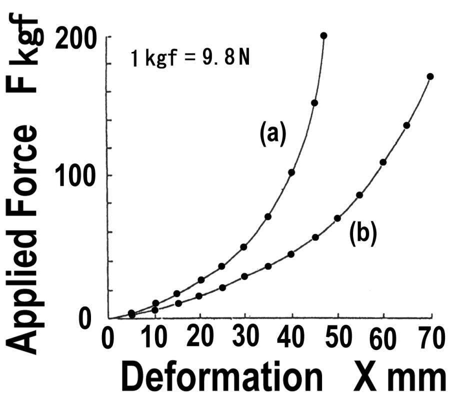

Fig.2 Results of a force-deformation test with pretension of strings

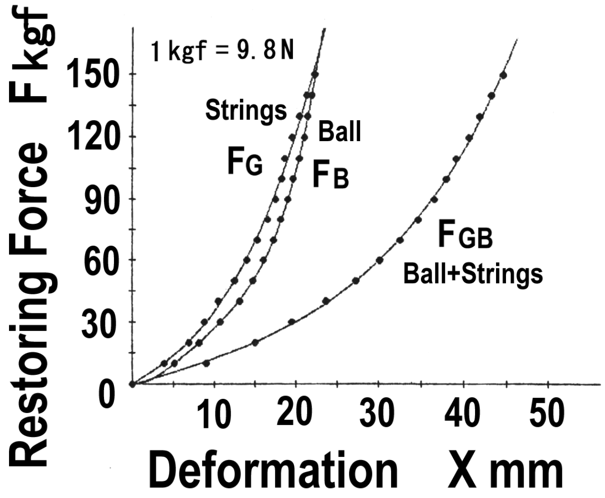

Fig.3 Restoring forces vs. deformation of a ball, strings, and a Composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

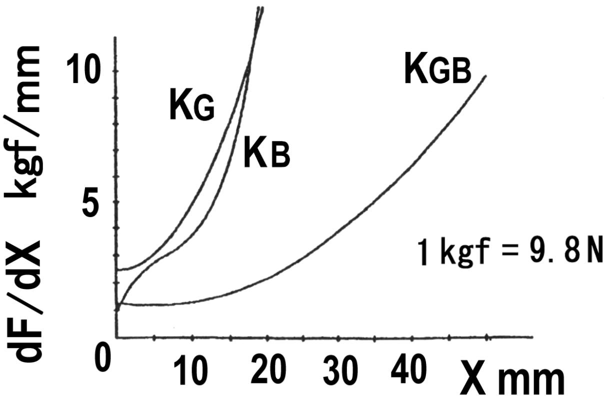

Fig.4 Stiffness vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

2.1.2 Energy loss in a collision between a ball and strings

The measured coefficient of restitution versus the incident velocity when a ball strikes the rigid wall is shown in Fig.5, while the measured coefficient of restitution e_{BG}, which is abbreviated as COR, when a ball strikes the strings with a racket head clamped is shown in Fig.6. Although the COR in Fig.5 decreases with increasing incident velocity, the coefficient e_{BG} with a racket head clamped is almost independent of ball velocity and strings tension. This value of COR can be regarded as being inherent to the materials of ball and strings, showing the important role of strings. This feature is due to the nonlinear restoring force characteristics of a composed ball/strings system (Kawazoe, 1992). Accordingly, the energy loss of a ball and strings due to impact can be related to the coefficient e_{BG}.

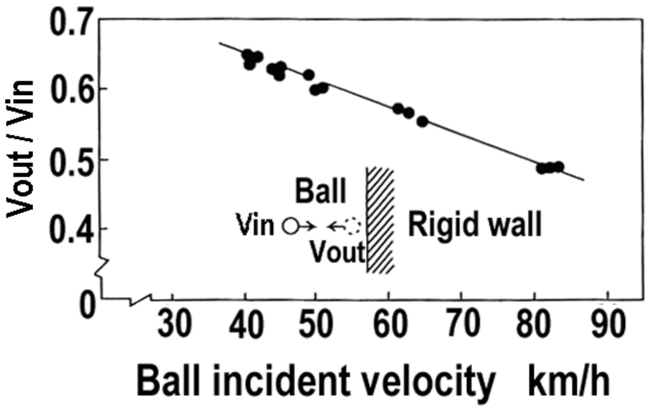

Fig.5. Measured coefficient of restitution (COR) between a ball and a rigid wall.

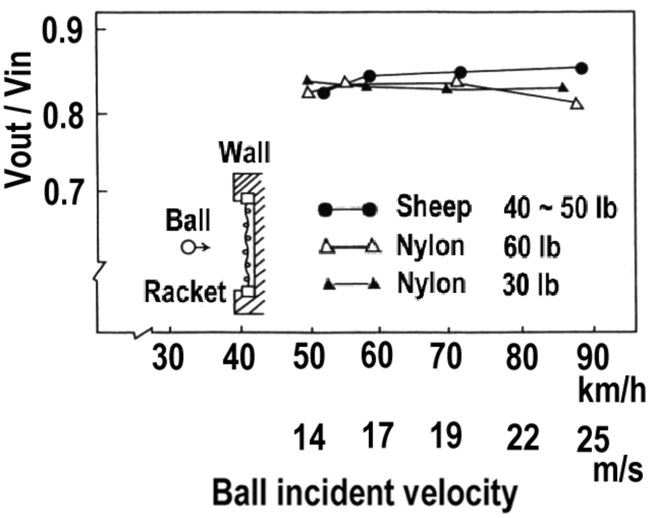

Fig.6 Measured COR between a ball and strings with frame clamped.

2.1.3 Remarks on the contact time between a racket and a ball during Impact

The result of measured contact time, which means how long the ball stays on the strings, with a normal racket and with a wide-body racket (stiffer) shows that the stiffness of the racket frame does not affect the contact time much (Kawazoe, 1992). Accordingly, the masses of a ball and a racket as well as the nonlinear stiffness of a ball and strings are the main factors in the deciding of a contact time. Therefore, the contact time can be calculated using a model assuming that a ball with a concentrated mass m_B and a nonlinear spring K_B, collides with the nonlinear spring K_G of strings supported by a frame without vibration, where the measured coefficient of restitution inherent to the materials of ball-strings impact is employed as one of the sources of energy loss.

2.1.4 Support condition of a racket handle

The result of the experimental modal analysis (Kawazoe, 1989, 1997) showed that the fundamental vibration mode of a conventional type racket supported by a hand has two nodes being similar to the mode of a freely supported racket. When we deal with the racket performance in terms of power, it can be assumed that the racket is freely suspended.

2.2 DERIVATION OF THE APPROXIMATE IMPACT FORCE AND THE CONTACT TIME

The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location and the center of gravity are given.

In case the vibration of the racket frame is neglected, the momentum equation and the coefficient restitution e_{BG} give the post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse could be described as the following equation, where m_B is the mass of a ball, M_r is the reduced mass of a racket at the hitting location, and (V_{BO}- V_{Ro}) is the pre-impact velocity.

$$\int F(t)\,dt = m_B V_{Bo} - m_B V_B = (V_{BO} - V_{Ro})(1+e_{BG}) m_B/(1+m_B/M_r). \qquad (1)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of m_B, K_{GB}, and M_r, it could be obtained as

$$T_c = \pi\, m_B^{1/2} / [\, K_{GB} (1+m_B/M_r)^{1/2}\,] \qquad (2)$$

In order to make the analysis simpler, the equivalent force F_{mean} can be introduced during contact time T_c, which is described as

$$\int^{Tc} F(t)\,dt = F_{mean} \cdot T_c \qquad (3)$$

Thus, from Eq.(10), Eq.(11) and Eq.(12), the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity (V_{BO}- V_{Ro}) is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1+e_{BG})\, m_B^{1/2} K_{GB}^{1/2} / [\, \pi\, (1+m_B/M_r)^{1/2}\,] \qquad (4)$$

On the other hand, from the approximated curves shown in Fig.3 and Fig.4, F_{GB} can be expressed as the function of K_{GB} in the form

$$F_{GB} = func.\,(K_{GB}). \qquad (5)$$

From Eq.(4) and Eq.(5), K_{GB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be determined against the pre-impact velocity by using Eq.(2). Figure 7 is a comparison between the measured contact times during actual forehand strokes (Nagarta, 1983) and the calculated ones when a ball hits the center of the string face of a conventional type racket (360 g), showing a good agreement between them.

Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force. The mathematical expression of impact force S_0 between a ball and the racket is written as

$$S_0 = F(t) = F_{max} \sin(\pi\, t / T_c) \quad (0 \leqq t \leqq T_c) \qquad (6)$$

where $F_{max} = \pi F_{mean}/2$. The fourier spectrum of Eq.(6) is represented as

$$S(f) = 2F_{max} T_c \mid \cos(\pi fT_c) \mid / [\pi \mid 1 - (2fT_c)^2 \mid] \qquad (7)$$

where f is the frequency [Hz]. Figure 8 shows the examples of the calculated shock shape during impact, where the ball strikes the center on the string face at a velocity of (a) 20 m/s and (b) 30 m/s with the racket strung at 55 lb, respectively.

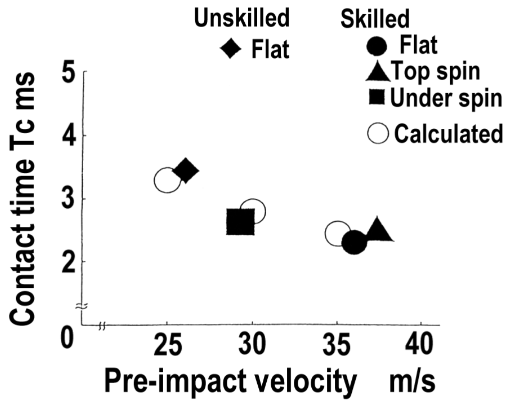

Fig.7 Comparison between the measured contact times during Strokes and the calculated results.

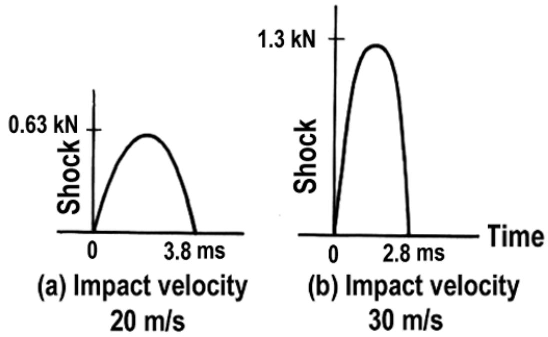

Fig.8 Calculated shock shape when a ball strikes the center on the String face of the racket at velocities of 20 m/s and 30 m/s.

3. PREDICTION OF THE RACKET VIBRATIONS

The vibration characteristics of a racket can be identified using the experimental modal analysis (Kawazoe, 1989, 1990) and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of a racket. When the impact force $S_j(2\pi f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i is expressed as

$$X_{ijk} = r_{ijk} \; S_j(2\pi f_k) \qquad (8)$$

where r_{ijk} denotes the residue of k-th mode between arbitrary point i and j, and $S_j(2\pi f_k)$ is the impact force component of k-th frequency f_k (Kawazoe,1993). Figure 12 shows the

predicted vibration amplitude of a racket struck by a ball at a velocity of 30 m/s.

4. ENERGY LOSS DUE TO RACKET VIBRATIONS INDUCED BY IMPACT

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along a racket frame. If the longitudinal mass distribution of racket frame is assumed to be uniform, the energy loss E_1 due to racket vibrations can be easily derived.

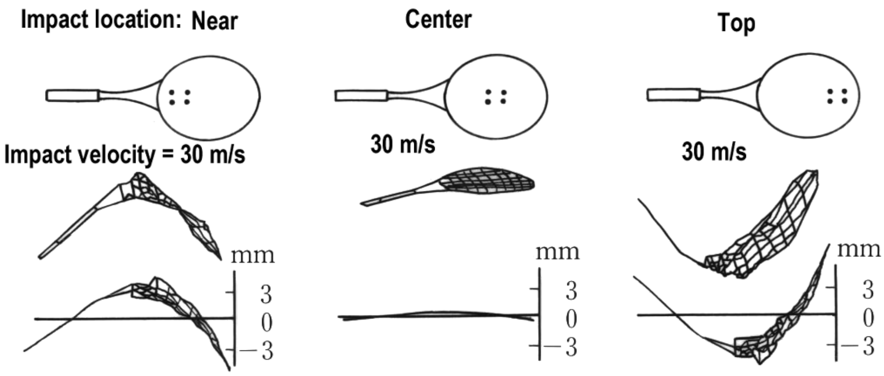

Fig.9 Predicted initial amplitude of 1st mode component of racket frame vibrations.

5. DERIVATION OF THE COEFFICIENT OF RESTITUTION

The coefficient of restitution (COR) can be derived considering the energy loss during impact. The main sources of energy loss is E_1 as well as E_2 due to the instantaneous large deformation of a ball and strings which is calculated by using the coefficient e_{BG}. If a ball collides with a racket at rest ($V_{Ro} = 0$), the energy loss E_2 could be easily obtained. The coefficient of restitution e_r corresponds to the total energy loss $E\ (=E_1+E_2)$ obtained as

$$e_r = (V_R - V_B)/V_{BO} = [1 - 2E\,(m_B + M_r)/(m_B M_r V_{BO}^2)]^{1/2} \qquad (9)$$

Figure 10 shows an example of predicted e_r at the longitudinal axis on the racket face when a player hits a coming ball with a velocity V_{BO} of 10 m/s, where a simple forehand ground stroke swing model (Kawazoe et al., 1993) is used as shown in Fig.11.
It is seen that e_r of a composite racket is higher than that of a wooden racket, particularly at the top of the string face

6. PREDICTION OF THE REBOUND POWER COEFFICIENT

The post-impact ball velocity V_B is represented as

$$V_B = -V_{Bo}(e_r - m_B/M_r)/(1+m_B/M_r) + V_{Ro}(1+e_r)/(1+m_B/M_r) \qquad (10)$$

Accordingly, if the ratio of rebound velocity against the incident velocity of a ball when a ball

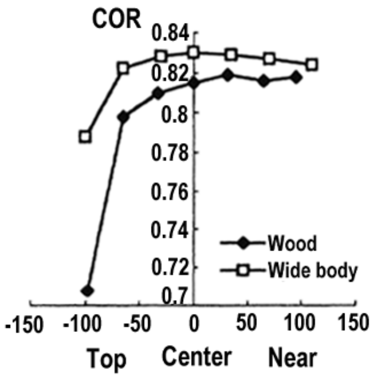

Fig.10 Examples of predicted e_r on the racket face when a player hits a ball.

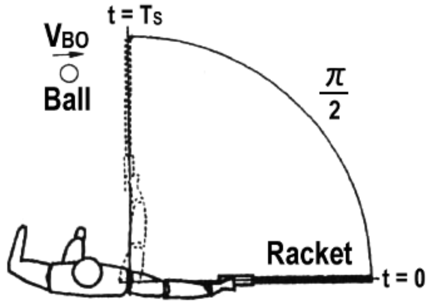

Fig.11 Simple forehand ground stroke swing model.

strikes the freely suspended racket (V_{Ro} = 0) is defined as the rebound power coefficient, it is written as Eq.(11) . The rebound power coefficient is often used to estimate the rebound power performance of a racket experimentally in the laboratory.

$$e = -V_B \;/\; V_{BO} \;=\; (e_r - m_B/M_r)/(1 + m_B/M_r) \qquad \textbf{(11)}$$

When a player hits a coming ball with a pre-impact racket head velocity V_{Ro}, the coefficient e can be expressed as

$$e = -(V_B - V_{Ro})\;/\;(V_{BO} - V_{Ro}) \qquad \textbf{(12)}$$

Figure 17 is a comparison between the measured e and the calculated e when a ball hits a freely-suspended racket (about 30 m/s), showing a good agreement between them.

7. PREDICTION OF THE POST-IMPACT BALL VELOCITY

The power of the racket could be estimated by the post-impact ball velocity V_B when a player hits a ball. The V_B can be expressed as Eq.(13). The V_{Ro} is given by $L_X\,(\pi\;N_s/\,I_s)^{1/2}$, where L_X denotes the holizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque around the shoulder joint, and I_s the

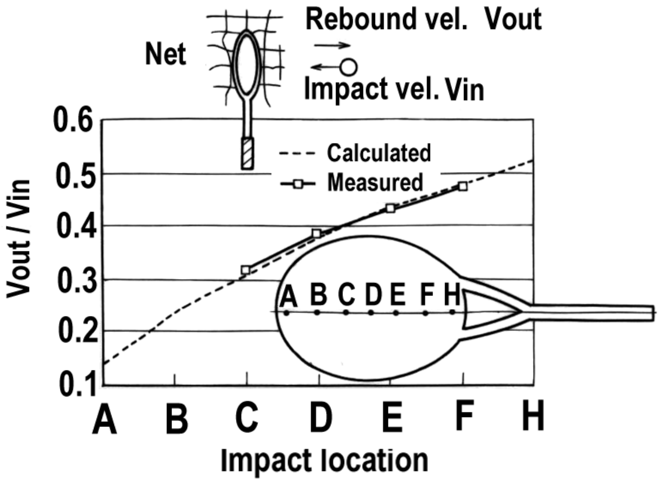

Fig.17 Comparison between the measured *e* and the predicted *e*.

moment of inertia of arm/racket system around the shoulder joint. Figure 12 shows the examples of the predicted V_B at each hitting location on the racket face (V_{Bo}= 10 m/s, N_s =56.9 Nm).

$$V_B = -V_{Bo}\ e + V_{Ro}(1+e) \qquad (13)$$

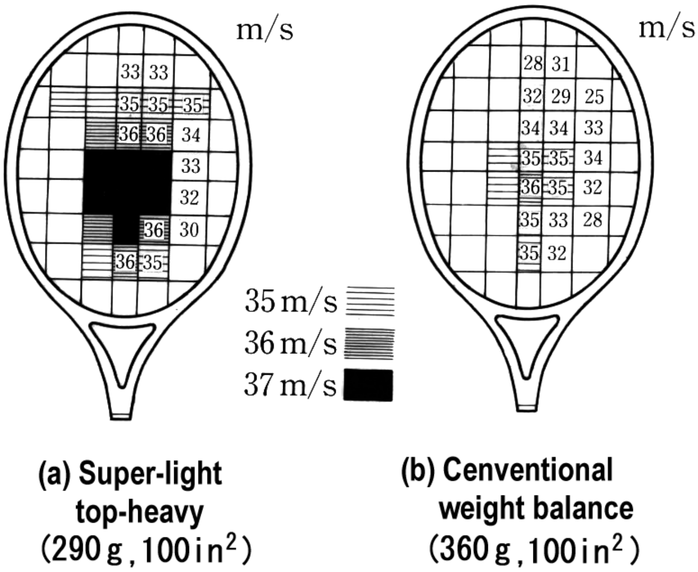

Fig.12 Examples of predicted V_B (V_{Bo}= 10 m/s, N_s =56.9 Nm).

8. PREDICTION OF THE WAVEFORMS OF SHOCK VIBRATIONS AT THE RACKET HANDLE AND THE WRIST JOINT

8.1 IMPACT SHOCK FORCES OF AN ARM JOINT SYSTEM

Figure 13 shows the situation of experiment where a male tournament player hits flat forehand drive and Fig.14 shows the locations of attached accelerometers at the wrist joint and

the elbow joint in the experiment. An accelerometer was also attached at 210 mm distance from the grip end on the racket handle. Figure 15 shows an impact model for the prediction of shock force transmitted to the arm joint from a racket [Casolo et al., 1991]. The impact force S_0 at P_0 causes a shock force S_1 on the player's hand P_1, a shock force S_2 on his elbow P_2, and finally a shock force S_3 on the player's shoulder P_3 during the impact at which the player hits the ball with his racket.

Since the shock forces S_0 ,S_1 ,S_2 , and S_3 should be one order of magnitude higher than the other forces in play during impact, the gravity force and muscular action are not taken into account in this paper. In other words, we consider the racket to be freely hinged to the forearm of the player, the forearm being freely hinged to the arm and the arm freely hinged to the player's body. This schematization only refers to the interval lasting no longer than one hundredth of a second. Both before and afterwards, in the absence of shock forces S_0 , S_1 ,S_2 , and S_3, all the movements depend on the intensity of the muscular forces and gravity forces in play. Therefore, the intensity with which the player grips the racket at the moment of impact with the ball has no effect on the phenomenon itself.

If we indicate the articulation of the hand by P_1, the articulation of the elbow by P_2 and the articulation of the shoulder by P_3, the limb is schematized by two straight line segments $P_1 P_2$ and P_2P_3.

The forearm length $a_a = P_1P_2$, with a mass m' to which the mass m'' of the hand is added, concentrated at P_1 : consequently, the total mass of the forearm is equal to $m_a = m' + m''$ and the distance b_a of the center of mass from elbow is

$$b_a = G_1P_2 = [\, m'(a_a/2) + m''a_a]/m_a \qquad (14)$$

Moreover, if we indicate the moment of inertia around the elbow P_2 by J_a, the mass of the arm with a length of $a_b = P_2P_3$ by m_b, the distance of the center of mass from the shoulder P_3 by $b_b = G_3P_3$, while the moment of inertia with respect to the shoulder P_3 by J_b, we obtain the following relationship from the equations of motion with respect to the arm P_2P_3 .

Fig.13 Experiment where a male player hits flat forehand drive.

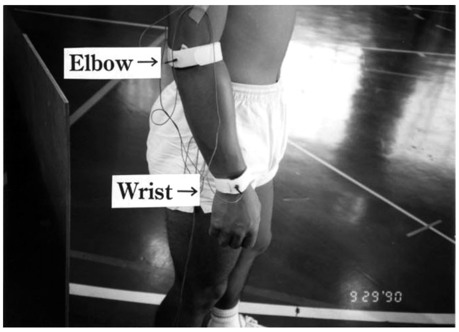

Fig.14 Accelerometers attached at the wrist and the elbow.

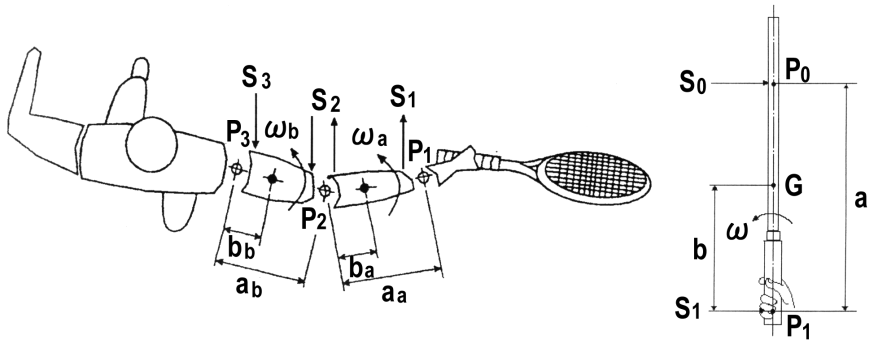

Fig.15 Impact model for the prediction of the shock force transmitted to the arm joints from a racket.

$$S_3 = S_2\,(m_b a_b b_b / J_b - 1) \qquad (15)$$

We can also derive the following relationship from the equations of motion for the forearm P_1P_2 and a few calculation steps.

$$S_2 = S_1\,(m_a a_a b_a / J_a - 1) / [\,1 + (m_a a_b^2 / J_b)(1 - m_b b_a^2 / J_a)] \qquad (16)$$

$$\mathrm{d}V_1/\mathrm{d}t = \mathrm{d}(\omega_a a_a + \omega_b a_b + V_3)/\mathrm{d}t = S_1\,[\mu_a a_a^2 / J_a - \chi_a a_b^2 / J_b] \qquad (17)$$

where

$$\mu_a = [1 + (m_a a_b^2 / J_b)(1 - b_a / a_a)] / [1 + (m_a a_b^2 / J_b)(1 - m_a b_a^2 / J_a)] \qquad (18)$$

$$\chi_a = (m_a a_a b_a / J_a - 1) / [\,1 + (m_a a_b^2 / J_b)(1 - m_b b_a^2 / J_a)\,] \qquad (19)$$

i.e. by assuming

$$M_H = 1 / [\,\mu_a a_a^2 / J_a - \chi_a a_b^2 / J_b\,] \qquad (20)$$

finally we have the acceleration A_{nv} at the grip portion and the wrist joint as

$$A_{nv} = \mathrm{d}V_1 / \mathrm{d}t = S_1 / M_H \qquad (21)$$

From Eq.(21) we can deduce that the inertia effect of the arm and the forearm can be attributed to a mass M_H concentrated in the hand; therefore the analysis of impact between ball and racket can be carried out by assuming that the racket is free in space, as long as the mass M_H is applied at point P_1 of the hand grip.

If the impact force S_0 between a ball and the racket is given when the ball hits the racket, the shock force S_1 can be obtained with a few steps as

$$S_1 = M_H \mathrm{d}V_1 / \mathrm{d}t = M_H A_{nv}$$
$$= S_0 (M_R ab/J - 1) / [\,1 + (M_R / M_H)(1 - M_R b^2 / J)] \qquad (22)$$

where we indicate the mass of the racket by M_R, the distance between the grip location on the handle and the impact location on the string face by a, the distance between the grip location on the handle and the center of mass of the racket by b, and the moment of inertia with respect to the articulation P_1 of the hand by J. Thus, the shock forces S_2 and S_3 also can be obtained from Eq.(16) and Eq.(15), respectively.

8.2 PREDICTION OF THE WAVEFORMS OF SHOCK VIBRATIONS AT THE RACKET HANDLE AND THE WRIST JOINT

The reduced mass M_r at the impact location with a racket-arm system can be derived as

$$M_r = 1/[1/(M_R+M_H)+c^2/I_G] = (M_R+M_H)I_G/[I_G+(M_R+M_H)c^2] \tag{23}$$

where

$$c = c_o + (L_{Go} - L_H)M_H/(M_R + M_H) \tag{24}$$

$$I_G = I_{Go} + M_R \triangle G^2 + M_H (L_{Go} - L_H - \triangle G)^2 \tag{25}$$

$$\triangle G = (L_{Go} - L_H)M_H/(M_R+M_H) \tag{26}$$

and L_{Go} denotes the distance between the center of mass and the grip end of the racket, I_{Go} the moment of inertia with respect to the center of gravity of the racket, c_o the distance between the center of gravity and the impact location of the racket, and L_H the distance of the point P_1 of the hand grip from the grip end The moment of inertia with respect to the center of gravity and the distance of the center of gravity from the impact location of the racket-arm system are indicated by I_G and c, respectively. The mass M_H denotes the equivalent mass of the arm system at the hand grip.

Since the force-time curve of impact between a ball and a racket considering the vibrations of a racket frame can be written as

$$S_0(t) = S_{0max} \sin(\pi t/T_c) \quad (0 \leqq t \leqq T_c) \tag{27}$$

by the same way as Eq.(4) and Eq.(6), where

$$S_{0max} = \pi F_{mean}/2 = (\pi/(2T_c))(V_{BO} - V_{Ro})(1+e_r)m_B/(1+m_B/M_r) \tag{28}$$

The shock acceleration at the hand grip considering the equivalent mass M_H of the arm system can be represented as

$$A_{nv}(t) = S_0(t)[1/(M_R+M_H) - (a/I_G)X] \tag{29}$$

where X denotes the distance between the center of mass of racket-arm system and the location of hand grip, a the distance between the center of mass of racket-arm system and the impact location of the racket, I_G the moment of inertia with respect to the center of mass of racket-arm system, respectively. The maximum shock force $S_{1\,max}$ transmitted to a wrist

joint corresponds to the maximum impact force S_{0max}.

Although the frequency drops slightly for the hand-held racket compared to the freely suspended racket, the positions of nodes on the string face are nearly identical. With a primary vibration mode, the position of the node on the handle for the hand-held racket shifts somewhat to the held position. Although the damping of frame vibrations is remarkably larger for the hand-held racket compared to the freely suspended racket, there is no big difference in the initial amplitude distributions of a racket frame between them.

The vibration acceleration component of *k*-th mode at the location *i* of hand grip is represented as

$$A_{ij,k}(t) = -(2\pi f_k)^2 r_{ijk} S_{0j}(2\pi f_k) exp(-2\pi f_k \zeta_k t) sin(2\pi f_k t) \qquad (30)$$

where *j* denotes the impact location between ball and racket on the string face, ζ_k the damping ratio of k-th mode, $S_{0j}(2\pi f_k)$ the fourier spectrum of Eq.(30).

The summation of Eq.(29) and Eq.(30) represents the shock vibrations at the hand grip.

Figure 18 shows the predicted shock vibrations of a wrist joint compared with the experimental ones when a ball is struck at the topside of the racket face. This racket is made of 75 % graphite, 20 % fiberglass and 5% others, with 685 mm of total length, 100 in^2 of face area, 342 g of mass including string mass, 310 mm of the center of mass from grip end, 14.2 gm^2 of moment of inertia about the center of racket mass, 60 lb. of string tension. The center of mass of racket-arm system shifts to the location of 131 mm from the grip end. In the figure, the first largest peak during the impact was caused by the shock and vibrations of a racket frame,

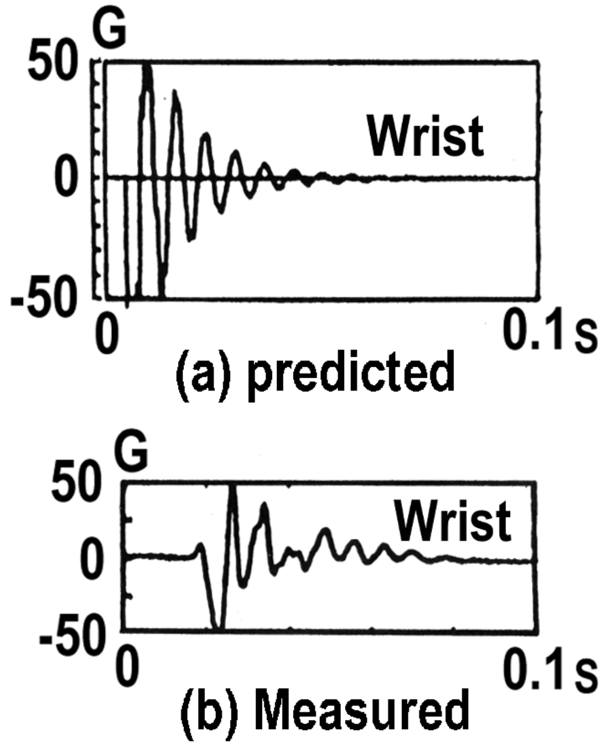

Fig. 16 Predicted shock vibrations of a wrist joint compared with the experimental.

followed by the residual vibrations. The shock vibrations are composed of the shock acceleration and the racket vibration components, and each component has its own time history and magnitude depending on the impact velocity, impact location, grip location of racket handle and the physical properties of a racket. The damping ratio of a hand-held racket during actual impact is estimated as about 2.5 times that of the one identified by the experimental modal analysis with small vibration amplitude. Furthermore, the damping of the waveform at the wrist joint was 3 times that at the grip portion of the racket handle. The predicted waveforms of the shock vibrations with the racket handle and the wrist joint agrees fairly well with the measured ones during actual forehand stroke by a player. The mechanism of the shock vibrations of the elbow joint, however, is left unsolved.

9. CONCLUSIONS

The predicted rebound power coefficient has shown a good agreement with the measured one. The predicted waveform of the shock vibrations at the wrist joint also agrees fairly well with the measured ones during actual flat forehand drive by a player. It was shown that the shock vibrations of the wrist joint was transmitted from the racket with an impulse at the impact location and the several vibrations mode components of a racket frame and strings. This work enables us to predict the difference in rebound power of the racket and the feel or comfort between the rackets and balls with different physical properties.

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