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Prediction of Various Factors Associated with Tennis Impact: Effects of Large Ball and Strings Tension

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ABSTRACT: At the current stage, the perception of performance of the tennis racket is based on the feel of an experienced tester or player. However, the optimum racket depends also on the physical and technical levels of each user. In addition, there are many unknowns regarding the relationship between the performance estimated by a player and the physical properties of a tennis racket. This paper describes the method that can be used to predict the various factors, such as the impact force, the contact time, the deformation of the ball and strings, the coefficient of restitution, the rebound power coefficient and the post-impact ball velocity, associated with the tennis impact when the impact velocity and the impact locations on the racket face are given. It is based on the experimental identification of the dynamics of ball-racket-arm system and the approximate nonlinear impact analysis. It enables us to predict the various factors associated with impact between various types of rackets with various strings and various types of balls. The predicted results with the large ball showed that the impact force is slightly smaller, the contact time is longer, the deformation of the ball and the strings are slightly smaller, compared to the conventional normal ball. The predicted results with the loosely strung racket showed that the contact time is longer below 20 m/s of impact velocities, the deformation of the strings is larger, and the contact time above 20 m/s. The impact force and the deformation of the ball are almost the same, compared to the tightly strung racket.

INTRODUCTION

Advanced engineering technology has enabled manufacturers to discover and synthesize new materials and new designs of sport equipment. There are rackets of all compositions, sizes, weights, shapes and strings tension, and very specific designs are targeted to match the physical and technical levels of each user. However, there are many unknowns regarding the relationship between the performance estimated by a player and the physical properties of a tennis racket.

Much engineering research has been conducted to determine an optimal tennis racket design, and numerous variables have been considered in order to assess the mechanical performance of the racket and string system along with their effect on the behavior of the ball after impact (Groppel et al. 1987). Nevertheless, the conventional research demonstrates the complexity of the interaction of ball, strings and racket. Accordingly, the perception of performance of a tennis racket is still based on the feel of an experienced tester or player.

This paper describes a method that can be used to predict various factors, such as the impact force, the contact time, the deformation of the ball and strings, the coefficient of restitution, the rebound power coefficient and the post-impact ball velocity, associated with the tennis impact when the impact velocity, swing model and the impact locations on the racket face are given. It is based on the experimental identification of the dynamics of ball-racket-arm system and the approximate nonlinear impact analysis (Kawazoe 1992, 1993, 1994a 1994b, 1997). It enables us to predict the various factors associated with impact between various types of rackets with various strings and various types of balls. In this paper, it is used to predict the effects of large balls and string tensions on various factors associated with tennis impact using the above impact model in order to see whether the large ball and the lower tensions increase the dwell time of the ball on the strings, and offer less impact on the arm (Pluim 2000).

DERIVATION OF IMPACT FORCE, CONTACT TIME, COEFFICIENT OF RESTITUTION, REBOUND POWER COEFFICIENT AND POST-IMPACT BALL VELOCITY

MAIN FACTORS ASSOCIATED WITH THE ENERGY LOSS AND COEFFICIENT OF RESTITUTION DURING IMPACT

Nonlinear restoring force characteristics of a ball and strings and a composed ball/ strings system

Fig. 1 shows schematically the test for obtaining the applied force-deformation curves, where the ball is deformed between two flat surfaces as shown in (a) and the ball plus strings is deformed with a racket head clamped as shown in (b). Assuming that a ball with concentrated mass deforms only at the side in contact with the strings (Kawazoe, 1992, 1994a), the curves of restoring force F_B vs. ball deformation, restoring force F_G vs. strings deformation, and the restoring force F_{GB} vs. deformation of the composed ball/strings system can be obtained. These restoring characteristics are determined so in order to satisfy a number of experimental data using the least square method. The curves of the corresponding stiffness K_B, K_G and K_{GB} are derived by differentiation of the equations of restoring force with respect to deformation. The stiffness K_B of a ball, K_G of strings and K_{GB} of a composed ball/strings system exhibit strong non-linearity.

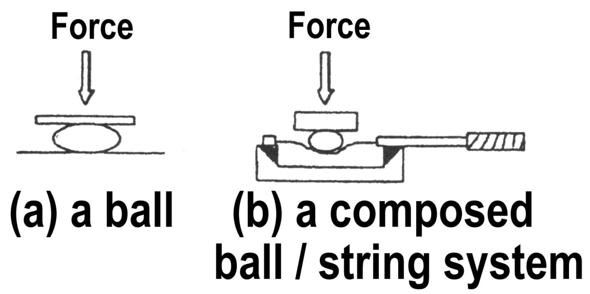

Fig.1 Illustrated applied force- deformation test

Energy loss in a collision between a ball and strings

The coefficient of restitution e_{BG}, when a ball strikes the strings with a racket head clamped is measured. This value can be regarded as being inherent to the materials of ball and strings, showing the important role of strings (Kawazoe, 1992). Accordingly, the energy loss of a ball and strings due to impact can be related to the coefficient e_{BG}.

Remarks on the contact time between a racket and a ball during impact

The result of measured contact time, (which means how long the ball stays on the strings), with a normal racket and a wide-body racket (stiffer) shows that the stiffness of the racket frame does not affect the contact time much (Kawazoe, 1992). Accordingly, the masses of the ball and the racket as well as the nonlinear stiffness of the ball and strings are the main factors affecting the contact time. Therefore, the contact time can be calculated using a model assuming that a ball with a concentrated mass m_B and a nonlinear spring K_B, collides with the nonlinear spring K_G of strings supported by a frame without vibrations, where the measured coefficient of restitution inherent to the materials of ball-strings is employed as one of the sources of energy loss.

Support condition of a racket handle

The result of the experimental modal analysis (Kawazoe 1989, 1997) showed that the fundamental vibration mode of a conventional racket supported by a hand has two nodes that are similar to the mode of a freely supported racket. When we deal with the racket performance in terms of power, it can be assumed that the racket is freely suspended.

DERIVATION OF THE APPROXIMATE IMPACT FORCE AND THE CONTACT TIME

The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location and the center of gravity are given.

In case the vibration of the racket frame is neglected, the momentum equation and the coefficient of restitution e_{BG} give the post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse could be described using the following equation, where m_B is the mass of a ball, M_r is the reduced mass of a racket at the hitting location, and (V_{BO} - V_{Ro}) is the pre-impact velocity.

$$\int F(t)\,dt = m_B V_{Bo} - m_B V_B = (V_{BO} - V_{Ro})(1 + e_{BG})m_B/(1 + m_B/M_r). \qquad (1)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of m_B , K_{GB} , and M_r , it could be obtained as

$$T_c = \pi\ m_B^{1/2}/[\,K_{GB}(1 + m_B/M_r)^{1/2}\,] \qquad (2)$$

In order to make the analysis simpler, the equivalent force F_{mean} can be introduced during contact time T_c , which is described as

$$\int^{Tc} F(t)\,dt = F_{mean} \cdot T_c \qquad (3)$$

Thus, from Eq.(1), Eq.(2) and Eq.(3), the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity (V_{BO} - V_{Ro}) is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1 + e_{BG})\, m_B^{1/2}\, K_{GB}^{1/2} / [\,\pi\ (1 + m_B/M_r)^{1/2}] \qquad (4)$$

On the other hand, from the approximated curves, F_{GB} can be expressed as the function of K_{GB} in the form

$$F_{GB} = f(K_{GB}). \qquad (5)$$

From Eq.(4) and Eq.(5), K_{GB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be determined against the pre-impact velocity by using Eq.(2). A comparison between the measured contact times during actual forehand strokes and the calculated ones when a ball hits the center of the string face of a conventional type racket (360 g) shows a good agreement between them (Kawazoe 1993a).

Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force. The mathematical expression of impact force S_0 between the ball and the racket is

$$S_0 = F(t) = F_{max} \sin(\pi\ t / T_c) \quad (0 \leqq t \leqq T_c) \qquad (6)$$

where $F_{max} = \pi F_{mean}/2$. The Fourier spectrum of Eq.(6) is represented as

$$S(f) = 2F_{max} T_c \,|\cos(\pi\ fT_c)|\, / \,[\,\pi\ |1 - (2fT_c)^2|\,] \qquad (7)$$

where f is the frequency (Kawazoe 1993a).

PREDICTION OF THE RACKET VIBRATIONS

The vibration characteristics of a racket can be identified using experimental modal analysis (Kawazoe, 1989) and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of the racket. When the impact force $S_j(2\pi\ f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i is expressed as

$$X_{ijk} = r_{ijk}\ S_j(2\pi\ f_k) \qquad (8)$$

where r_{ijk} denotes the residue of k-th mode between arbitrary point i and j, and $S_j(2\pi\ f_k)$ is the impact force component of k-th frequency f_k (Kawazoe 1993a, 1997).

ENERGY LOSS DUE TO RACKET VIBRATIONS INDUCED BY IMPACT

The energy loss E_1 due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along the racket frame. If the longitudinal mass distribution of the racket frame is assumed to be uniform, the energy loss E_1 due to racket vibrations can be easily derived.

DERIVATION OF THE COEFFICIENT OF RESTITUTION

The coefficient of restitution (COR) can be derived considering the energy loss during impact. The main sources of energy loss are E_1 and E_2 due to the instantaneous large deformation of a ball and strings which is calculated by using the coefficient e_{BG}. If a ball collides with a racket at rest ($V_{Ro} = 0$), the energy loss E_2 could be easily obtained. The coefficient of restitution e_r corresponds to the total energy loss E ($= E_1 + E_2$) obtained as

$$e_r = (V_R - V_B)/V_{BO} = [1 - 2E(m_B + M_r)/(m_B M_r V_{BO}^2)]^{1/2.} \qquad (9)$$

PREDICTION OF THE REBOUND POWER COEFFICIENT

The post-impact ball velocity V_B is represented as

$$V_B = -V_{Bo}(e_r - m_B/M_r)/(1+m_B/M_r)+V_{Ro}(1+e_r)/(1+m_B/M_r) \qquad (10)$$

Accordingly, if the ratio of rebound velocity against the incident velocity of a ball when a ball strikes the freely suspended racket ($V_{Ro} = 0$) is defined as the rebound power coefficient, it is written as Eq.(11) . The rebound power coefficient is often used to estimate the rebound power performance of a racket experimentally in the laboratory.

$$e = -V_B / V_{BO} = (e_r - m_B/M_r)/(1+ m_B/M_r) \qquad (11)$$

When a player hits a coming ball with a pre-impact racket head velocity V_{Ro}, the coefficient e can be expressed as

$$e = -(V_B - V_{Ro})/(V_{BO} - V_{Ro}) \qquad (12)$$

PREDICTION OF THE POST-IMPACT BALL VELOCITY

The power of the racket could be estimated by the post-impact ball velocity V_B when a player hits the ball. The V_B can be expressed as Eq.(13). The V_{Ro} is given by $L_X(\pi N_s/I_s)^{1/2}$, where L_X denotes the horizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque about the shoulder joint, and I_s the moment of inertia of arm/racket system about the shoulder joint

$$V_B = -V_{Bo}\, e + V_{Ro}(1 + e) \qquad (13)$$

EFFECTS OF LARGE BALLS

Fig. 2 shows the new large ball and conventional normal ball. Fig. 3 shows the reduced mass of the racket and the racket-arm system at the impact locations on the string face. The equivalent mass of 1.0 [kgf] is added at the grip from 70 mm from the grip end. The effect of the mass of the arm is small except at the near off-center.

Fig. 4, Fig.5, Fig.6, and Fig.7 are the predicted contact time, the maximum impact force, the deformation of the strings, and the deformation of the ball, respectively, against impact velocity at the center on the string face.

The contact time of the larger ball is slightly longer and the impact force is slightly smaller. Accordingly, there is no big difference in the deformation of the string and the ball between the larger ball and the normal ball.

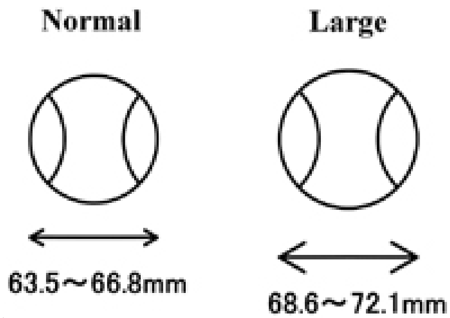

Fig.2 New large ball and conventional normal ball

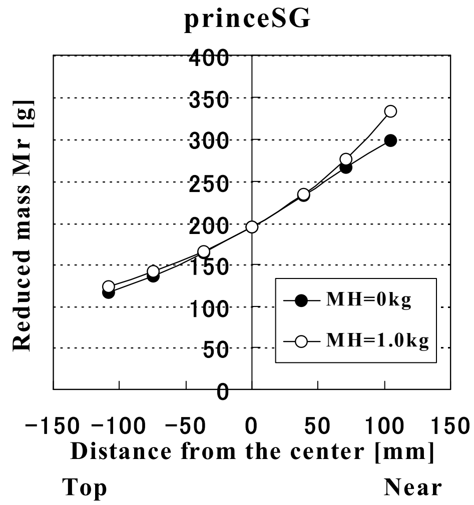

Fig.3 Reduced mass of the racket arm system

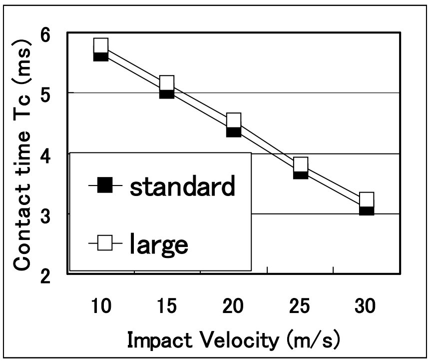

Fig.4 Predicted contact times against impact velocity at the center on the string face (45 lbs).

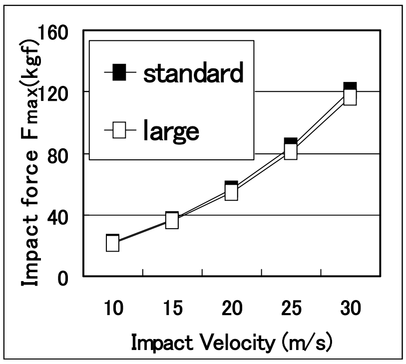

Fig.5 Predicted impact force against impact velocity at the center on the string face (45 lbs).

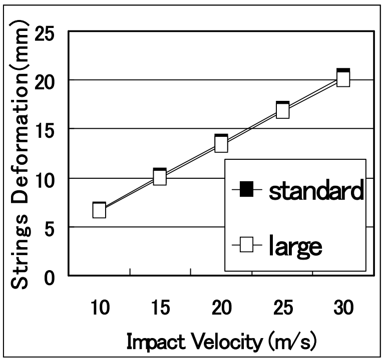

Fig.6 Predicted string deformation against impact velocity (45 lbs).

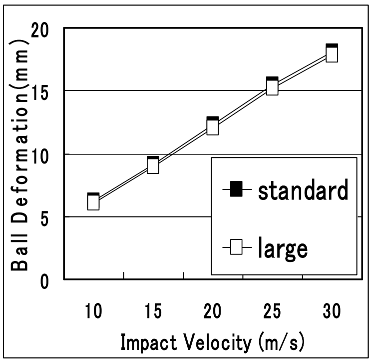

Fig.7 Predicted ball deformation against impact velocity (45 lbs).

EFFECT OF STRING TENSION

Fig. 8, Fig.9, Fig.10, and Fig.11 are the predicted contact time, the maximum impact force, the deformation of the strings, and the deformation of the ball, respectively, against impact velocity at the center of the string face strung at different tensions.

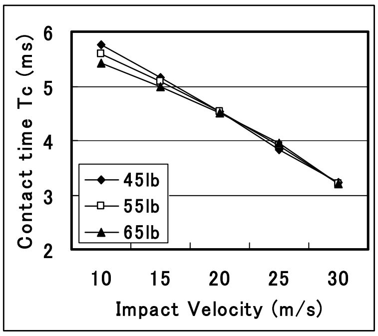

Fig.8 Contact time

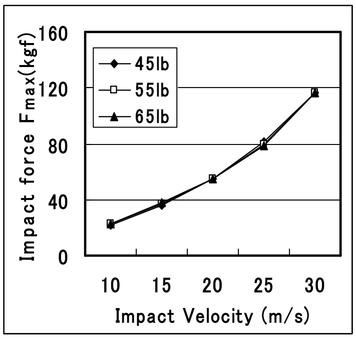

Fig.9 Max. Impact force

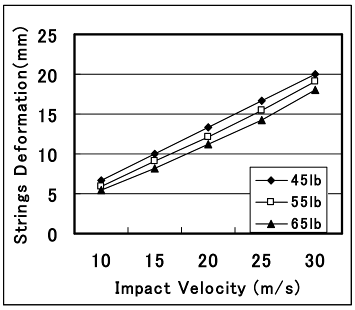

Fig.10 with large ball
String deformation

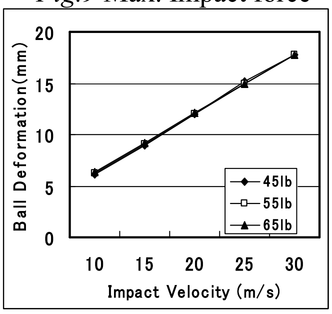

Fig.11 with large ball
Ball deformation

CONCLUSIONS

This paper described a method that can be used to predict the various factors associated with tennis impact when the impact velocity or swing model and the impact locations on the racket face are given. It enables us to predict the various factors associated with impact and performance of the racket. The predicted results for the large ball showed that the impact force is slightly smaller, the contact time is longer, the deformation of the ball and the strings are slightly smaller, and the rebound power coefficient and the post-impact velocity are almost the same, compared to the conventional normal ball. It can explain the reason why there is no big difference between the effects of large ball and the normal ball on the shock vibrations of wrist and elbow described in the separate paper (Kawazoe 2002). The details are omitted owing to limited space, the predicted results with the string tensions can also explain the experimental results that the lower tensions give no increase of the contact time above 20 m/s and do not offer less impact on the arm compared to higher tensions, which will be separately reported.

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