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Experimental Identification of a Hand-Held Tennis Racket and Prediction of Rebound Ball Velocity in an Impact

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It is difficult to construct a theoretical model of the hand holding of a tennis racket upon the interaction of a tennis ball with a racket. The coefficient of restitution during the impact is related to a main source of the energy loss caused by the collision, such as due to the impact between a ball and strings, the rotation of the racket and the vibration of the racket frame. This study investigates conditions of a hand-held racket and the mechanism of the impact, providing a simple impact model for the estimation of the coefficient of restitution of a hand-held racket, on the basis of an idea that the contact duration is determined by the natural period of the entire system which comprises the mass of a ball, the nonlinear stiffness of the ball and strings, and the reduced mass of racket at the impact point on the string face. By using an estimated impact force and contact time to an experimentally identified vibration model of a hand-held racket, the vibration of the racket during the impact can be predicted. Furthermore, the predicted restitution coefficient of a hand-held racket is compared with that of a freely supported racket.

1. INTRODUCTION

The implementation of material composites has led to increased flexibility in the design and production of sporting goods. The increased freedom has enabled manufacturers to tailor goods to match the different physical characteristics and techniques of users. If the racket dynamics from the time of ball contact at a certain speed and angle until the time of separation (contact time), and the resulting ball velocity and spin can be determined, the racket can be evaluated with respect to its physical characteristics such as mass distribution, rigidity distribution, racket head size, and string tension.

However, impact between a ball and racket is an instantaneous event in which the contact time decreases with increasing impact velocity (approximately 3 – 6ms). The event involves a nonlinear phenomenon accompanied by frame vibration and large deformations in the ball and strings. In addition, the human system is involved in each stroke, further complicating the model. As a result, many points regarding ball and racket dynamics at the time of impact and racket characteristics remain unclear [1)-17)].

Because engineering can contribute little to the current design of rackets, these points are evaluated by players who use the equipment extensively.

A previous report [15)] examined the close relationship between energy loss and the coefficient of restitution during impact. In addition, a method for predicting the coefficient of restitution that accounts for energy loss due to the substantial ball and string deformation and energy loss due to racket frame vibration was proposed. That study demonstrated that the predicted rebound velocity of a ball that impacts a racket hanging in mid-air was in good agreement with the experimental values obtained using a free hanging racket.

When racket rebound characteristics are predicted in actual play, the contribution of the handle portion (commonly called the grip), which is the point of connection between the player and the racket, becomes unclear as a boundary parameter.

The present study investigates the influence of a hand-held racket grip on racket vibration amplitude at the time of impact and on rebound characteristics. Based on the results, a method for predicting rebound characteristics when the grip is held is proposed.

Ball rotation (spin), which occurs when a ball impacts a racket at a given angle, is an important consideration with respect to ball control characteristics. However, the present report focuses on the rebound mechanism, and therefore only frontal impact is considered in order to simplify the analysis. In order to simplify the terminology, a racket hanging in mid-air without the handle held will be called "freely supported" or "grip released" and a racket handle held by hand will be called "hand-held" throughout this report.

2. EXPERIMENTAL MODAL ANALYSIS OF HAND SUPPORTED RACKET

First, the racket vibration characteristics were investigated using the experimental modal analysis for a racket placed horizontally on a soft sponge (corresponding to mid-air hanging, freely supported racket) and a racket with the handle held firmly.

Figure 1 shows the impact points of a test racket during experimental modal analysis using the impulse hammer method [7, 18-20]. The black circle represents the attachment point of the accelerometer. The test racket, including the strings, had a mass of 360 g, a total length of 680 mm, a ball striking surface area of 628 cm^2 {97 in^2}, and a center of gravity at 308 mm from the grip end. The inertial moments around the center of gravity and the grip (70mm from the end) were 13.1 gm^2 and 33.5 gm^2, respectively. The string tension was 246 N {55 lbs} and the materials used were fiberglass, graphite, ceramic, and Kevlar.

Figure 2 shows an example of the frequency response functions (compliances) and coherences found using the average of 10 measurements for impact force and acceleration with an impact on the racket frame (accelerometer position). Figure 2(a) shows the results obtained when the racket was placed horizontally on a soft sponge during impact (freely supported) and Figure 2(b) shows the results obtained when the racket was held firmly (hand-held). The sampling period was 200 μs and the frequency resolution was 4.8 Hz. Plastic was used in the hammer tip [19]. Figure 3 shows the results obtained when the impact was at the center of the string surface. The primary mode of the string membrane showed a remarkably pronounced peak. When analyzing frame impact, care must be given in order to avoid a double strike. However, the impact force spectrum stretches to a high frequency. On the other hand, in the case of a string surface impact, the contact time between the hammer and strings is extended, and there the impact force spectrum does not reach a high frequency.

Figure 4 shows the vibration modal analysis results for freely supported and hand-held rackets. Three-dimensional figures are also included, showing the racket from an angle and from the side. The points that cross the horizontal axis correspond to nodes of the frame vibration mode. Figure 5 shows the nodal lines on the string surface. The boundaries between the black and white regions represent nodes. The racket in the figure shows that starting from a low frequency, the modes proceed from 2 node frame bending (primary vibration mode of a racket) through 3 node bending, 2 node twisting, and a one-dimensional membrane mode for the strings. When the strings are not present, the natural frequencies of the frame increase by approximately 4%. However, strings have almost no effect on the position of the frame vibration mode nodes [7]. In addition, the 4% increase is equal to the proportion of mass held by the strings relative to the total racket mass. Figure 6 shows the results of a detailed

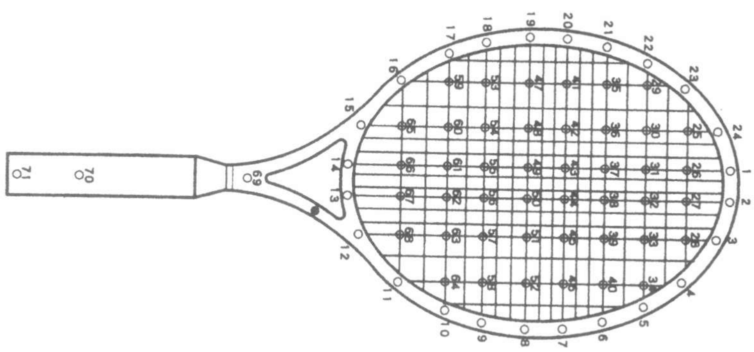


Fig.1 Racket impact Points and acceleration measurement positions.

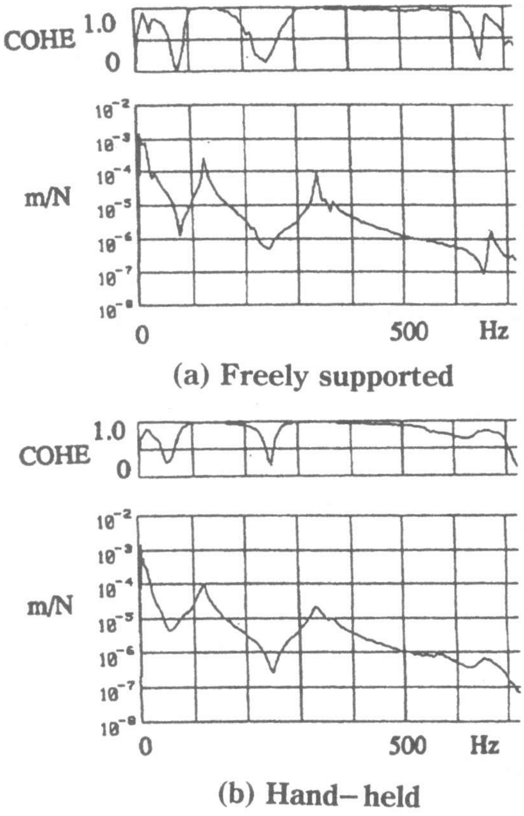


Fig.2 Frequency response function during frame impact.

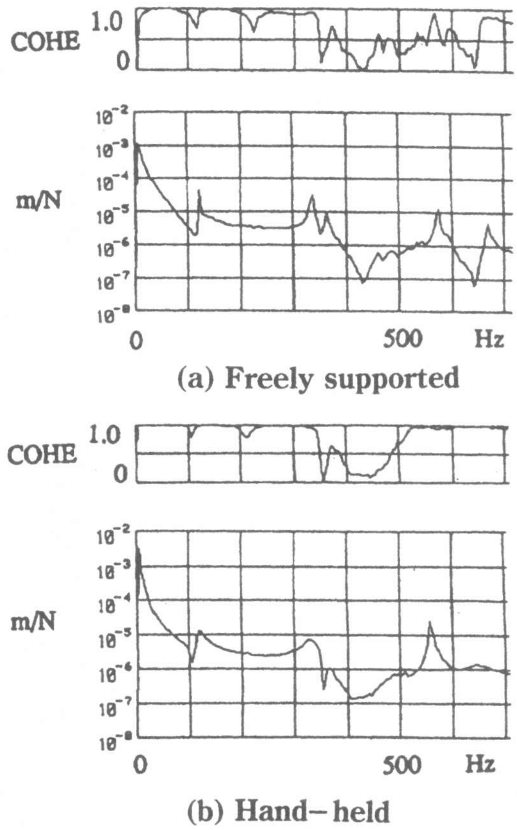


Fig.3 Frequency response function during impact at string center.

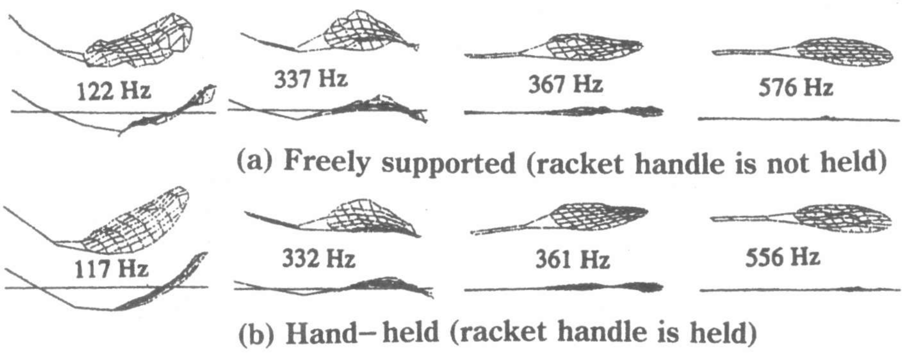


Fig.4 Racket vibration modes.

investigation of the positions of the primary vibration mode nodes on freely supported and hand-held rackets. At a low frequency, the cantilever mode [20] that appear when the grip is secured in a vise does not appear with the hand-held racket. The resulting vibration modes are similar to the freely supported racket modes. In addition, although the frequency drops slightly for the hand-held racket compared to the freely supported racket, the positions of nodes on the string surface are nearly identical.

With a 2 node mode, primary vibration, the position of the node on the handle for the hand-held racket changes somewhat from the held position. In addition, the frame vibration damping for the hand-held racket was larger than that for the freely supported racket. These phenomena should be investigated in the future as they clearly play a role when vibration transmitted to the hand is considered.

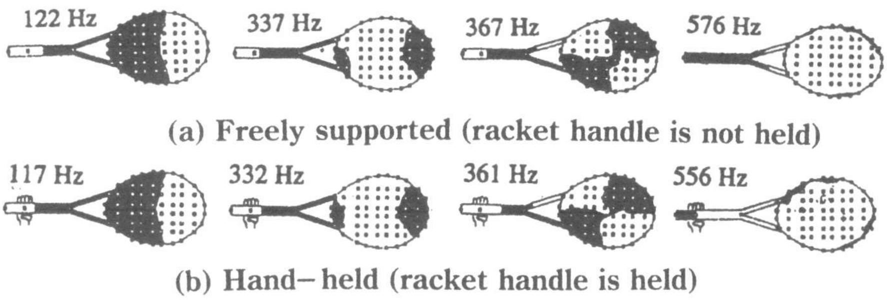


Fig.5 Racket vibration modes (Node positions on string surface)

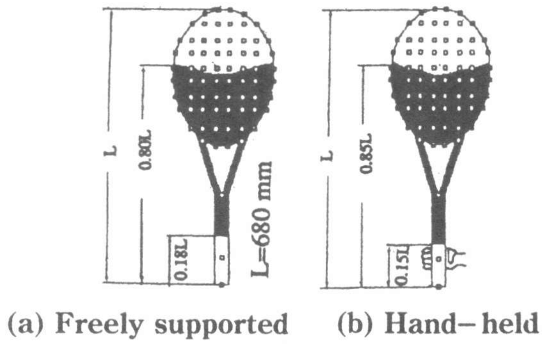


Fig.6 Node positions for primary racket vibration mode.

3. IMPACT MODEL OF HAND-HELD RACKET AND BALL

3.1 Measurement of Contact Time Using Hammer Strike

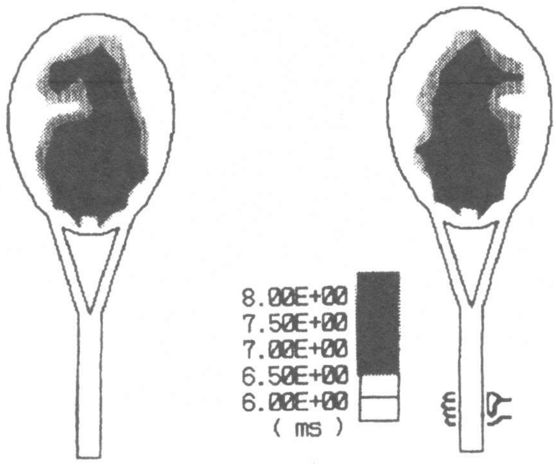
In the case of impact sports, the contact time (time between ball contact and separation) is an important parameter for determining the specific characteristics of the sport. The behaviors of the ball after impact are determined during this short contact period.

In order to investigate the effect of a hand-held grip on the contact time of a racket, the string surfaces of hand-held and freely supported rackets were struck using an impulse hammer (impact points 25 – 68 in Figure 1). The contact time was then determined from the impact force waveform (similar to a sine half-wave). Figure 7(a) shows the contact time for a freely supported racket and Fig. 7(b) shows that for a hand-held racket. The contour lines are also shown in these figures.

The contact time did not change greatly when the grip was held. Thus, the contact time appears to remain relatively constant upon impact, regardless of the grip condition.

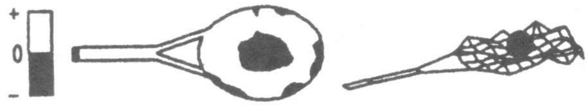
3.2 Experimental Modal Analysis of Compound Ball and Racket System

Next, an experimental modal analysis was applied to a compound ball and racket system in which the ball was adhered to the center of the racket string surface. Figure 8(a) shows the primary vibration modes of the compound system. The ball and frame show mutual pushing and pulling modes through the strings (black and white regions move opposite each other). The 1/2 cycle for the primary vibration mode of this ball and racket system appear to correspond to the contact time during impact. Figure 8(b) shows the impact force waveform when a point, slightly away from the contact point of the ball on the string surface, is struck by an impulse hammer. Figure 8(c) shows the response acceleration waveform of the frame neck, measured at the same time as the impact force in Fig. 8(b). Figure 9(a) shows the compliance of the racket alone (without the ball adhering), whereas Fig. 9(b) shows the compliance of the compound racket and ball system. A new natural frequency appears at 103Hz in Fi. 9(b). This

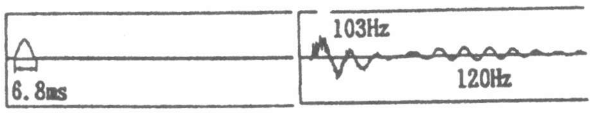


(a) Freely supported (b) Hand-held

Fig.7 Contact times Measured using Hammer

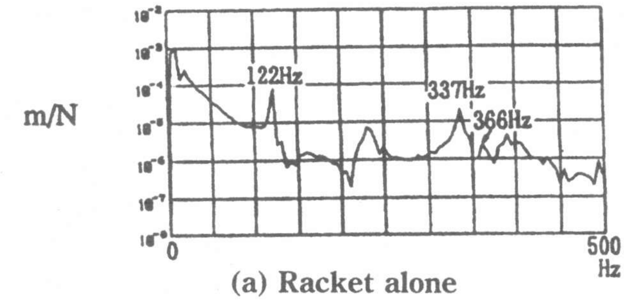


(a) Primary vibration mode of compound system

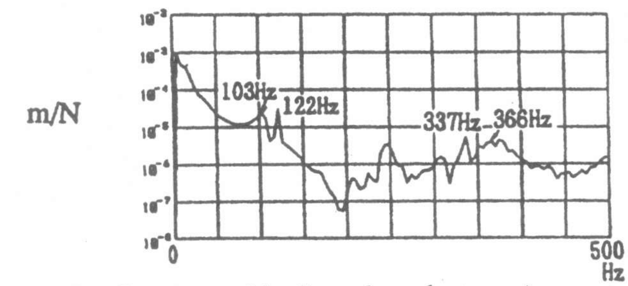


(b) String surface impact force (c) Frame response

Fig.8 Measurement of primary vibration mode in compound ball and racket system



(a) Racket alone



(b) Compound ball and racket system

Fig.9 Frequency response functions of compound ball and racket system peak is not found in Fig. 9(a). The natural frequencies of the frame do not change even when the ball is attached. The vibration seen in the initial period of the frame response in Fig. 8(c) is the 103Hz vibration mode when the ball moves back and force in the direction opposite the frame through the string surface. After this mode has decreased (after approximately 2 cycles), the mode becomes a 2 node bending vibration of the frame at approximately 120Hz.

Similarly, a racket with almost identical mass, but approximately 4 times the frame rigidity was tested by the same protocol (data not shown). Similar results were obtained with respect to the reciprocating motion of the ball on the string surface. Thus, in the case of a ball adhering to the center of the string surface, the vibration associated with lifting and dropping of the ball does not appear to be affected by frame rigidity. These results are in agreement with a previous study [11] that reported that contact time hardly affected by frame rigidity during actual impact tests using a ball and racket.

Thus, frame rigidity and frame vibration effects can be considered negligible with respect to contact time, and therefore only mass distribution need to be considered. As is discussed in Section 3.5, when contact time is considered to be independent of frame rigidity, approximate nonlinear impact analysis can be performed.

3.3 Considerations Associated with Contact Time between Ball and Hand– Held Racket

Consider a ball that directly impacts a stationary racket with a rigid frame at a velocity of V_{BO}. Assume that the racket rotates around the grip position. The distance between the racket support point (grip position) and the impact point is denoted L_A, the inertial moment around the racket support point is denoted I_A ($=M_R k_A{}^2$, M_R : racket mass, k_A: rotation radius), and the angular velocity immediately a fter impact is called ω . Under these conditions, the following equation can be expressed if the law of angular momentum conservation is applied relative to the grip support point.

$$m_B V_{BO} L_A = m_B V_B L_A + I_A \omega \quad \cdots\cdots (1)$$

Based on the geometric relationships, the velocity V_R of the racket impact point immediately after impact can be expressed as follows:

$$V_R = \omega L_A \quad \cdots\cdots (2)$$

When ω is removed from equations (1) and (2), the following equation can be written:

$$m_B V_{BO} = m_B V_B + (I_A / L_A^2) V_R \quad \cdots\cdots (3)$$

When the relationship in equation (4) is applied, equation (3) becomes equation (5).

$$M_r = I_A / L_A^2 = M_R k_A^2 / L_A^2 \quad \cdots\cdots (4)$$

$$m_B V_{BO} = m_B V_B + M_r V_R \quad \cdots\cdots (5)$$

The symbol M_r expressed in equation (4) refers to the reduced mass at the impact point for a racket rotating around the grip position.

When the racket rotates around a support point 70mm from the grip end (center of the standard hand –holding position), the results for the reduced mass differ from that of freely supported racket at the impact near the ends of the string surface [11], [17] . However, impact in the normal strike area shows little difference.

In Section 3.2, the contact time during impact was assumed to correspond to 1/2 the primary vibration cycle for a ball and racket system with the ball in contact with the strings. Under this assumption, the contact time at impact is assumed to depend on the reduced mass of the racket. It is seen that the contact times did not differ remarkably between the hand– held and freely supported rackets in Section 3.1 because the reduced mass of the rackets did not differ.

3.4 One Degree of Freedom Ball and String Model and Outline of Impact Analysis

As shown in the high– speed video picture in Fig.10, in general, only the side of the ball contacting the strings is deformed during impact. Based on this finding, a ball with a point mass concentrated in

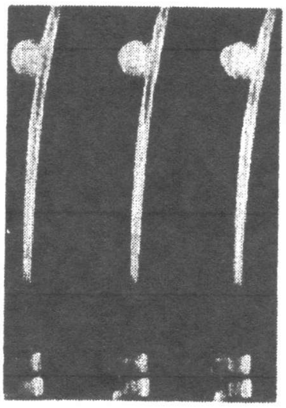


Fig.10 Ball deformation during impact with racket

the center was assumed to deform on only one side during string contact. Applying this assumption and using actual measurements of load and deformation of a ball and ball and string compound system, the ball restoring force characteristic F_B, string restoring force characteristic F_G, and compound ball and string system restoring force characteristic F_{GB} were determined by the least squares method. Next, the derivatives were found for the equations related to displacement X of restoring forces F_G, F_B, and F_{GB} and nonlinear spring rigidity K_G, K_B, and K_{GB} were calculated as a function of displacement [11].

The energy loss during impact between the ball and strings was considered to be the rebound coefficient e_{BG} (ball rebound velocity/incident velocity) under impact conditions in which the racket head is fixed around the strings. Within the impact velocity range during normal play, the actual e_{BG} is almost independent of string tension and impact velocity. Furthermore, frame material does not appear to influence this value [11], [17].

Based on considerations outlined in Section 3.3, the grip portion of a hand-held grip was assumed to be a pin joint. When the reduced mass M_r at the racket impact point is introduced and the above rebound coefficient e_{BG} is used, the system can be analyzed using the nonlinear spring K_{GB} of the compound ball and string system as the impact between a ball with mass m_B in the center and a racket with reduced mass M_r [11] [17]. In other words, the ball and string impact factors accompanying large deformations(nonlinear deformation) can be analyzed using an approximation with a one degree of freedom model and assuming a rigid frame, as described in Section 3.5. Impact forces found by approximation analysis are then applied to the vibration model of a racket identified experimentally as a linear system with small vibration amplitudes, as shown in Section 4.

3.5 Approximate Nonlinear Analysis of Impact Force and Contact Time

Consider an impact between a racket with a hand-supported grip and a ball with mass m_B. The strike velocity of racket head is V_{R0} and the pre-impact ball velocity is V_{B0}. Using the rebound coefficient e_{BG} (corresponding to the energy loss of the ball and strings), when the law of momentum conservation is applied to the impact between the ball and racket, the ball velocity V_B after impact can be determined if the racket frame vibration is ignored. The impulse during impact can be calculated from the following equation:

$$\int F(t)dt = m_B V_{Bo} - m_B V_B \quad \cdots\cdots (6)$$

The contact time can be expressed using equation (7) when 1/2 the natural period of the system, consisting of ball mass m_B, compound ball and string system spring rigidity K_{GB}, and reduced racket mass M_r at the strike point, is assumed to be the contact time.

$$T_C = \pi m_B^{1/2} / [K_{GB}(1 + m_B/M_r)]^{1/2} \quad \cdots\cdots (7)$$

However, the spring rigidity K_{GB} of the compound ball and string system is nonlinear and changes over time during impact, depending on the extent of deformation and impact velocity [11), 15)].

To simplify the analysis, the change in impact force over time is discussed later. In this section, the equivalent impact force F_{MEAN}, which is constant during impact time T_C, is introduced. Thus, the left part of equation (6) can be expressed as:

$$\int F(t)dt = F_{MEAN} \cdot T_C \quad \cdots\cdots\cdots\cdots (8)$$

From equations (6),(7), and (8), the relationship between F_{MEAN} and the corresponding spring rigidity K_{GB} can be determined when the impact velocity is given as follows.

$$F_{MEAN} = (V_{BO} - V_{RO})(1 + e_{BG}) m_B^{1/2} \times K_{GB}^{1/2} / \{\pi(1 + m_B/M_T)^{1/2}\} \quad \cdots\cdots\cdots (9)$$

In this calculation, the ball is assumed to impact a stationary racket and the racket velocity before impact is $V_{RO} = 0$.

On the other hand, based on the restoring force characteristics determined experimentally, the equivalent force can be expressed as a function of the compound spring rigidity as shown below [11)].

$$F_{MEAN} = \text{func.}(K_{GB}) \quad \cdots\cdots\cdots\cdots (10)$$

The equivalent impact force F_{MEAN} and K_{GB} are determined from equations (9) and (10) when the impact force is given, and the contact time is obtained from equation (7).

Figures 11 and 12 show the calculated results obtained for F_{MEAN} and T_C when the ball velocity was varied and the impact occurred at the top of the string surface, in the middle, and at the near.

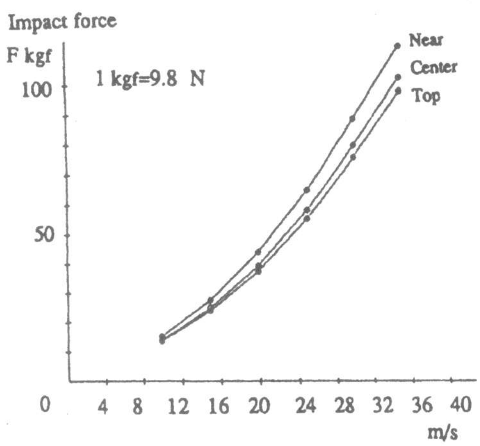


Fig.11 Impact velocity and calculated equivalent impact force (Hand-held racket)

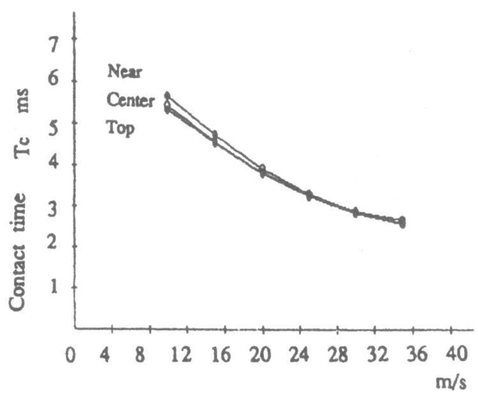


Fig.12 Impact velocity and calculated contact time(Hand-held racket)

3.6 Approximation of Impact Force Wave form Acting on Racket Surface

The impact force waveform (variation over time) affects racket vibration. Therefore, under constant impulse condition (equation (8)), the impact force waveform is approximated using the half sine pulse shown at the left in Figure 13. This waveform is similar to the actual measured waveform and is expressed as follows:

$$F(t) = F_{MAX} \sin(\pi t / T_C) \quad (0 \leqq t \leqq T_C) \quad \cdots\cdots (11)$$

The integration ranges from zero to T_C and the maximum F_{MAX} of the impact force is $F_{MAX} = \pi F_{MEAN}/2$ [15)].

4. VIBRATION AMPLITUDE AND REBOUND COEFFICIENT PREDICTION FOR HAND SUPPORTED RACKET FRAME

4.1 Vibration Amplitude for Hand–Supported Racket Frame

The Fourier spectrum $S(f)$ (f: vibration frequency Hz) is shown at the right in Fig. 13. Within the frequency range, when the impact force component at a point j on the racket surface corresponding to the kth vibration mode of the racket (natural frequency $\omega_k = 2\pi f_k$) is expressed as $S_j(\omega_k)$, the response amplitude component X_{ijk} of the kth mode at point i on the racket can be approximated by equation [22]

$$X_{ijk} = r_{ijk} * S_j(\omega_k) = \frac{r_{i0k} \cdot r_{0jk}}{r_{00k}} * S_j(\omega_k) \quad \cdots (12)$$

Here r_{ijk} is the residue of the kth vibration mode between some point i and some point j. The symbol r_{i0k} is the known residue, obtained from the experimental modal analysis, between impact point i on the racket and fixed response point(reference point) 0. The symbol $r_{0jk}(=r_{j0k})$ is the known residue between strike point j and response point 0, and, r_{00k} is the known drive point residue at the reference point (pick up).

Figure 14 shows the estimated vibration displacement amplitude (only for primary vibration mode) immediately after ball impact on a freely supported racket (a) and a hand–held racket (b). These examples demonstrate an impact at the very end of the string surface at impact velocities of 10, 20, and 30 m/s. Although the amplitude at the end of the frame is somewhat larger for the hand–held racket compared to the freely supported racket, the vibration displacement amplitude distribution is similar in both cases. Furthermore, the impact at the center of the string surface resulted in small vibration amplitudes for both hand–held and freely supported rackets.

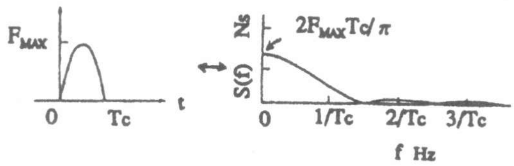


Fig.13 Approximation of ball and racket impact force wave form and fourier spectrum.

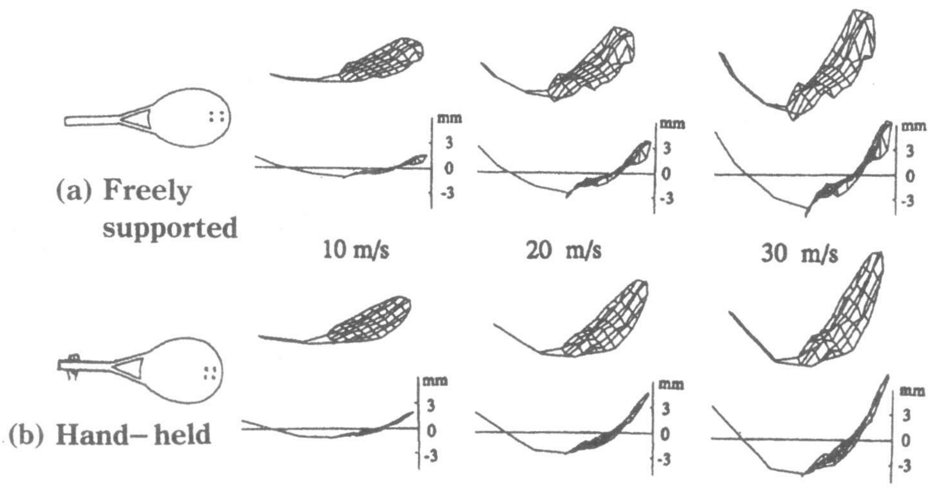


Fig.14 Predicted vibration displacement for racket frames during ball racket impact (primary vibration mode only).

4.2 Rebound Capabilities of Hand-Held Racket

Rebound capabilities can be estimated by considering the energy loss due to large instantaneous deformation of the ball and string, energy loss due to rotational rigid body movement of the racket, and energy loss due to frame vibration. The estimated rebound coefficient for a ball impacting a freely supported racket are in good agreement with the experimental results obtained using a suspended racket [15].

Figure 15 shows the estimated rebound capabilities when a ball with a velocity of 20 m/s impacts a freely supported racket and a hand-held stationary racket. The rebound capability in this case is expressed as a ratio between ball rebound velocity and incident velocity. The black squares represent the hand-held racket and the black diamonds represent the freely supported racket. Although the open squares represent a hand-held racket, the vibration characteristics were identified under the condition of freely supported. The black and open squares almost completely overlapped. The hand-held racket (black squares) demonstrated slightly better rebound characteristics than the freely supported racket (black diamonds) at the end of the string surface.

In Fig.15, the lack of difference between the black and open squares demonstrates that vibration characteristics concerning rebound capability used to evaluate energy loss due to frame vibration for a hand-held racket can be determined by substituting the freely supported racket. Thus, the problems associated with holding onto the racket during the hammering experiment can be eliminated. In addition, the improved rebound at the end of the string surface demonstrates that the reduced mass at that location is greater for the hand-held racket than the freely supported racket.

In an actual swing, the racket velocity differs according to the impact location [11]. Furthermore, the ball velocity after impact is also related to stroke type and play style. Therefore, physical body movements must also be considered during analysis.

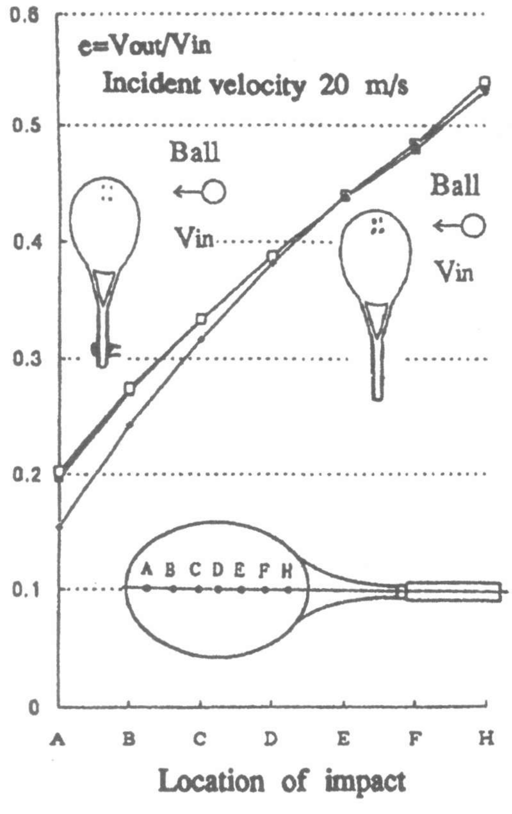


Fig.15 Predicted ball rebound velocity(comparison between hand-held and freely supported rackets)

5. CONCLUSIONS

The primary findings of the present study can be summarized as follows:

(1) The frequencies associated with hand-held rackets is slightly lower than that of freely supported rackets. Although the position of the vibration nodes on the handle change slightly, vibration mode does not differ remarkably between the two.

(2) String surface hammering demonstrated little difference in contact time between the hand-held and freely supported rackets.

(3) When an experimental modal analysis was applied to a compound ball and racket system, a pulling and pushing vibration mode through the strings was discovered between the ball and frame. The natural period of this vibration mode was not dependent on the frame rigidity.

(4) Estimated racket vibration amplitude during impact with a ball did not differ remarkably between the hand-held and freely supported rackets.

(5) The estimated rebound capability was slightly higher for the hand-held racket at the end of the string surface. This is likely due to the greater reduced mass of the racket at the impact location.

(6) The hand-held racket can be replaced by a freely supported racket in order to identify racket vibration characteristics related to rebound capability without greatly influencing the results.

ACKNOWLEDGMENTS

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