Impact Phenomena between Racket and Ball during Tennis Stroke

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The performance of racket such as the coefficient of restitution and the shock-vibration of racket handle is closely related to the impact phenomena. Nevertheless, there are a number of unclarified points regarding impact phenomenon between a ball and a racket as well as regarding optimum design of rackets. It is the purpose of this study to investigate the impact phenomena during tennis stroke as a step toward the proposal of a computer aided evaluation system for optimum design of rackets. In this paper, the mechanism of impact has been investigated, and a simple impact model has been proposed on the basis of the idea that the contact duration is determined by the natural period of a whole system composed of the mass of the ball, the nonlinear stiffness of the ball and strings, and the reduced mass of the racket at the impact point on the face. This simple model considering the main source of energy loss during impact such as the impact between a ball and strings, the rotation of a racket and the vibration of a racket-frame can explain the experimental results fairly well.

I. INTRODUCTION

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In this paper, the mechanism of impact has been investigated, and a simple impact model has been proposed on the basis of the idea that the contact duration, which has a strong influence on the racket vibration, is determined by the natural period of a whole system composing the mass of ball, the nonlinear stiffness of ball and strings, and the reduced mass of racket at the impact point on the face.

Although the spin of ball seems to be very important for ball control, it is not dealt with here for the simplification of impact phenomena.

This simple model considering the main source of energy loss during impact such as the impact between a ball and strings, the rotation of racket and the vibration of racket-frame can explain the experimental results fairly well.

II. MODELLING OF IMPACT BETWEEN BALL AND RACKET

2.1 Outline of Modelling

The result of measured contact durations, which means how long the ball stays on the strings, with a normal racket and with a wide-body racket(stiffer) shows that the stiffness of the racket frame does not affect the contact duration much[1].

Accordingly, the masses of a ball and a racket as well as the nonlinear stiffness of a ball and strings are the main factors in the deciding of a contact time.

Therefore, the impact force and contact time can be calculated using a model assuming that a ball with a concentrated mass and nonlinear stiffness collides with the nonlinear spring of strings supported by a frame without vibration, where the measured coefficient of restitution inherent to the materials of ball-strings impact is employed as one of the sources of energy loss.   By applying the derived impact force and contact time to the experimentally identified racket vibration model, the response of racket can be simulated. The larger the energy loss due to the frame vibrations caused by the collision, the lower the coefficient of restitution of impact might be.

2.2 Reduced Mass of Racket at the Impact Point on the Face.

In case a racket rotates around the grip portion, the reduced mass of racket at the impact point on the face can be derived from the racket mass distribution.   The reduced mass of racket depends on hitting locations[1].

2.3 Coefficient Of Restitution Inherent to the Materials of Ball and Strings

The measured Coefficient Of Restitution(COR) e_{BG} with a racket head clamped shows that it is almost independent of ball velocity and string tension. Thus, this constant value of COR can be regarded as COR inherent to the materials of ball and strings. In other words, the energy loss due to impact between a ball and strings with a racket head firmly clamped is almost independent of impact velocity and tension of strings.

2.4 Nonlinear Characteristics of Restoring Force of Ball and Strings

Figure 1 shows the curves of restoring force F_B vs. ball deformation, restoring force F_B vs. string deformation, and the restoring force F_{GB} vs. deformation of the compound system of ball and strings, where the curves are determined so as to satisfy a number of experimental data using the least square method assuming that a ball with concentrated mass deforms only at the side in contact with the strings. Figure 2 shows the curves of the corresponding stiffness K_G, K_B, K_{GB} derived by differentiation of the restoring force with respect to deformation.

The nonlinear springs of both ball and strings become stiffer for larger deformation.
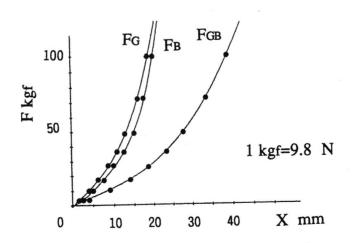

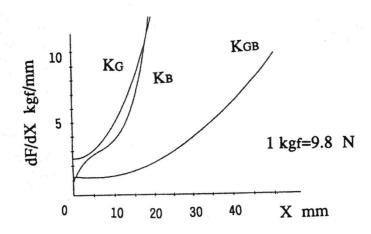Fig.1. Restoring forces vs. deformation of a ball, strings, and a composed system of ball and strings.

Fig.2. Stiffnesses vs. deformation of a ball, strings,
and a composed system of ball and strings.

2.5 Impact Time and Impact force against Impact Velocity

The coefficient of restitution e_{BG} in the collision between a ball and strings with the racket head clamped could be regarded as the case the mass of racket frame is infinitely large. In case the vibration of racket frame is neglected, the momentum equation and e_{BG} give the post-impact velocity of a ball. When a ball collides with a racket standing still before impact, the coefficient of restitution e_1, defined as the ratio between the relative velocity after and before impact, is

$$e_1 = -V_B/V_{Bo} = (e_{BG} - m_B/M_r)/(1 + m_B/M_r) \qquad (1)$$

where m_B is the mass of a ball, M_r is the reduced mass of racket at the hitting point considering the rotation of the racket, and V_{Bo} and V_B are the velocity before and after impact.

The impulse could be described

$$\int F(t)dt = (V_{Bo} - V_{Ro})(1 + e_{BG})m_B /(1 + m_B/M_r) \qquad (2)$$

The contact duration between a ball and strings might be

$$T_c = \pi\sqrt{m_B}/\sqrt{K_{GB}(1 + m_B/M_r)} \qquad (3)$$

where K_{GB} is the stiffness of the ball/strings compound system. The stiffness K_{GB} has a strong nonlinearity and its value changes during impact also depending on the impact velocity. In order to make the analysis simpler, the mean force F_{MEAN} can be introduced during impact time T_C. Thus, if the velocity before impact is given, the relationship between F_{MEAN} and corresponding K_{GB} is represented as Eq.(4) from Eq.(2) and Eq.(3).

$$F_{mean} = (V_{Bo} - V_{Ro})(1 + e_{BG})\sqrt{m_B}\sqrt{K_{GB}}/(\pi\sqrt{1 + m_B/M_r}) \qquad (4)$$

On the other hand, the relationship between F_{GB} and K_{GB} can be derived from Fig.1 and Fig.2. From this relation and Eq.(4), the parameters K_{GB}, F_{MEAN}, T_C and deformation of ball and strings can be determined against the velocity before impact[1].

The calculated impact force F_{MEAN} and contact duration T_C against the velocity before impact are shown in Fig.3 and Fig.4 respectively.
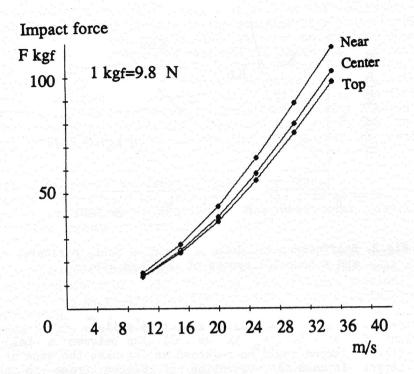
Impact force

F kgf

1 kgf=9.8 N

100

                                                    Near
                                                    Center
                                                    Top

50

0   4   8   12   16   20   24   28   32   36   40
                                                    m/s

Fig.3. Impact force vs. impact velocity, where the hitting
locations are shown beside the curve.

7

6   Near

5   Center

4   Top

Tc ms

3

Contact time

2

1

0   4   8   12   16   20   24   28   32   36   40
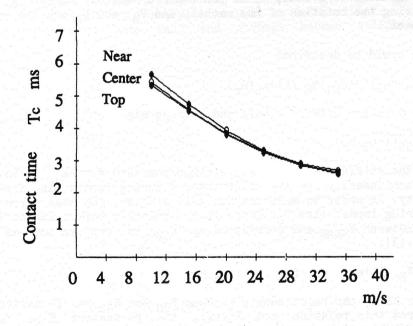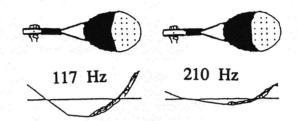   m/s

Fig.4. Impact time vs. impact velocity, where the hitting
locations are shown beside the curve.

2.6 Approximation of Impact Force-Time Curve

Since the force-time curve of impact has an influence on the magnitude of racket frame vibration, it is approximated as a half-sine pulse, which is similar in shape to the actual impact force. The mathematical expession is

$$F(t)=F_{max}\sin(\pi t/T_c) \qquad (0 \le t \le Tc) \qquad\qquad (5)$$

where $F_{max}= \pi F_{mean}/2$

2.7 Vibration Mode of a Racket supported by a hand

Figure 5(a),(b) shows the fundamental mode of normal racket and wide-body(stiffer) racket supported by a hand respectively, where each racket has two nodes being similar to the mode of a racket freely supported, though the frequency of hand-held racket is lower than that of freely supported racket a little.

117 Hz   210 Hz

(a) Normal racket   (b) Wide-body racket

Fig.5. Fundamental vibration mode of hand-held racket.

2.8 Vibration Model of Racket and Prediction of Racket Response against the Impact Velocity

The response of racket can be simulated by applying the impact force to the experimentally identified racket vibration model[1)2)3)].

When the impact force $S_j(\omega_k)$ applies to the point j on the racket face, the summation of amplitudes X_{ij} at point i is expressed as

$$X_{ij}= \sum_{k=1}^{n} r_{ijk}S_j(\omega_k) \qquad\qquad (6)$$

(a) Vo=10 m/s   (b) Vo=20 m/s   (c) Vo=30 m/s
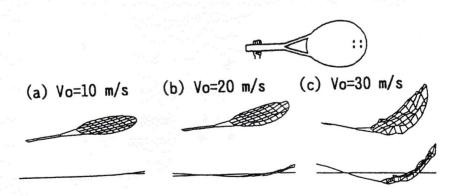
Fig.6. Prediction of amplitude of racket displacement when a ball hits the top off-center.

where $r_{ij,k}$ denotes the residue of k-th mode between arbitrary point i and j, and $S_j(\omega_k)$ is the impact force component of k-th frequency ω_k in the frequency region. When the hitting position and the pre-impact velocity are given, the vibration of the racket can be simulated by using Eq.(6) and the fourier transform of Eq.(5). Figure 6 shows the examples of amplitude distribution of racket displacement when a ball strikes the top of the racket face at a speed of 10, 20, 30 m/s respectively.

III. ESTIMATION OF THE RACKET PERFORMANCE WITH REBOUND VELOCITY OF A BALL

The racket performance with respect to the coefficient of restitution (COR) should be estimated considering the energy loss due to the post-impact vibrations of racket frame.

Figure 7 shows the measured COR when a ball strikes the strings at a speed of 26.4 m/s at various places with racket handle firmly clamped or freely supported. It is found that the COR tends to maximize along the longitudinal axis of the racket and peaks close to the throat. The COR curve tends to become lower at the tip and at the near of the racket face.

When the center of strings is hit in Fig.7, the measured COR is very close to the calculated COR in Fig.8 without consideration of frame vibrations. On the other hand, when the off-center is struck, the value of measured COR is lower than that of calculated one.

Figure 9 is the calculated results of energy loss due to frame vibrations versus hitting locations when the pre-impact velocity is given. The energy loss is very small when a ball hits the center, whereas it is large when a ball hits the top or the near of the racket face. This means that the larger the energy loss due to frame vibration, the lower the coefficient of resitution is. The energy loss due to frame vibration depends on the mass distribution and stiffness of racket frame, tension of strings as well as hitting location and impact velocity.

The effects of impact velocity, mass balance, tension of strings, and stiffness of frame on the impact behavior could be estimated using this simple model.
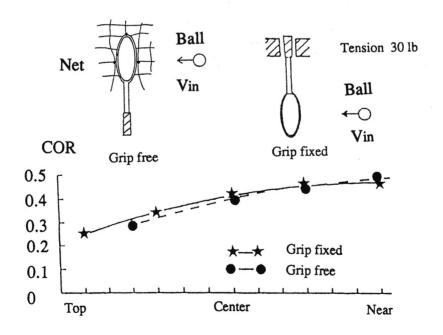

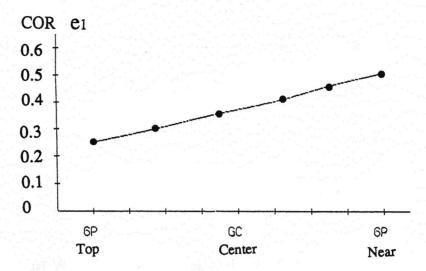Fig.7. Measured distribution of coefficient of restitution on the string face.

Fig.8. Calculated distribution of coefficient of restitution on the string face.
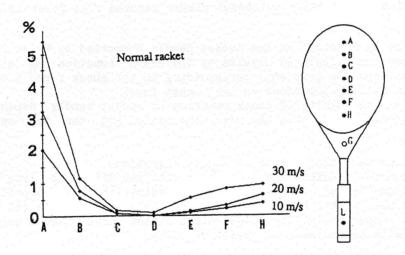

Fig.9. Calculated energy loss due to racket frame vibrations versus hitting point when the pre-impact velocity is given.

IV. SHOCK AND VIBRATIONS OF RACKET HANDLE AND WRIST JOINT

4.1 Measured Shock and Vibrations of Racket Handle and Wrist Joint during Forehand Stroke

Figure 10(a)(b) show the wave form of measured acceleration of racket handle and wrist joint when a male tournament player stroked flat forehand drives, where one pick-up is located near the throat of racket 210 mm from the grip end and the other pick-up on the wrist joint.

The first largest peak during the impact was caused by the shock and vibrations of racket frame, followed by the residual vibrations of the racket frame.

Although the slow frequency fluctuation caused by the swing is also seen in the acceleration wave of the wrist joint in Fig.10(a), its amplitude is rather small.
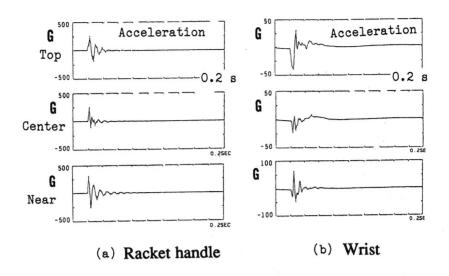
(a) Racket handle   (b) Wrist

Fig.10. The wave form of measured acceleration of racket handle and wrist
        joint when a male tournament player stroked flat forehand drives.

4.2 Predicted Shock Reaction at the Racket Handle Supported by Wrist Joint.

Figure 11 shows the calculated results of the shock reaction S_R at the racket handle(at 70 mm from the grip end) coresponding to the shock force S on the racket face against the hitting locations on the racket face.

Figure 12 shows the results of shock reaction at racket handle depending on the grip position when the ball hits the near, the center and the top on the racket face.

4.3 Prediction of the Acceleration of Racket Vibrations

Figure 13 shows the predicted accelerations with two different type of rackets, which are the summation of the amplitudes considering the components of four vibration modes when a ball hits the top of the racket face at a speed of 10m/s, 20 m/s, and 30 m/s respectively. The acceleration of the racket handle supported by the hand increases with an increase of impact velocity.

4.4 Mechanism of the Shock-Vibrations of the Racket Handle

From Fig.12, it is seen that the shock reaction of the racket handle at 210 mm from the grip end, which is the same location as Fig.10(a), is the largest when the near on the racket face is struck, and the second largest when the center is struck. On the other hand, the closer the hitting point to the node of the bending vibration mode of lowest frequency located nearly at the center on the face, the smaller is the magnitude of the racket vibration. It means that the vibrations are violent when the ball hits the off-center of the racket.

These are the reasons why the amplitude of the shock and vibration of racket handle 210 mm from the grip end in case a ball hits the near of the racket face in Fig.10(a) is the largest and the amplitude of racket handle when a ball hits the top is nearly the same as the amplitude when it hits the center.

It is also seen that the shock reaction at the racket handle 210 mm from the grip end in Fig.12 is small when the ball hits the top, accordingly the peak value of the acceleration of top hitting in Fig.10(a) seems to be caused by the vibration of the racket frame.

The wave form of the wrist acceleration in Fig.10(b) is similar to that of the racket handle, which means that the shock and vibrations of the racket handle transmits to the wrist joint. If the location of the grip is assumed to be 70 mm from the grip end, the magnitude of the shock and vibration at the grip position is
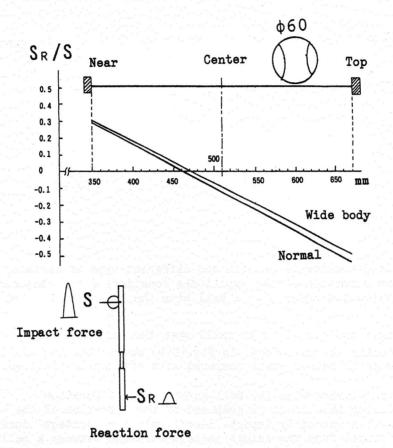
Fig.11. Calculated shock reaction S_R at the racket handle(at 70 mm from the grip end) corresponding to the shock force S on the racket face against the hitting locations on the racket face.

Grip position from the grip end

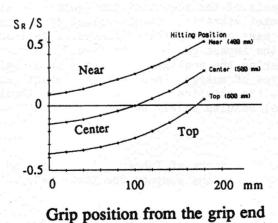Fig.12. Calculated shock reaction at racket handle depending on the grip position when a ball hits the near, the center and the top on the racket face.
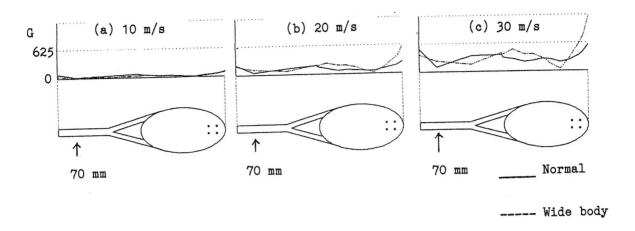
Fig.13. Predicted accelerations with two different type of rackets, which are the summation of the amplitudes considering the components of four vibration modes when a ball hits the top of the racket face.

predicted from Fig.5 and Fig.11 to be small when the ball hits the center of the racket face. Actually the wave form in Fig.10(b) shows that the amplitude when a ball hits the center is rather small compared with off-center hittings.

4.5 Feeling of the players with the Ball-Racket Contact Duration

The wave form during the impact is composed of the vibration of the racket frame and the shock reaction caused by impact. Accordingly the contact duration players feel could be different from the actual contact duration between a ball and strings. The frequency of the frame vibration mode of a wide-body racket(stiffer) is much higher than that of normal racket. This seems to be a reason why players feel the contact duration with a wide-body racket to be shorter than that with a normal racket.

V. CONCLUSIONS

The mechanism of impact has been investigated, and a simple impact model in tennis has been proposed on the basis of the idea that the contact duration, which has a strong influence on the racket vibration, is determined by the natural period of a whole system composing the mass of ball, the stiffness of ball and strings, and the reduced mass of racket at the impact point on the face.

This simple model considering the nonlinear stiffness of a ball and strings as well as the three main source of energy loss during impact such as the impact between a ball and strings, the rotation of racket and the vibration of racket-frame can explain the experimental results fairly well.

Acknowledgments

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