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Dynamic Analysis of Sweet Spot of a Tennis Racket in Terms of Feel

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Abstract

This paper investigated the feel or comfort of the arm or hand in an impact. It derived the shock vibrations of the wrist joint caused by the impact when a player hits flat forehand drive. Furthermore, it predicted the sweet spots of a racket in terms of feel. It was based on the identification of the racket-arm system and the predicted coefficient of restitution between a racket and a ball. The predicted waveform of the shock vibrations at the wrist joint agreed fairly well with the measured ones during actual forehand stroke by a player, showing that the shock vibrations of the wrist joint are transmitted from an impulse at the impact location and the several vibrations mode components of the racket. The predicted results could also explain the difference in sweet spots of a racket in terms of feel or comfort of rackets with different weight and weight balance.This study enables us to predict the various factors associated with impact and performance of the various racket.

1. Introduction

At the current stage, the performance of tennis racket is based on the feel of an experienced tester or a player. However, the optimum racket depends on the physical and technical levels of each user. Accordingly, there are many unknowns regarding the relationship between the performance estimated by a player and the physical properties of a tennis racket.

This paper investigated the feel or comfort of the arm or hand in an impact. It derives the shock vibrations of the wrist joint caused by the impact when a player hits flat forehand drive. Furthermore, it predicts the sweet spots of a racket in terms of feel or comfort of rackets with different weight and weight balance. It is based on the identification of the racket-arm system and the predicted coefficient of restitution between a racket and a ball.

2. Prediction of Shock Vibrations at the Racket Handle and the Wrist Joint

2.1 Derivation of Impact Shock Forces of an Arm Joint System [1[-[3]

Figure 1 shows the situation of experiment where a male tournament player hits flat forehand drive and

Fig.1 Experiment where a male player hits flat forehand drive.

Fig.2 shows the locations of the attached accelerometers on the wrist joint and the elbow joint in the experiment. In this experiment an accelerometer was also attached at 210 mm distance from the grip end on the racket handle as shown in Fig.3.

Figure 4 shows an impact model for the prediction of shock force transmitted to the arm joint from a racket. The impact force S_0 at P_0 causes a shock force S_1 on the player's hand P_1, a shock force S_2 on his elbow P_2, and finally a shock force S_3 on the player's shoulder P_3 during the impact at which the player hits the ball with his racket. Since the intensity of the impulse decreases with the distance from the point of impact with the ball, it can be assumed that the shoulder does not basically alter its velocity, despite the presence of the shock force S_3. Furthermore, the shock forces S_0, S_1, S_2, and S_3 are assumed to be one order of magnitude higher than the other forces in play during the impact; consequently the gravity force and muscular action are not taken into account in the impact. In other words, we consider the racket to be freely hinged to the forearm of the player, the forearm being freely hinged to the arm and the arm freely hinged to the player's body. This schematization only refers to the interval lasting no longer than one hundredth of a second: both before and afterwards, in the absence of shock forces S_0 , S_1 ,S_2 , and S_3, all the movements depend on the intensity of the muscular forces and gravity forces in play.

Let the forearm length be $a_a = P_1P_2$, with a mass m' to which the mass m'' of the hand is added: consequently, the total mass of the forearm is equal to $m_a = m' + m''$ concentrated at P_1, and the distance of the center of mass from the elbow be b_a. Moreover, let the moment of inertia around the elbow P_2 be J_a, the mass of the arm, with a length of $a_b = P_2P_3$ be m_b, the distance of the center of mass from the shoulder P_3 be $b_b = G_3 P_3$, while the moment of inertia with respect to the

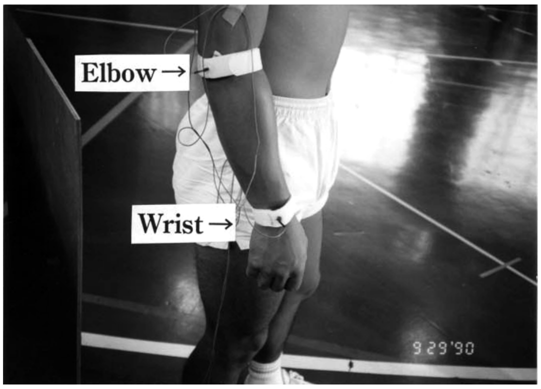


Fig.2 Accelerometers attached at the wrist and the elbow.

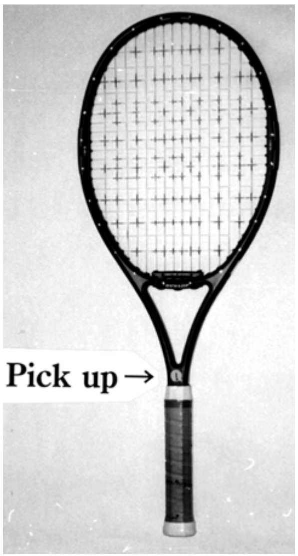


Fig.3 Accelerometer attached at a racket handle

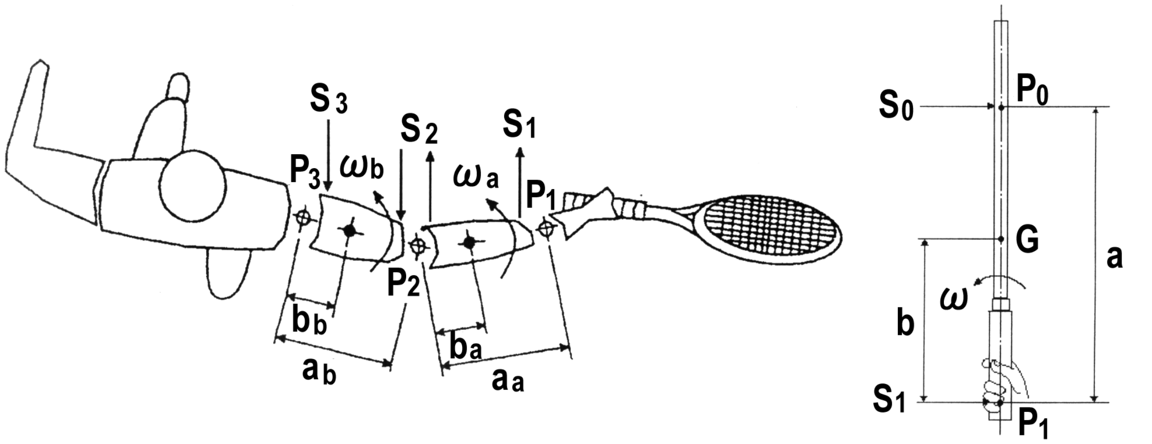


Fig.4 Impact model for the prediction of the shock force transmitted to the arm joints from a racket.

shoulder P_3 be J_b. We can derive the following relationship between the acceleration dV_1/dt of point P_1 and the shock force S_1 from the equations of motion for the forearm P_1P_2 and a few calculation steps.

$$dV_1/dt = [\mu_a a_a^2/J_a - \chi_a a_b^2/J_b]S_1 \quad (1)$$

where

$$\mu_a = [1+(m_a a_b^2/J_b)(1 - b_a/a_a)]/[1+(m_a a_b^2/J_b)(1 - m_a b_a^2/J_a)] \quad (2)$$

$$\chi_a = (m_a a_a b_a/J_a - 1)/[1 + (m_a a_b^2/J_b)(1 - m_b b_a^2/J_a)] \quad (3)$$

i.e. by assuming

$$M_H = 1/[\mu_a a_a^2/J_a - \chi_a a_b^2/J_b] \quad (4)$$

finally we have the acceleration A_{nv} at the grip portion and the wrist joint as

$$A_{nv} = dV_1/dt = S_1 / M_H \quad (5)$$

From Eq.(5) we can deduce that the inertia effect of the arm and the forearm can be attributed to a mass M_H concentrated in the hand; therefore the analysis of impact between ball and racket can be carried out by assuming that the racket is free in space, as long as the mass M_H is applied at point P_1 of the hand grip.

If the impact force S_0 between a ball and the racket is given when the ball hits the racket, the shock force S_1 can be obtained with a few steps as

$$S_1 = S_0(M_R ab/J - 1)/[1 + (M_R/M_H)(1 - M_R b^2/J)] \quad (6),$$

where we let the mass of the racket be M_R, the distance between the grip location on the handle and the impact location on the string face be a, the distance between the grip location on the handle and the center of mass of the racket be b, and the moment of inertia with respect to the articulation P_1 of the hand be J.

2.2 Derivation of Restitution Coefficient, Impact Force and the Contact Time at the Impact [4]-[7]

The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum when the moment of inertia and the distance between an impact location and a center of gravity are given. The reduced mass M_r at the impact location with a racket-arm system can be derived as

$$M_r = 1/[1/(M_R + M_H) + c^2/I_G] = (M_R + M_H)I_G/[I_G + (M_R + M_H)c^2] \quad (7)$$

where

$$c = c_o + (L_{Go} - L_H)M_H/(M_R + M_H) \quad (8)$$

$$I_G = I_{Go} + M_R \triangle G^2 + M_H(L_{Go} - L_H - \triangle G)^2 \quad (9)$$

$$\triangle G = (L_{Go} - L_H)M_H/(M_R + M_H) \quad (10)$$

and L_{Go} denotes the distance between the center of mass and the grip end of the racket, I_{Go} the moment of inertia with respect to the center of gravity of the racket, c_o the distance between the center of gravity and the impact location of the racket, and L_H the distance of the point P_1 of the hand grip from the grip end The moment of inertia with respect to the center of gravity and the distance of the center of gravity from the impact location of the racket-arm system are indicated by I_G and c, respectively.

In case the vibration of the racket frame is neglected, the post-impact velocity V_B of a ball and V_R of a racket head at the impact location are derived using the momentum equation and the measured coefficient restitution e_{BG} with a ball striking the a racket head clamped. The impulse at impact between ball and racket could also be obtained.

It is assumed that the contact time T_c during impact is half the natural period of a whole system composed of the mass m_B of a ball, the equivalent stiffness K_{GB} of ball/strings, and the reduced mass M_r of the racket. If we introduce the equivalent force F_{mean} during contact time T_c, the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity is derived. On the other hand, from the measured restoring force characteristics of a ball and strings, the restoring force can be expressed as a function of K_{GB}. Thus, the parameters K_{GB} and F_{mean} against the pre-impact velocity can be obtained. Accordingly the contact time T_C can also be determined against the pre-impact velocity. Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is the more likely impulse waveform.

The vibration characteristics of a racket can be identified using the experimental modal analysis and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of the racket. When the impact force component of k-th mode frequency f_k in the frequency region applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i can be derived using the residue r_{ijk} of k-th mode between arbitrary point i and j.

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along a racket frame when an impact location on the string face and the impact velocity are given.

The coefficient of restitution e_r (COR) can be derived considering the energy loss $\triangle E$ during impact [6]. The main sources of energy

loss is $\angle E_1$ due to racket vibrations as well as $\angle E_2$ due to the instantaneous large deformation of a ball and strings corresponding to the coefficient e_{BG}.
Furthermore, the force-time curve of impact between a ball and a racket considering the vibrations of a racket frame can be derived as

$$S_0(t)= S_{0max}\sin(\pi t/T_c) \quad (0\leqq t \leqq T_c) \qquad (11)$$

where

$$S_{0\,max}= \pi F_{mean}/2 = (\pi/(2T_c))(V_{BO}-V_{Ro})(1+e_r)m_B/(1+m_B/M_r). \qquad (12)$$

The contact time T_c during impact can be determined against the pre-impact velocity ($V_{BO}-V_{Ro}$) between a ball and a racket assuming the contact time to be half the natural period of a whole system composed of the mass m_B of a ball, the equivalent stiffness K_{GB} of ball/strings, and the reduced mass M_r of the racket.

2.3 Shock Accelerations Transmitted to the Wrist joint From a Racket

The shock acceleration A_{nv}(t) at the handgrip considering the equivalent mass M_H of the arm system can be represented as

$$A_{nv}(t) = S_0(t)[\,1/(M_R+M_H) - (a/I_G)X\,] \qquad (13)$$

where X denotes the distance between the center of mass of racket-arm system and the location of hand grip, a the distance between the center of mass of racket-arm system and the impact location of the racket, I_G the moment of inertia with respect to the center of mass of racket-arm system, respectively. The maximum shock force $S_{1\,max}$ transmitted to a wrist joint corresponds to the maximum impact force $S_{0\,max}$.

2.4 Shock Vibrations at the Grip

The natural frequency of racket frame drops slightly and the position of the node on the handle shifts somewhat to the held position for the hand-held racket compared to the freely suspended racket. Furthermore the damping of frame vibrations is remarkably larger for the hand-held racket compared to the freely suspended racket. Nevertheless, there is no big difference in the initial amplitude distributions of a racket frame between the hand-held racket and the freely suspended racket.
The vibration acceleration component $A_{ij,k}(t)$ of k-th mode at the location i of handgrip is represented as

$$A_{ij,k}(t) = -(2\pi f_k)^2 r_{ijk} S_{0j}(2\pi f_k) exp(-2\pi f_k \zeta_k t) \, sin(2\pi f_k t) \qquad (14)$$

where j denotes the impact location between ball and racket on the string face, ζ_k the damping ratio of k-th mode, $S_{0j}(2\pi f_k)$ the fourier spectrum of Eq.(11). The summation of Eq.(13) and Eq.(14) represents the shock vibrations at the handgrip.

2.5 Shock Vibrations at the Wrist Joint during Forehand Stroke

Figure 5 shows the center of gravity in a racket-arm system. Figure 6 is the result of the predicted accelerations of the shock vibrations of a wrist joint

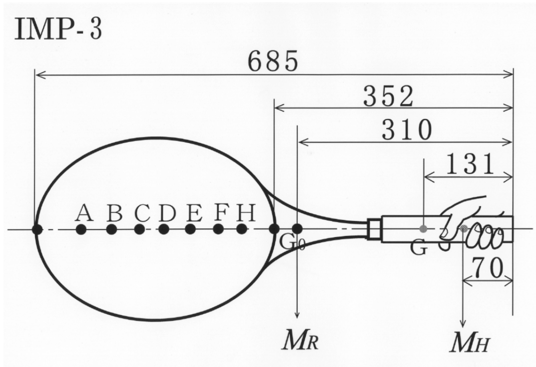


Fig.5 Center of gravity in a racket-arm system

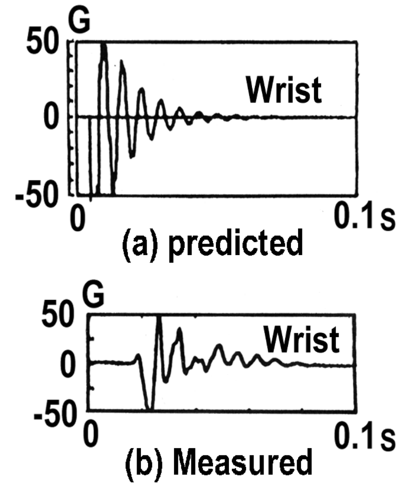


Fig. 6 Predicted shock vibrations of a wrist joint compared with the experimental.

compared with the experimental ones when a ball is struck at the topside of the racket face. This racket is made of 75 % graphite, 20 % fiberglass and 5% others, with 685 mm of total length, 100 in^2 of face area, 342 g of mass including string mass, 310 mm of the center of mass from grip end, 14.2 gm^2 of moment of inertia about the center of racket mass, 60 lbs. of strings tension. The center of mass of racket-arm system shifts to the location of 131 mm from the grip end. The first largest peak in Fig.6 was caused by the initial shock and vibrations during the impact, followed by the residual vibrations of a racket frame. The shock vibrations are composed of the impact shock component and the vibration components, and each component has its own time history and magnitude depending on the impact velocity, impact location, grip location of racket handle and the physical properties of a racket. The damping ratio of a hand-held racket during actual impact has been estimated as about 2.5 times that of the one identified by the experimental modal analysis with small vibration amplitude. Furthermore, the damping of the waveform at the wrist joint has been 3 times that at the grip portion of the racket handle. The predicted waveform of the shock vibrations with the wrist joint agrees fairly well with the measured one during actual forehand stroke by a player.

Table 1 physical properties

Racket	EOS100	PROTO-02
Total length	680 mm	680 mm
Face area	606 cm^2	606 cm^2
Mass	290 g	370 g
Center of gravity from grip end	350 mm	317 mm
Moment of inertia I_{GY} about Y axis	34.1 gm^2	36.6 gm^2
Moment of inertia I_{GX} about X axis	1.121 gm^2	1.620 gm^2
1st frequency	171 Hz	215 Hz
Strings tension	55 lb	55 lb

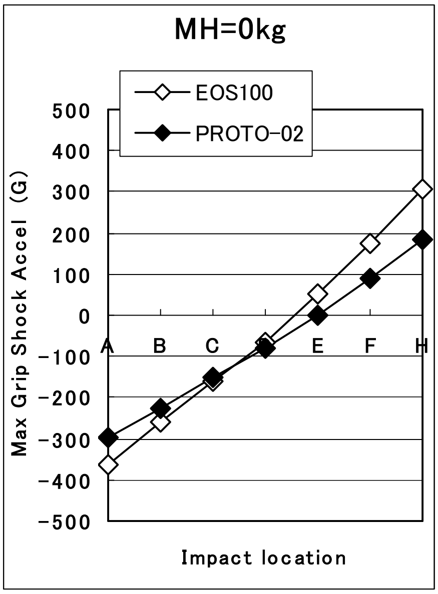


Fig.8 Predicted Maximum shock acceleration at the grip of freely suspended racket.

3. Estimation of the Sweet Spots in terms of Feel for Tennis Rackets having different weight and Weight Balance

Now we can predict the shock vibrations at the grip and the wrist joint during the impact when impact velocity or swing model besides the impact locations on the racket face are given. Furthermore we can estimate the sweet spots in terms of feel for the various rackets with different physical properties.

Figure 8 shows a comparison of the predicted maximum shock accelerations at the racket grips (70 mm from the grip end)

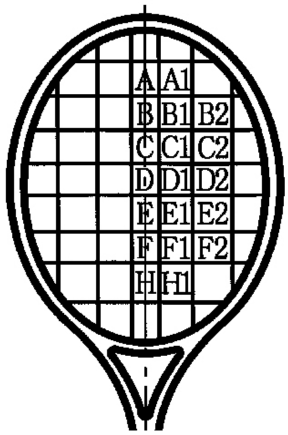


Fig.9 Hitting locations on the string face.

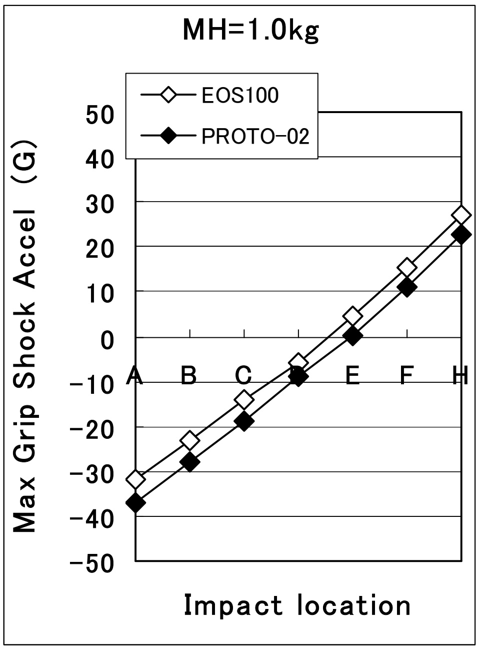
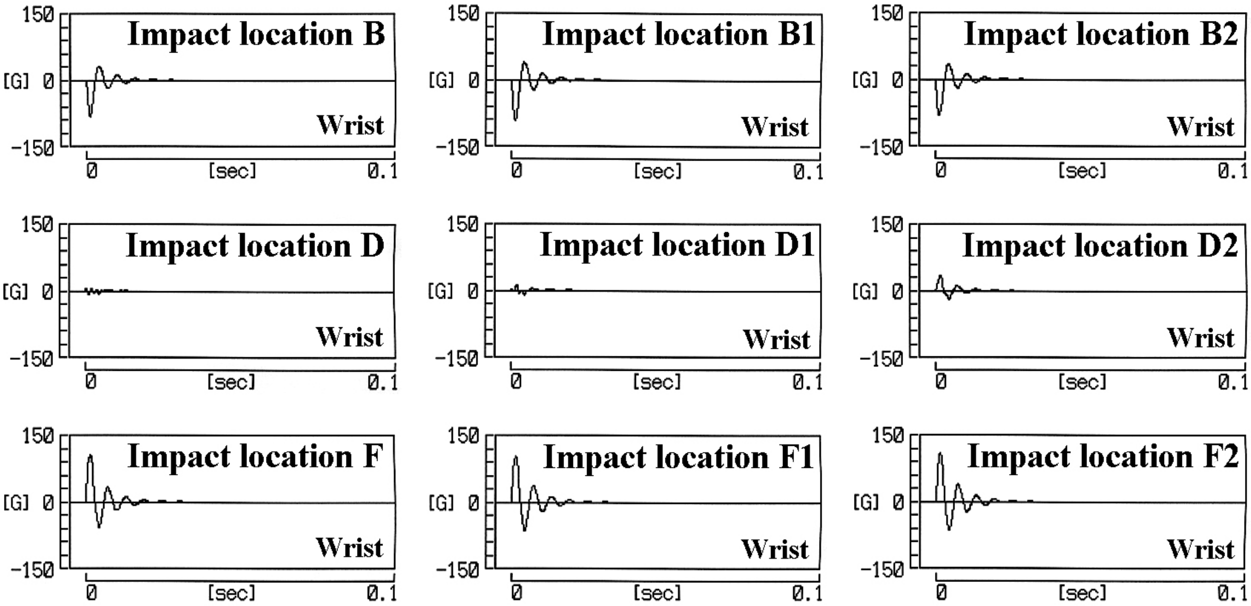


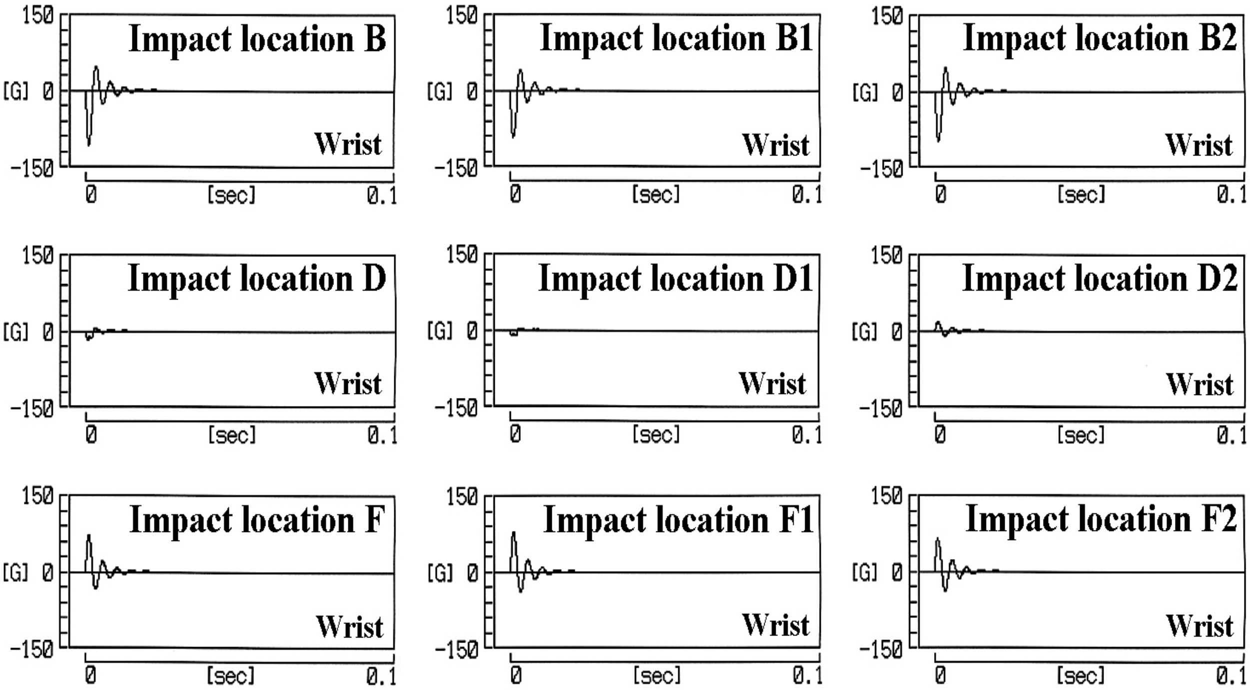
Fig.10 Predicted Maximum shock accelerations at the grip of hand-held racket

between the super-light weight racket (EOS100: 290 g) and the conventional weight and weight balanced racket (PROTO-02: 370 g) when a ball strikes the freely suspended rackets.

Table 1 shows the specifications and the main physical properties of the two rackets, where the sign I_{GY} denotes the moment of inertia about the center of mass and the sign I_{GX}

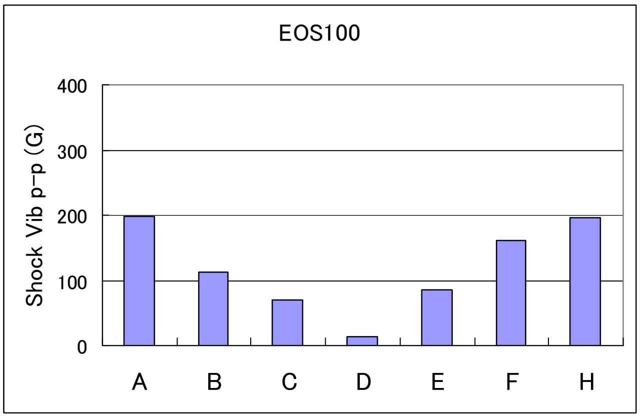


(a) Racket EOS100 (Super light)

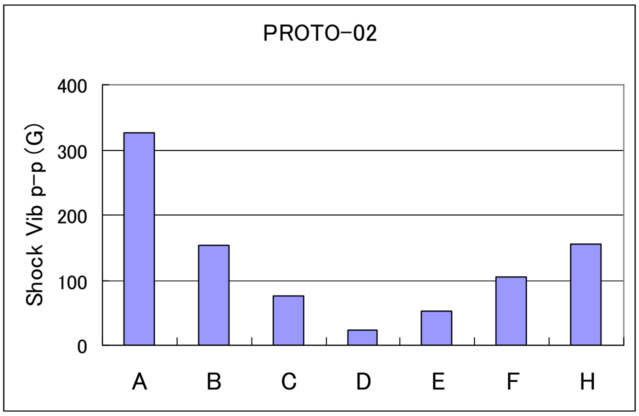


(b) Racket Proto-02 (Conventional)

Fig.11 Predicted waveform of the shock vibrations of the player's wrist joint (impact velocity: 30 m/s).

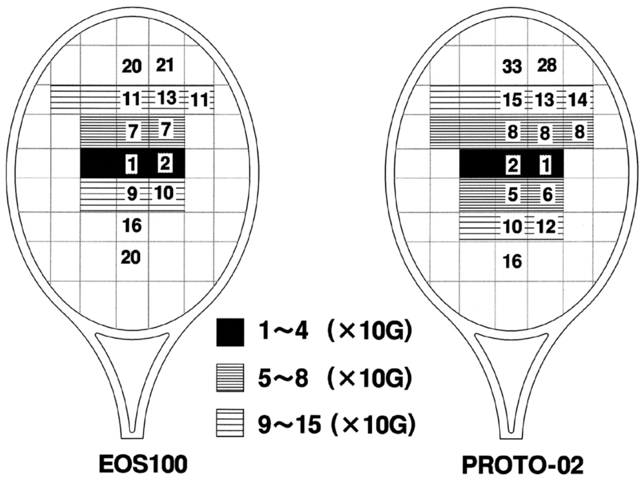


(a) Racket EOS100 (Super light)



(b) Racket Proto-02 (Conventional)

Fig.12 Comparison of the predicted shock vibrations between the super-light weight racket (EOS100: 290 g) and conventional weight and weight balanced racket (PROTO-02: 370 g) estimated by the initial peak-peak value of wrist acceleration waveforms when the ball strikes the each hitting location along the longitudinal axis on the string face of hand-held rackets.



(a) Super light (b) Conventional

Fig.13 Comparison of the predicted sweet area in terms of feel or comfort between the super-light weight racket (EOS100: 290 g) and conventional weight and weight balanced racket (PROTO-02: 370 g) estimated by the initial peak-peak value of wrist acceleration waveforms, where the ball hits the string face at each hitting location on the racket face.

the moment of inertia about the longitudinal axis of racket head. Figure 9 shows the impact locations on the string face of the racket.

accelerations at the grips of hand-held rackets between them.

The equivalent mass of an arm is estimated as M_H= 1.0 kg [1]-[3]. The equivalent mass of an arm reduces remarkably the maximum shock acceleration of a racket grip on comparing with Fig.8.

Figure 11 shows a comparison of the predicted waveforms of shock vibrations at the wrist joint between the super-light weight racket (290 g).

Figure 12 shows a comparison of the predicted shock vibrations between the super-light weight racket (EOS100: 290 g) and conventional weight and weight balanced racket (PROTO-02: 370 g) estimated by the initial peak-peak value of wrist acceleration waveforms when the ball strikes the each hitting location along the longitudinal axis on the string face of hand-held rackets.

Figure 13 shows a comparison of the predicted sweet area in terms of feel or comfort of tennis rackets estimated by the initial peak-peak value of wrist acceleration waveforms, where the ball hits the string face at each hitting location on the racket face.

The predicted results could explain the difference in racket performance in terms of feel or comfort between the rackets with different physical properties. It is seen that the sweet area of both rackets in terms of feel or comfort lies near the center of the string face, but the shock vibrations of the wrist using the conventional weight and weight balanced racket is smaller at the near side and the t near しで higher than that of a super-light weight racket anywhere on the string face.

4. CONCLUSIONS

At the current stage, the terms used in describing the performance of a tennis racket are based on the feel of an experienced tester or a player.

This paper investigated the feel or comfort of the arm or hand in an impact. It derived the shock vibrations of the wrist joint caused by the impact when a player hits flat forehand drive. Furthermore, it predicted the sweet spots of a racket in terms of feel. It was based on the identification of the racket-arm system and the predicted coefficient of restitution between a racket and a ball.

The predicted waveform of the shock vibrations at the wrist joint agreed fairly well with the measured ones during actual forehand stroke by a player, showing that the shock vibrations of the wrist joint are transmitted from an impulse at the impact location and the several vibrations mode components of the racket. The predicted results could also explain the difference in sweet spots of a racket in terms of feel or comfort of rackets with different weight and weight balance.

This study enables us to predict the various factors associated with impact and performance of the various racket.

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