in THEORETICAL AND APPLIED MECHANICS JAPAN Volume 51 (2002) pp.177-187

Mechanism of High-Tech Tennis Rackets Performance

Yoshihiko KAWAZOE,

Department of mechanical Engineering, Saitama Institute of Technology, saitama

With sport equipment, engineering technology has advanced to enable manufacturers to discover and synthesize new materials and new design. There are rackets of all compositions, sizes, weights, shapes and string tensions. At the current stage, very specific designs are targeted to match the physical and technical levels of each user. However, ball and racket impact in tennis is an instantaneous phenomenon creating large deformations of ball/strings and vibrations in the racket. The problem is further complicated by the involvement of humans in the actual strokes. Therefore, there are many unknowns involved in the mechanisms explaining how the specifications and physical properties of the high-tech rackets influence the racket capabilities. The terms used in describing the performance of a tennis racket are still based on the feel of an experienced tester or a player. This paper has investigated the physical properties, predicting the coefficient of restitution, the rebound power coefficient, the post-impact ball velocity and the feel of various high-tech rackets, and estimating the overall racket performances of them using scientific method. It is based on the experimental identification of the racket dynamics and the approximate nonlinear impact analysis with a simple forehand swing model. The predicted results could explain the mechanism of high-tech rackets performance and the difference in performance between the rackets with different physical properties.

1. BACKGROUNDS AND PURPOSE OF THE STUDY

With sport equipment, engineering technology has advanced to enable manufacturers to discover and synthesize new materials and new design. There are rackets of all compositions, sizes, weights, shapes and string tensions. At the current stage, very specific designs are targeted to match the physical and technical levels of each user. However, ball and racket impact in tennis is an instantaneous phenomenon creating large deformations of ball/strings and vibrations in the racket. The problem is further complicated by the involvement of humans in the actual strokes. Therefore, there are many unknowns involved in the mechanisms explaining how the specifications and physical properties of the racket frame influence the racket capabilities.

Traditional wooden frame tennis rackets were used for a century, but they have evolved through several technology advancement over the past 30 years. In 1967 steel was introduced and aluminum soon followed in 1968. These materials made the racket frame stiffer and stronger. Composite materials made their debut in 1974 but did not become popular until 6-8 years later. Rackets, which were produced with composite materials in the seventies, were made by compression molding a prepeg tube over an expanding foam core. This created large amount of voids and inconsistencies in the laminate, which reduced stiffness, strength, and increased frame weight. For the next 20 years, composite materials and processing continued to improve the stiffness of the racket while reducing the overall weight. [1)]

The most evolutionary advancement in tennis rackets was in 1976 when the "oversize racket" was introduced with a head size of 110 square inches compared to the traditional head size of 68 square inches. The next revolution in tennis rackets was in 1987 when the "wide body racket" increased the frame cross sectional height from the traditional 19 mm beam to over 30 mm to increase bending and torsional stiffness.

The wide body racket became lighter and very popular for the recreational market, but has soften in recent years due to mainly decreased participation of recreational tennis players. Tour professionals are reluctant to use the very stiff rackets. The latest revolution in tennis rackets is longer length. First introduced in 1995, these "long body rackets" were originally introduced at length 1-2 inches longer than traditional rackets. Longer tennis rackets uses the lightest graphite fiber/epoxy prepegs for maneuverability. In the two years since their introduction (in 1997), longer rackets have captured over 40 % of the tennis racket market in the United States. However, people who used to play with traditional-length rackets seem to be going back to them. [1), 2)] Despite all the innovation in racket design, traditional racket types are still very popular.

The terms used in describing the performance of a tennis racket are still based on the feel of an experienced_tester or a player. Accordingly, there are many unknowns in the relationship between the performance estimated by a player and the physical properties of a tennis racket.

This paper has investigated the physical properties, predicting the coefficient-of-restitution (COR), the rebound power coefficient, the post-impact ball velocity and the feel of various high-tech rackets, and estimating the overall racket performances of them including the feel using scientific method. It would explain the mechanism of high-tech rackets performance and the difference in performance between the rackets with different physical properties. It is based on the experimental identification of the racket dynamics and the approximate nonlinear impact analysis[3)-14)] with a simple forehand swing model.

2. PREDICTION OF COEFFICIENT OF RESTITUTION BETEEEN BALL AND RACKET

2.1 MAIN FACTORS ASSOCIATED WITH ENERGY LOSS

2.1.1 Nonlinear restoring force characteristics of a ball and strings and a composed ball/strings system

Figure 1 shows the test for obtaining the applied force- deformation curves schematically, where the ball was deformed between two flat surfaces as shown in (a) and the ball plus strings were deformed with a racket head clamped as shown in (b). The results for the ball and racket strung at a tension of 246 N (55 lbs) are shown in Fig.2. According to the pictures of a racket being struck by a ball, it seems that the ball deforms only at the side, which contact to the strings.

Assuming that a ball with concentrated mass deforms only at the side in contact with the strings [7)], the curves of restoring force F_B vs. ball deformation, restoring force F_G vs. strings deformation, and the restoring force F_{GB} vs. deformation of the composed ball/strings system are obtained from Fig.2 as shown in Fig.3. These restoring characteristics are determined as mathematical expressions so as to satisfy a number of experimental data using the least square method in order to be used in the impact analysis. The curves of the corresponding stiffness K_B, K_G and K_{GB} are derived as shown in Fig.4 by differentiation of the equations of restoring force with respect to deformation, respectively. The stiffness K_B of a ball, K_G of strings and K_{GB} of a composed ball/strings system exhibit the strong nonlinearity.

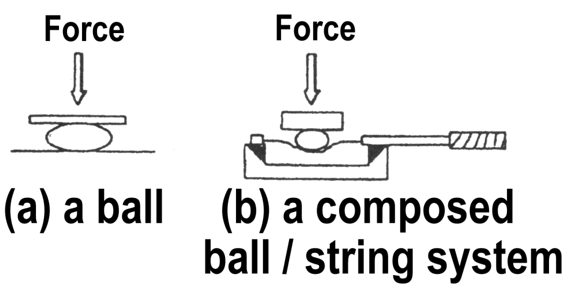


Fig.1 Illustrated applied force-Deformation test

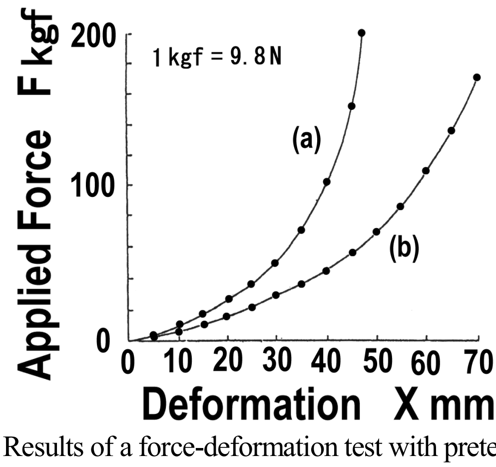


Fig.2 Results of a force-deformation test with pretension of strings 55 lbs(246 N)

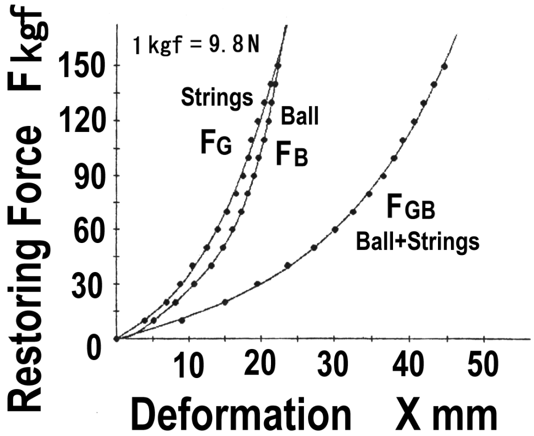


Fig.3 Restoring forces vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings [5)6)].

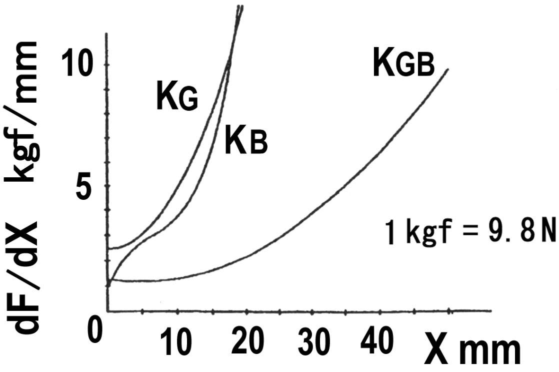


Fig.4 Stiffness vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings[5)6)].

2.1.2 Energy loss in a collision between a ball and strings

The measured coefficient of restitution versus the incident velocity when a ball strikes the rigid wall is shown in Fig.5, while the measured coefficient of restitution e_{BG}, which is abbreviated as COR, when a ball strikes the strings with a racket head clamped is shown in Fig.6. Although the COR in Fig.5 decreases with increasing incident velocity, the coefficient e_{BG} with a racket head clamped is almost independent of ball velocity and strings tension. This value of COR can be regarded as being inherent to the materials of ball and strings, showing the important role of strings. This feature is due to the nonlinear restoring force characteristics of a composed ball/strings system [4)6)]. Accordingly, the energy loss of a ball and strings due to impact can be related to the coefficient e_{BG}.

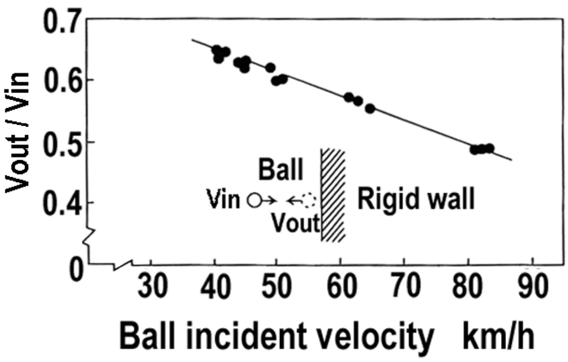


Fig.5. Measured coefficient of restitution (COR) between a ball and a rigid wall.

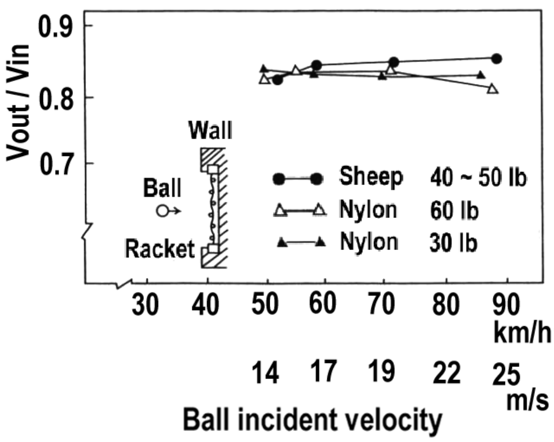


Fig.6. Measured COR between a ball and strings with frame clamped.

2.1.3 Remarks on contact time between racket and ball during impact

The result of measured contact time, which means how long the ball stays on the strings, with a normal racket and with a wide-body racket (stiffer) shows that the stiffness of the racket frame does not affect the contact time much [5)6)]. Accordingly, the masses of a ball and a racket as well as the nonlinear stiffness of a ball and strings are the main factors in the deciding of a contact time. Therefore, the contact time can be calculated using a model assuming that a ball with a concentrated mass m_B and a nonlinear spring K_B, collides with the nonlinear spring K_G of strings supported by a frame without vibration, where the measured coefficient of restitution inherent to the materials of ball-strings impact is employed as one of the sources of energy loss.

2.1.4 Support condition of a racket handle

The result of the experimental modal analysis [3),4),12)13)] showed that the fundamental vibration mode of a conventional type racket supported by a hand has two nodes being similar to the mode of a freely supported racket. The racket is assumed to be freely suspended in terms of the performance of power.

2.2 DERIVATION OF APPROXIMATE IMPACT FORCE AND CONTACT TIME

The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location and a center of gravity are given.

In case the vibration of the racket frame is neglected, the momentum equation and the coefficient restitution e_{BG} give the post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse could be described as the following equation, where m_B is the mass of a ball, M_r is the reduced mass of a racket at the hitting location, and (V_{BO} - V_{Ro}) is the pre-impact velocity.

$$\int F(t)\,dt = m_B V_{Bo} - m_B V_B = (V_{BO} - V_{Ro})(1+e_{BG})m_B/(1+m_B/M_r). \quad (1)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of m_B , K_{GB} , and M_r according to the vibration theory, it could be obtained as

$$T_c = \pi\, m_B^{1/2}/[K_{GB}(1+m_B/M_r)]^{1/2} \quad (2)$$

In order to make the analysis simpler, the equivalent force F_{mean} can be introduced during contact time T_c , which is described as

$$\int^{Tc} F(t)\,dt = F_{mean} \cdot T_c \quad (3)$$

Thus, from Eq.(1), Eq.(2) and Eq.(3), the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity (V_{BO} - V_{Ro}) is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1+e_{BG})\, m_B^{1/2} K_{GB}^{1/2}/\pi(1+m_B/M_r)^{1/2} \quad (4)$$

On the other hand, from the least square approximated mathematical expressions of the curves shown in Fig.3 and Fig.4, F_{GB} can be expressed as the function of K_{GB} in the form

$$F_{GB} = f(K_{GB}). \quad (5)$$

From Eq.(4) and Eq.(5), K_{GB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be determined against the pre-impact velocity by using Eq.(2). Figure 7 is a comparison between the measured contact times during actual forehand strokes [15)] and the calculated ones when a ball hits the center of the strings face of a conventional type racket (360 g), showing a good agreement between them. This means that the rigidity or vibrations of racket frame does not affect much on the contact time between the ball and the strings and the assumptions are reasonable.

Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force. The mathematical expression is

$$F(t) = F_{max} \sin(\pi t / T_c) \quad (0 \leqq t \leqq T_c) \qquad (6)$$

where $F_{max} = \pi F_{mean}/2$. The fourier spectrum of Eq.(6) is represented as

$$S(f) = 2F_{max} T_c \mid \cos(\pi f T_c) \mid / [\pi \mid 1 - (2fT_c)^2 \mid] \qquad (7)$$

where f is the frequency.

Figure 8 shows the examples of the calculated shock shape during impact, where the ball strikes the center on the string face of the racket strung at 55 lb at a velocity of (a) 20 m/s and (b) 30 m/s, respectively.

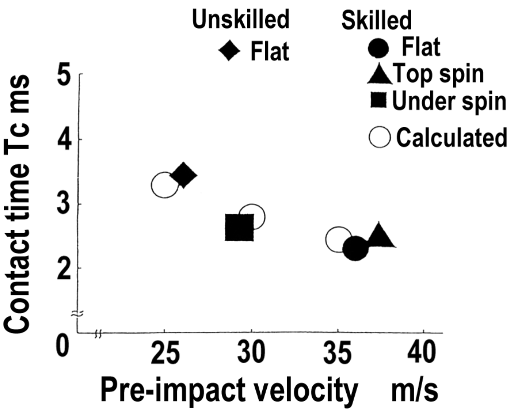


Fig.7. Comparison between the measured contact times during strokes and the calculated results.

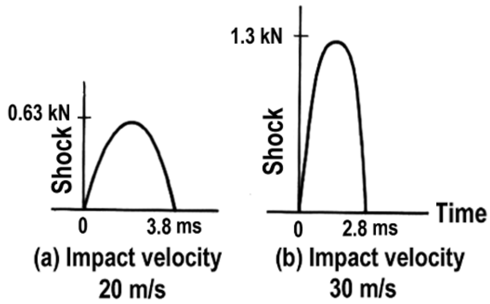


Fig.8 Calculated shock shapes when a ball strikes the center on the string face of the racket at a velocity of (a) 20 m/s and (b)30 m/s, respectively.

2.3 PREDICTION OF RACKET VIBRATIONS

The vibration characteristics of a racket can be identified using the experimental modal analysis [3),4),12),13)] and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of a racket. When the impact force $S_j(2\pi f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i is expressed as

$$X_{ijk} = r_{ijk} \; S_j(2\pi f_k) \qquad (8)$$

where r_{ijk} denotes the residue of k-th mode between arbitrary point i and j, and $S_j(2\pi f_k)$ is the impact force component of k-th frequency f_k. [7)]

Figure 9 shows the predicted vibration amplitude of a racket struck by a ball at a velocity of 30 m/s.

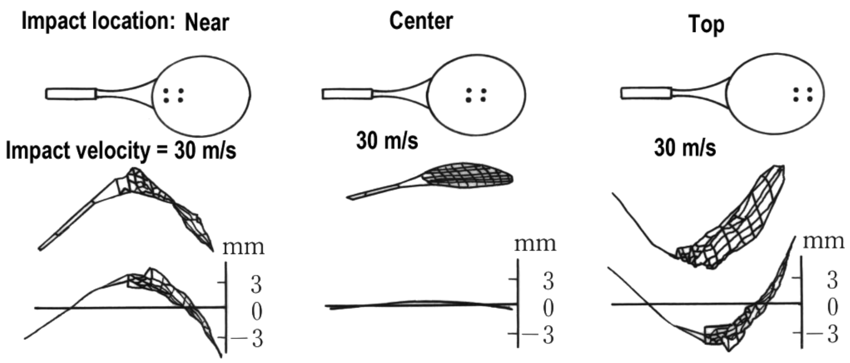


Fig.9 Predicted initial amplitude of 1st mode component of racket frame vibrations

2.4 ENERGY LOSS DUE TO RACKET VIBRATIONS INDUCED BY IMPACT

The energy loss E_1 due to the racket vibrations induced by impact can be derived from the ampli tude distribution of the vibration velocity and the mass distribution along the racket frame. If the lon gitudinal mass distribution of the racket frame is assumed to be uniform, the energy loss E_1 due to r acket vibrations can be easily derived.

2.5 DERIVATION OF COEFFICIENT OF RESTITUTION

The coefficient of restitution (COR) can be derived considering the energy loss during impact. The main sources of energy loss are E_1 due to the racket vibrations and E_2 due to the instantaneous large deformation of a ball and strings which is calculated by using the coefficient e_{BG}. If a ball collides with a racket at rest (V_{Ro} = 0), the energy loss E_2 could be easily obtained. The coefficient of restitution e_r corresponds to the total energy loss E (= $E_1 + E_2$) obtained as

$$e_r = (V_R - V_B)/V_{BO} = [1 - 2E(m_B + M_r)/(m_B M_r V_{BO}^2)]^{1/2.} \quad (9)$$

Figure 10 shows an example of predicted coefficient of restitution e_r at the longitudinal axis on the racket face when a player hits a coming ball with a velocity V_{BO} of 10 m/s, where a simple forehand ground stroke swing model [16),17)] shown in Fig.11 and denoted in Section 3 was used. It is seen that e_r of a wide body racket with very stiff composite frame is higher than that of an old wooden racket, particularly at the top of the string face.

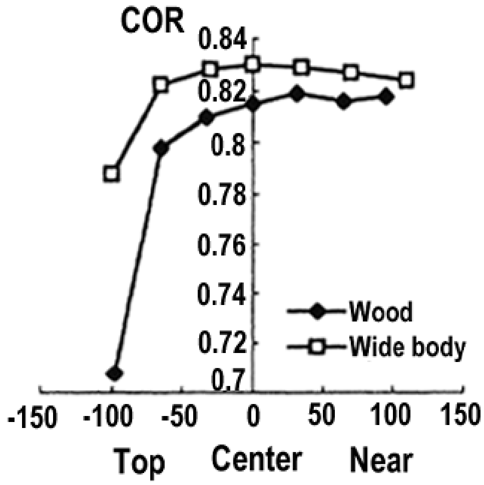


Fig.10 Examples of predicted e_r on the racket face when a player hits a ball (comparison between a wide body racket with very stiff composite frame and an old wooden racket).

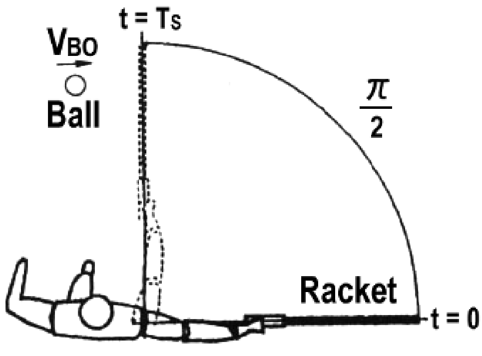


Fig.11 Simple forehand ground stroke swing model.

3. PREDICTION OF REBOUND POWER COEFFICIENT

The post-impact ball velocity V_B is represented as

$$V_B = -V_{Bo}(e_r - m_B/M_r)/(1+m_B/M_r) + V_{Ro}(1+e_r)/(1+m_B/M_r) \quad (10)$$

Accordingly, the ratio e of rebound velocity against the incident velocity of a ball when a ball strikes the freely suspended racket (V_{Ro} = 0) is written as Eq.(11). We call this coefficient e the rebound power coefficient. The rebound power coefficient e is often used to estimate the rebound power performance of a racket experimentally in the laboratory.

$$e = -V_B / V_{BO} = (e_r - m_B/M_r)/(1 + m_B/M_r) \quad (11)$$

When a player hits a coming ball with a pre-impact racket head velocity V_{Ro}, the rebound power coefficient e can be expressed as

$$e = -(V_B - V_{Ro}) / (V_{BO} - V_{Ro}) \qquad (12)$$

Figure 12 is a comparison between the measured e and the predicted e when a ball hits a freely-suspended racket with conventional weight and weight distribution (about 30 m/s), showing a good agreement between them [5), 6)].

4. PREDICTION OF POST-IMPACT BALL VELOCITY

The power of the racket could be estimated by the post-impact ball velocity V_B when a player hits a ball. The V_B can be expressed as Eq.(13). The V_{Ro} is given by $L_X (\pi N_s / I_s)^{1/2}$, where L_X denotes the holizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque around the shoulder joint, and I_s the moment of inertia of arm/racket system around the shoulder joint [18)- 20)].

Figure 13 shows the examples of the predicted V_B at each hitting location on the racket face (V_{Ro}= 10 m/s, N_s =56.9 Nm).

$$V_B = - V_{Bo} \; e + V_{Ro} (1 + e) \qquad (13)$$

We can see the difference in sweet area in terms of racket power between a lighter weighted composite racket (Super light weight type racket: EOS100, 290 g with strings) and heavier weighted composite racket (Conventional weight type racket: EX-II, 360 g with strings).

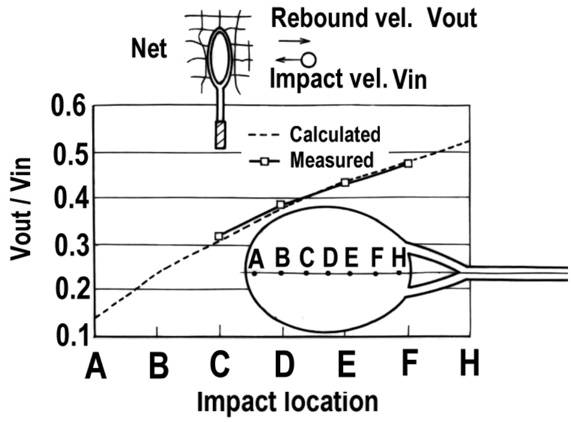


Fig.12 Comparison between the measured rebound power coefficient e and the predicted rebound power coefficient e [7)].

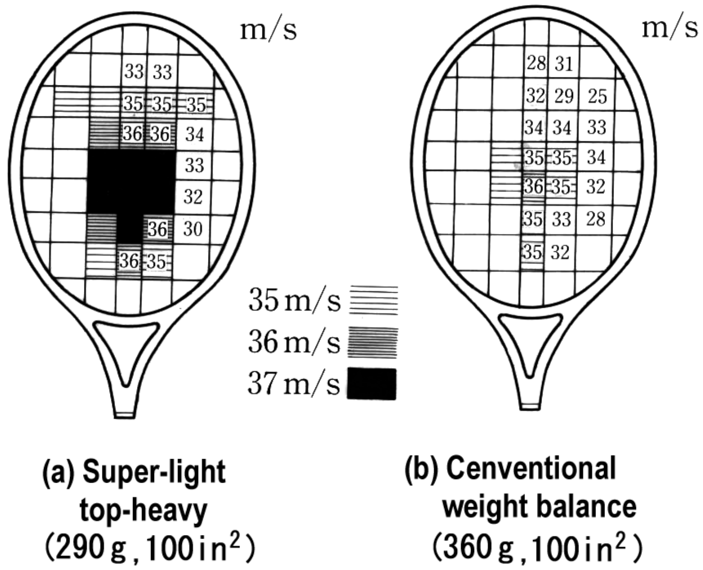


Fig.13 Examples of predicted racket power V_B (coming ball velocity V_{Bo}= 10 m/s, shoulder torque N_s =56.9 Nm).

5. REMARKS ON BALL CONTROL AND RACKET STABILITY

Control is simply being able to put the ball where desired, but it is the most difficult to analyze. Designing equipment to optimize control for all types of players is nearly impossible. Tennis players have a wide variety of styles from smooth stroking to whippy and wristy. These style differences require different equipment to optimize control. However, one characteristic required for control is stability. Stability refers to the ability at impact to maintain its swing path without deviation. Stability is also defined as the ability to resist off center hits. It is desirable to maximize stability [1].

Figure 14 illustrates the twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket. We can calculate the amount of the racket twist using racket physical properties and the predicted impact factors.

Figure 15 shows the predicted amount of the racket twist vs. distance of the impact location from the long axis, assuming that there is no friction between the hand and the racket grip because the twist torque is rather large when a racket turn about the long axis according to the calculation of impact. This analysis assumed a frontal impact between the ball and racket with no ball rotation (spin). It is the comparison between a lighter weighted racket (Super light weight racket: EOS100, 290 g with strings) and heavier weighted wide body racket (Conventional weight and stiffer racket: PROTO-02, 370 g with strings) at the topside, the center and the near side on the racket face away from the long axis. There is no twist about long axis at the topside away from the long axis, because the racket turns about the location near the grip. There is big difference in twist angles at the near side on the racket face but there is no big difference at the topside and the center away from long axis between the lighter racket and the heavier racket. The conventional heavier racket seems to be desirable in stability, but actually the hitting at the topside and the center is preferable for the groundstroker.

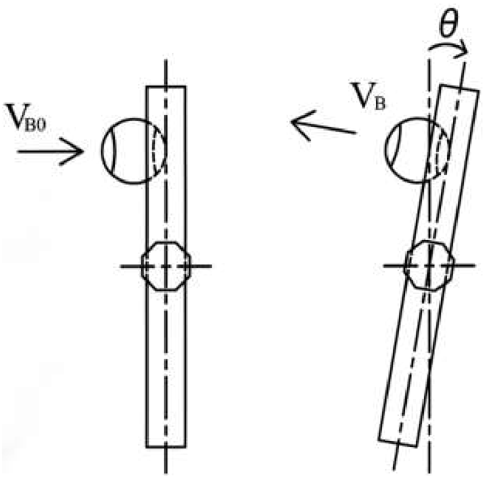
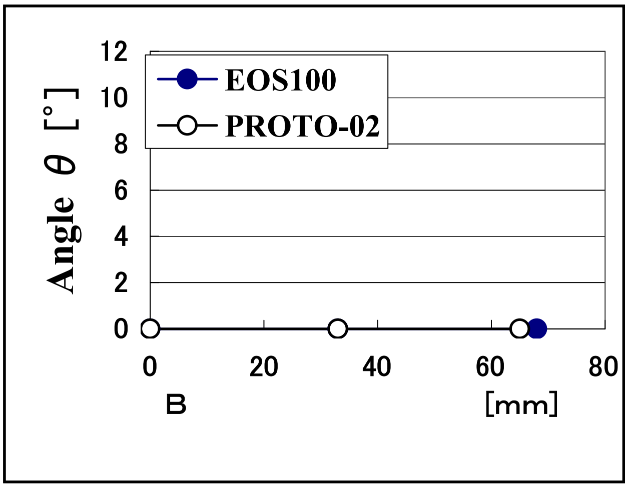
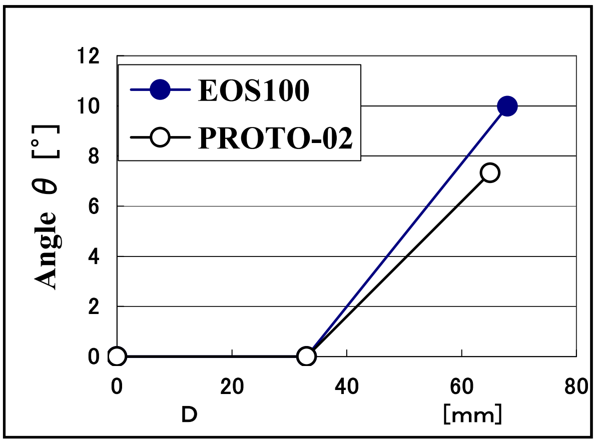
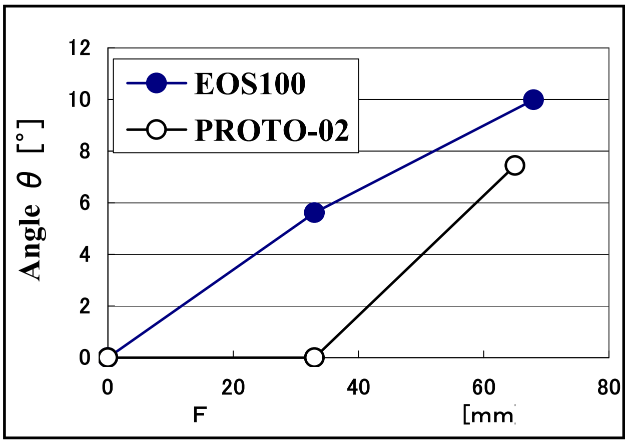


Fig.14 Twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket.



(a) Top side on the racket face





(b) Center on the racket face **(c) near side on the racket face**

Fig.15 Calculated amount of the racket twist vs. distance of the impact location from the long axis, assuming that there is no friction between the hand and the racket grip. (Super light weighted racket: EOS100, 290 g, Conventional weighted wide body racket: PROTO-02, 370 g with strings).

6. ESTIMATED TENNIS RACKETS PERFORMANCE

The terms used in describing the performance of a tennis racket are still based on the feel of an experienced tester or a player. Accordingly, there are many unknowns in the relationship between the performance estimated by a player and the physical properties of a tennis racket.

Now we can predict the various factors associated with the tennis impact, such as the coefficient-of-restitution, the rebound power coefficient, the post-impact ball velocity and the feel of various high-tech rackets except control, and also estimate the overall racket performances of them including the feel using scientific method. It would explain the mechanism of high-tech rackets performance and the difference in performance between the rackets with different physical properties.

Figure 16 shows the estimated overall performance of various types of tennis rackets available in the market shown in Table 1 when the impact velocity or swing model and the impact locations on the racket face are given. In table 1, the sign I_{GY} denotes the moment of inertia about the center of mass, the I_{GR} the moment of inertia about the grip portion 70 mm from the grip end, the I_{GX} the moment of inertia about the longitudinal axis of racket head. Rackets A, B, C, D, E, F, G, H in Table 1 correspond to the Rackets EX-2, PROTO-02, EOS100, Ex-110, EOS110, EOS120H, EOS120A, Wilson, respectively. They are produced with composite materials except Racket H (Wilson) produced with wood. The rackets EX-2, PROTO-02, Ex-110, EOS120H are conventional weight- balanced and the rackets EOS100, EOS110, EOS120A are super-lighter weight-balanced.

The performance was estimated when a ball hits the racket at the center of string face except the feel.

The sign e_r denotes the coefficient of restitution, the sign e the rebound power coefficient, the sign V_{Ro} the pre-impact racket head velocity, and the sign V_B the post-impact ball velocity.

Table 1 Physical properties of different type of tennis rackets

Racket	A	B	C	D	E	F	G	H
Face area	100 in^2	100 in^2	100 in^2	110 in^2	110 in^2	120 in^2	120 in^2	68 in^2
Total length	27 in 680 mm	27 in 680 mm	27 in 680 mm	27 in 685 mm	27 in 685 mm	27 in 685 mm	27 in 690 mm	27 in 685 mm
Mass (+Strings)	360 g	370 g	290 g	366 g	283 g	349 g	292 g	375 g
Center of Gravity	308 mm	317 mm	350 mm	325 mm	361 mm	323 mm	363 mm	335 mm
I_{GY}	13.1 g・m^2	14.0 g・m^2	11.4 g・m^2	16.9 g・m^2	12.0 g・m^2	16.0 g・m^2	14.0 g・m^2	14.8 g・m^2
I_{GR}	33.5 g・m^2	36.6 g・m^2	34.1 g・m^2	40.7 g・m^2	35.9 g・m^2	38.0 g・m^2	39.0 g・m^2	41.2 g・m^2
I_{GX}	1.29 g・m^2	1.62 g・m^2	1.12 g・m^2	1.68 g・m^2	0.99 g・m^2	2.21 g・m^2	1.78 g・m^2	0.94 g・m^2
1st freq	122 Hz	215 Hz	171 Hz	132 Hz	176 Hz	142 Hz	137 Hz	103 Hz
Strings tension	55 lbs	55 lbs	55 lbs	63 lbs	50 lbs	79 lbs	79 lbs	50 lbs
Reduced mass	170 g	196 g	175 g	220 g	183 g	205 g	206 g	188 g
Real Name	EX-II	PROTO-02	EOS100	EX110	EOS110	EOS120H	EOS120A	Wilson (wood)

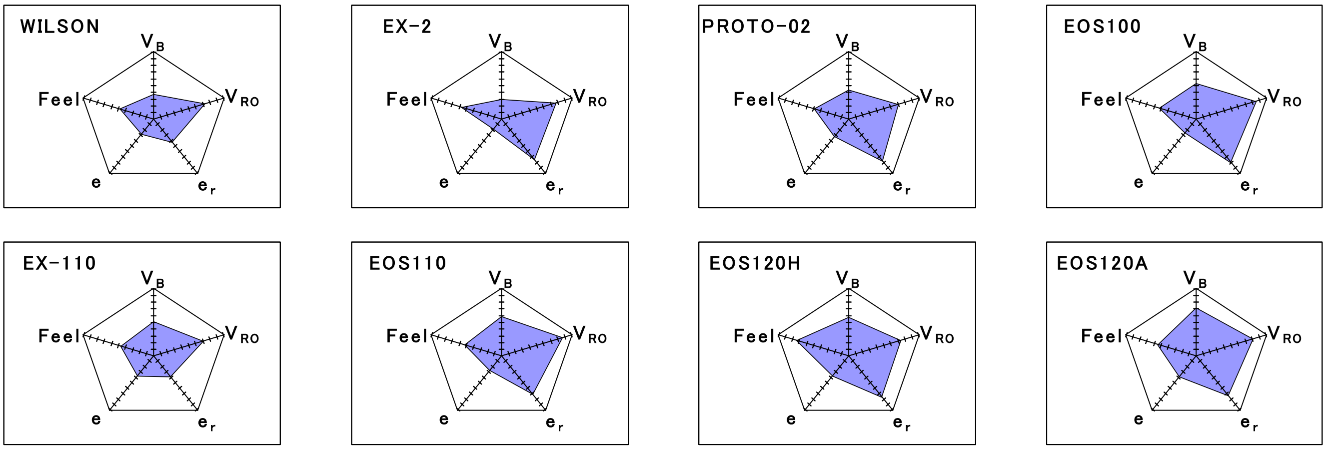
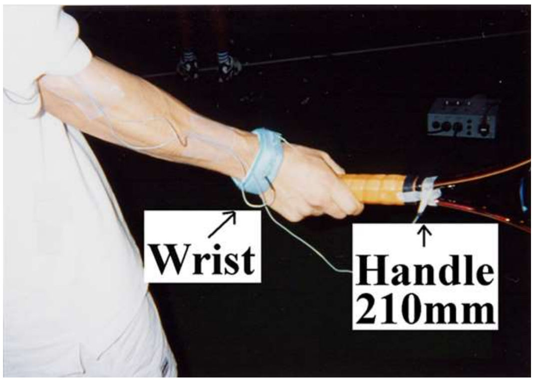


Fig.16 Estimation of rackets

The V_{Ro} is given by $L_X\,(\pi N_s / I_s)^{1/2}$, where L_X denotes the horizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque about the shoulder joint, and I_s the moment of inertia of arm/racket system about the shoulder joint. In Fig.16, N_S=56.9Nm and V_{Bo}=10m/s which are the values in the women pro-player's ground stroke rally were given. With the estimation of the performance in Fig.16, the lowest value among the rackets available in the market corresponds to 40 % and the highest value corresponds to 95 %, and the scales in Fig.16 were selected from 30 % to 100 %. The predicted performance in terms of feel was estimated by the initial peak-peak values of acceleration waveforms at the wrist joint as shown in Fig. 17 and Fig. 18, when the ball hits the top side off center on the string face [14].

The predicted results on the various rackets could explain the mechanism of different type of high-tech rackets performance and the difference in performance between the rackets with different physical properties.



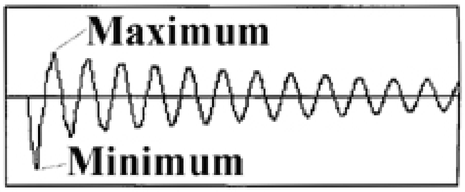


Fig.18 Peak value of shock vibrations[14].

Fig.17 Location of accelerometers at the wrist joint and the racket handle in the forehand ground stroke.

7. CONCLUSIONS

The terms used in describing the performance of a tennis racket have been based on the feel of an experienced tester or a player. Accordingly, there are many unknowns in the relationship between the performance estimated by a player and the physical properties of a tennis racket.

This paper has investigated the physical properties of various rackets, predicting the performance in terms of the coefficient of restitution, the rebound power coefficient, the post-impact ball velocity and the feel, and estimating the overall racket performances of various types of high-tech rackets using scientific method. It is based on the experimental identification of the racket dynamics and the approximate nonlinear impact analysis with a simple forehand swing model. The predicted results could explain the mechanism of high-tech rackets performance with different physical properties.

ACKNOWLEDGMENTS

The author would like to thank many students in his laboratory for their help in carrying out the study as senior students during the academic year. He would also like to thank the International Tennis Federation (ITF) for funding the research. This work was supported by a Grant-in-Aid for Science Research of the Ministry of Education, Culture, Sports, Science and Technology of Japan, and a part of this work was also supported by the High-Tech Research Center of Saitama Institute of Technology.

REFERENCES

1) Davis S. "Rackets science applied to golf", Proc. 5th Japan International SAMPE Symposium, (1997), pp.1329-1334.
2) Ashley S., "High-tech rackets hold court ", Mechanical Engineering, August (1993), pp.50-55.
3) Kawazoe,Y., "Dynamics and computer aided design of tennis racket", Proc. Int. Sympo. on Advanced Computers for Dynamics and Design'89, (1989) , pp.243-248.
4) Kawazoe,Y., "Dynamics and computer aided design of tennis racket: The Influence of Some Factors on the Mode Characteristics and a View of Optimum Design", Trans. JSME, 56-526, (1990), pp.1511-1517. (in Japanese)
5) Kawazoe,Y. (1992) Impact phenomena between racket and ball during tennis stroke, Theoretical and Applied Mechanics, Vol.41, pp.3-13.
6) Kawazoe,Y., "CAE of Tennis Racket with Impact Phenomena: Prediction of Racket Response and a View of Restitution in Racket-Ball Impact", Trans. JSME, 58-552, (1992), pp.2467-2474. (in Japanese)
7) Kawazoe,Y., Coefficient of restitution between a ball and a tennis racket, Theoretical and Applied Mechanics, Vol.42, (1993), pp.197-208.
8) Kawazoe,Y., Analysis of coefficient of restitution during a nonlinear impact between a ball and strings considering vibration modes of racket frame, Trans. JSME, 59-562, (1993), pp.1678-1685. (in Japanese)
9) Kawazoe,Y., "CAE of Tennis Racket with Impact Phenomena: The effects of String tension on a Ball-Racket Impact", Trans. JSME, 59-558, (1993), pp.521-528. (in Japanese)
10) Kawazoe,Y., Effects of String Pre-tension on Impact Between Ball and Racket in Tennis, Theoretical and Applied Mechanics, Vol.43, (1994), pp.223-232.
11) Kawazoe,Y., Computer Aided Prediction of the Vibration and Rebound Velocity Characteristics of Tennis Rackets with Various Physical Properties, Science and Racket Sports, (1994) , pp.134 -139. E & FN SPON.
12) Kawazoe, Y. , Experimental Identification of Hand-held Tennis Racket Characteristics and Prediction of Rebound Ball Velocity at Impact, Theoretical and Applied Mechanics, Vol.46, (1997), pp.165-176.
13) Kawazoe,Y. and Tomosue, R., "Impact Vibrations of a Racket Handle and the Wrist Joint in Tennis", Trans. JSME, 59-560, (1993), pp.1001-1008. (in Japanese)
14) Kawazoe, Y., "Mechanism of Tennis Racket Performance in terms of Feel", Theoretical and Applied Mechanics, Vol.49, (2000), pp.11-19.
15) Nagata,A., "Analysis of tennis movement", J. J. Sports Sci., 2-4, (1983), pp.245-259. (in Japanese)
16) Kawazoe, Y. and Kanda, Y., Analysis of Impact Phenomena in a Tennis Ball-Racket System (Effects of Frame Vibrations and Optimum Racket Design), JSME International Journal, Series C, Vol.40, No.1, (1997), pp.9-16.
17) Kawazoe, Y. and Kanda, Y., Analysis of Impact Phenomena with a Ball-Racket System in Tennis (Effects of Frame Mass Distribution on the Rebound Velocity of a Racket), Trans. JSME, 59-566, (1993), pp.3108-3115. (in Japanese)
18) Kawazoe, Y. and Tomosue, R., "Sweet Area Prediction of Tennis Rackets Estimated by Ball Post-Impact Velocity (comparison between two rackets with different frame mass distributions)", Proc. Symp. on Sports Engineering, (1996), pp.55-59. Japan Society of Mechanical Engineers, Tokyo. (in Japanese)
19) Kawazoe, Y. and Tomosue, R., "Prediction of a Sweet Area on a Racket Face in a Tennis Impact (Restitution Coefficient, Rebound Power Coefficient and Ball Post-Impact Velocity)", Trans. JSME, 64-623, (1998), pp.2382-2388. (in Japanese)
20) Kawazoe, Y. and Sakurai, T, "Effect of Player's Arm on the Rebound Power Coefficient and the Power of a Tennis Racket, ", Proc. Symp. on Sports Engineering, (1999), pp.207-211. Japan Society of Mechanical Engineers, (in Japanese)