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Coefficient of Restitution between a Ball and a Tennis Racket

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The performance of tennis racket in terms of the coefficient of restitution (CO R) is closely related to the impact phenomena. In the present paper, the distribution of the coefficient of restitution of a tennis racket when the racket vibrates is predicted using an approximate nonlinear impact model relating to an experimentally identified vibration model of a tennis racket on the basis of the idea that the COR can be derived from the energy loss during ball-racket impact. The calculated COR, considering the main sources of energy loss such things as the impact between the ball and strings, rotation of the racket and vibration of the racket-frame, can explain the experimental results very well. It was found that each racket having different physical property has its own characteristics with respect to the coefficient of restitution.

I. INTRODUCTION

With sport equipment, engineering technology has advanced to enable manufacturers to discover and synthesize new materials and new design. There are rackets of all compositions, sizes, weights, shapes and string tension. However, little is ever said about what equipment type is best suited for what individual.

The performance of the tennis racket in terms of the coefficient of restitution (COR) is closely related to the impact phenomena. Nevertheless, there are a number of unclarified points regarding the impact phenomenon between a ball and a racket as well as regarding optimum design of rackets[1),2)], because the impact is a complicated strong nonlinear phenomenon with an extremely short contact time and a very large contact force and deformation. It is the purpose of this study to investigate the impact phenomena for the optimum design of tennis rackets.

The previous paper[3)] has investigated the mechanism of a ball/racket impact, such as the impact force, the contact duration and the racket vibrations, on the basis of the idea that the contact duration, which has a strong influence on the racket-frame vibration and is not much affected by the frame stiffness, is determined by the natural period of a whole system composed of the mass of a ball, the nonlinear stiffness of a ball and strings, and the reduced mass of a racket at the impact location on the string face. The present paper proposes a simple nonlinear impact model for the prediction of the distribution of the coefficient of restitution of a tennis racket during impact using an experimentally identified vibration model of tennis rackets, considering the main sources of energy loss such as the impact between the ball and strings, rotation of the racket and vibration of the racket-frame.

Although the spin of ball seems to be very important for ball control, it is not dealt with here for the simplification of impact phenomena concerning the coefficient of restitution.

II. MAIN FACTORS ASSOCIATED WITH THE ENERGY LOSS AND COEFFICIENT OF RESTITUTION DURING IMPACT

2.1 Energy Loss in a Collision between a Ball and Strings

The measured coefficient of restitution versus the incident velocity when a ball strikes the rigid wall is shown in Fig.1, while the measured coefficient of restitution (COR) e_{BG} when a ball strikes the strings with a racket head clamped is shown in Fig.2. Although the COR in Fig.1 decreases with increasing incident velocity, the coefficient e_{BG} with a racket head clamped is almost independent of ball velocity and string tension. Thus, this constant value of COR can be regarded as COR inherent to the materials of ball and strings. This is due to the nonlinear characteristics of a composed ball/strings system3)4), and the experimental value agrees well with that of the literature5) being independent of the materials such as glassfibers, aluminum and wood.

Accordingly, the energy loss due to impact between a ball and strings with a racket head firmly clamped can be related to the coefficient e_{BG}.

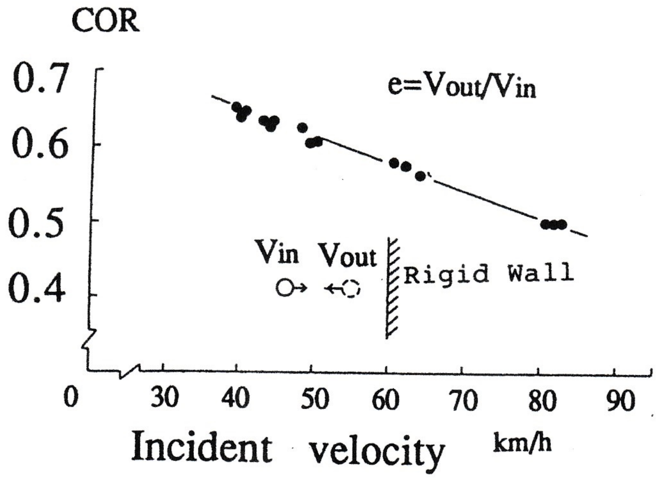


Fig.1. Measured COR between a ball and a rigid wall.

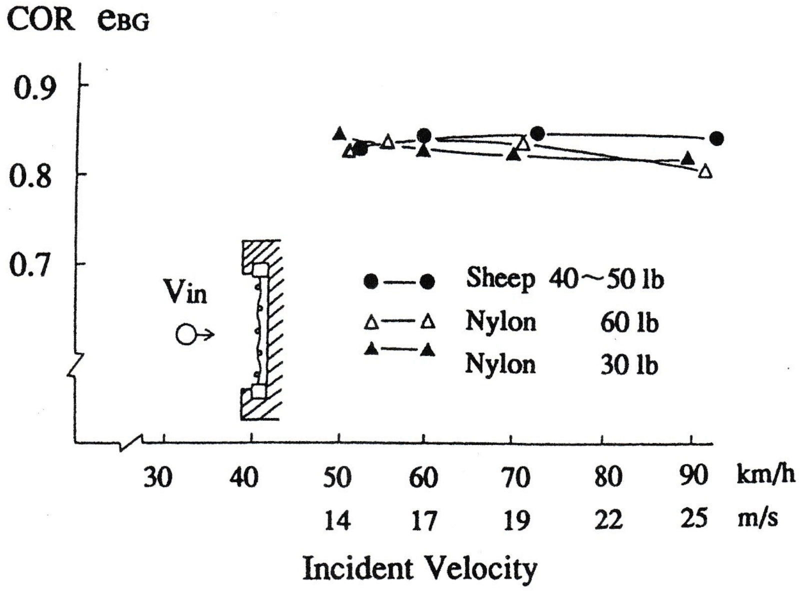


Fig.2. Measured COR between a ball and strings with racket head clamped.

2.2 Nonlinear Restoring Force Characteristics of a Ball and Strings and a Composed Ball/String System

Figure 3 shows a racket being struck by a ball where the racket handle can be rotated. It seems that the ball deforms only at the side which contact to the strings. Figure 4 shows the curves of restoring force F_B vs. ball deformation, restoring force F_G vs. string deformation, and the restoring force F_{GB} vs. deformation of the composed ball/string system, where the curves are determined so as to satisfy a number of experimental data using the least square method assuming that a ball with concentrated mass deforms only at the side in contact with the strings[2]. Figure 5 shows the curves of the corresponding stiffness K_B, K_G, K_{GB} derived by differentiation of the restoring force with respect to deformation. The nonlinear springs K_B of a ball, K_G of strings and K_{GB} of a composed ball/strings system become stiffer for larger deformation.

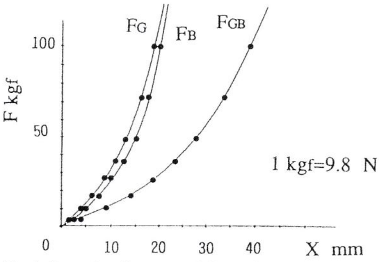


Fig.4. Restoring forces vs. deformation of a ball, strings, and a composed system of a ball and strings.

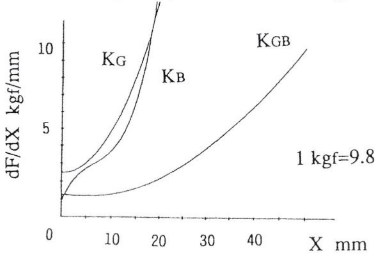


Fig.5. Stiffnesses vs. deformation of a ball, strings, and a composed system of ball and strings.

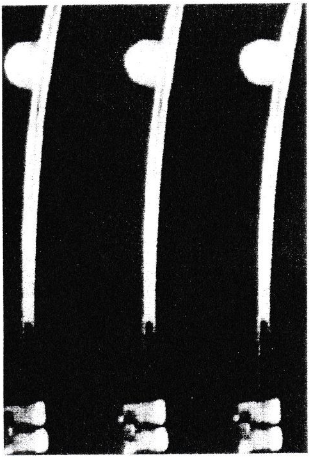


Fig.3. A racket being struck by a ball where the racket handle can be rotated.

2.3 Contact Duration between a Racket and a Ball during Impact

The result of measured contact duration, which means how long the ball stays on the strings, with a normal racket and with a wide-body racket (stiffer) shows that the stiffness of the racket frame does not affect the contact duration much[2]. Accordingly, the masses of a ball and a racket as well as the nonlinear stiffness of a ball and strings are the main factors in the deciding of a contact time.

Therefore, the contact duratin(contact time) can be calculated using a model assuming that a ball with a concentrated mass m_B and nonlinear stiffness K_B collides with the nonlinear spring K_G of strings supported by a frame without vibration, where the measured coefficient of restitution inherent to the materials of ball-strings impact is employed as one of the sources of energy loss.

2.4 Energy Loss due to the Rotation of the Racket and an Introduction of the Reduced Mass of a Racket

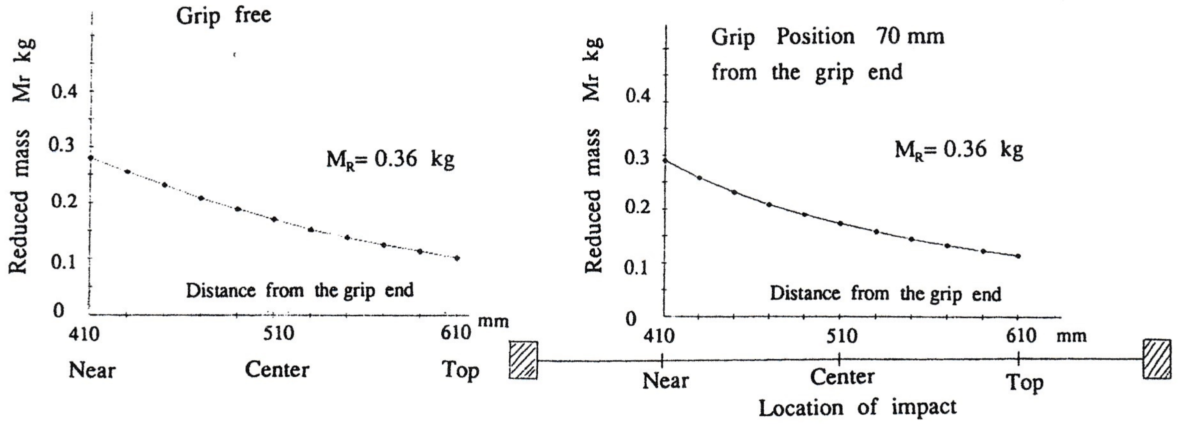
In case a racket rotates around the grip portion at the racket handle, the reduced mass Mr at the impact location on the string face can be derived as Eq.(1.a) from the principle of the conservation of angular momentum, where I_A is the moment of inertia about the grip portion and L_A is the distance between a impact location and a grip portion[2)].

$$M_r = I_A/L_A^2 \tag{1a}$$

Also in case a freely-supported racket (not constrained at the grip) rotates around the center of gravity, the reduced mass can be calculated in the same way as Eq.(1.b), where $I_G(= M_R k_G^2)$ is the moment of inertia about the center of grvity, L_{GA} is the distance between the impact location and the center of gravity, and M_R is the total mass of a racket[2)].

$$M_r = I_G/(L_{GA}^2 + k_G^2) \tag{1b}$$

Fig.6 shows an example of reduced mass of a normal (standard) racket when the racket rotates about the grip portion and about the center of gravity. There is no big difference between them.



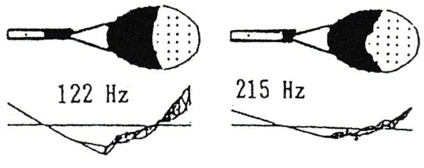
(a) Freely-supported racket
(Grip free)

(b) Grip is supported at 70 mm from the grip end

Fig.6. Reduced mass M_r of a normal racket (M_R=0.36 kg) vs. location of impact.

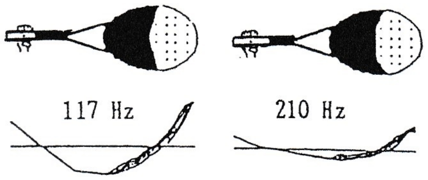
2.5 Support Condition of a Racket Handle

The result of the experimental modal analysis[6)7)] showed that the fundamental vibration mode of a racket supported by a hand has two nodes being similar to the mode of a freely-supported racket, although the frequency of hand-held racket is a little lower than that of freely supported racket as shown in Fig.7 and Fig.8 with two different type of rackets[8)]. Furthermore,



(a) Normal (b) Wide-body

Fig.7. Fundamental Vibration mode of freely supported rackets. (a) Normal racket , (b) Wide body racket



(a) Normal (b) Wide-body

Fig.8 Fundamental Vibration mode of hand-held rackets. (a) Normal racket, (b) Wide body racket there is no significant difference between the reduced mass of a freely-supported racket and that of a hand-held racket.

Accordingly, the racket is assumed to be supported freely or pin-jointed at the grip portion.

III. APPROXIMATE NONLINEAR ANALYSIS FOR DERIVATION OF THE IMPACT FORCE AND CONTACT DURATION

3.1 Outline of Approximate Nonlinear Impact Model

In order to predict the coefficient of restitution, the vibrational response of a racket has to be calculated. The impact force-time curve at the hitting portion on the string face has a strong influence on the amplitude of the vibrations of a racket[2)4)6)]. Therefore, in this chapter, the impact force and contact time will be calculted using an approximate nonlinear impact model assuming that a ball with a concentrated mass m_B and nonlinear stiffness k_B collides with the nonlinear spring k_G of strings supported by a rigid frame without vibrations, where the measured coefficient of restitution inherent to the materials of ball-strings is employed as one of the sources of energy loss.

3.2 Nonlinear Analysis of Impact Force and Contact Duration

The coefficient of restitution e_{BG} in the collision between a ball and strings with the racket head clamped could be regarded as the case in which the mass of racket frame is infinitely large. In case the vibration of racket frame is neglected, the momentum equation and the coefficient e_{BG} give the post-impact velocity V_B of a ball and V_R of a racket at the impact location. When a ball collides with a racket, post-impact velocities V_B and V_R are given as

$$V_B= -V_{Bo}(e_{BG}- m_B/M_r)/(1+ m_B/M_r) + V_{Ro}(1+ e_{BG})/(1+ m_B/M_r) \qquad (2)$$

$$V_R= V_{Ro}(M_r/m_B- e_{BG})/(1+ M_r/m_B) + V_{Bo}(1+ e_{BG})/(1+ M_r/m_B) \qquad (3)$$

where m_B is the mass of a ball, M_r is the reduced mass of a racket at the hitting location considering the rotation of the racket, and V_{Bo} is the ball velocity before impact.

The impulse could be described as

$$\begin{aligned}\int F(t)dt &= m_BV_{Bo} - m_BV_B \\ &=(V_{Bo}-V_{Ro})(1+e_{BG})m_B /(1+m_B/M_r) \qquad (4)\end{aligned}$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of the mass m_B of a ball, the stiffness K_{GB} of ball/strings, and the reduced mass of the racket, it could be obtained as

$$T_c= \pi\sqrt{m_B}/\sqrt{K_{GB}(1+ m_B/M_r)} \qquad (5)$$

Actually, the stiffness K_{GB} has a strong nonlinearity and its value changes during impact also depending on the impact velocity. In order to make the analysis simpler, the equivalent force F_{mean} can be introduced during impact time T_C, which is described as

$$\int F(t)dt= F_{mean}T_c. \qquad (6)$$

Thus, from Eq.(4), Eq.(5) and Eq.(6), the relationship between F_{mean} and corresponding K_{GB} against the velocity before impact is given by

$$F_{mean}= (V_{Bo}-V_{Ro})(1+ e_{BG})\sqrt{m_B}\sqrt{K_{GB}} /(\pi\sqrt{1+m_B/M_r}) \qquad (7)$$

On the other hand, from Fig.4 and Fig.5 using the least square method, the relationship between F_{GB} and K_{GB} can be expressed in the form

$$F_{mean}= \mathrm{func.}(K_{GB}). \qquad (8)$$

From Eq.(7) and Eq.(8), the parameters K_{GB} and F_{mean} against impact velocity can be obtained as shown graphically in Fig.9, accordingly the contact duration T_C can also be determined against the velocity before impact from Eq.(5).

Figure 10 is a comparison between the measured contact times during actual forehand strokes[9] and the calculated ones when a ball hits the center of the strings face, showing a good agreement between them.

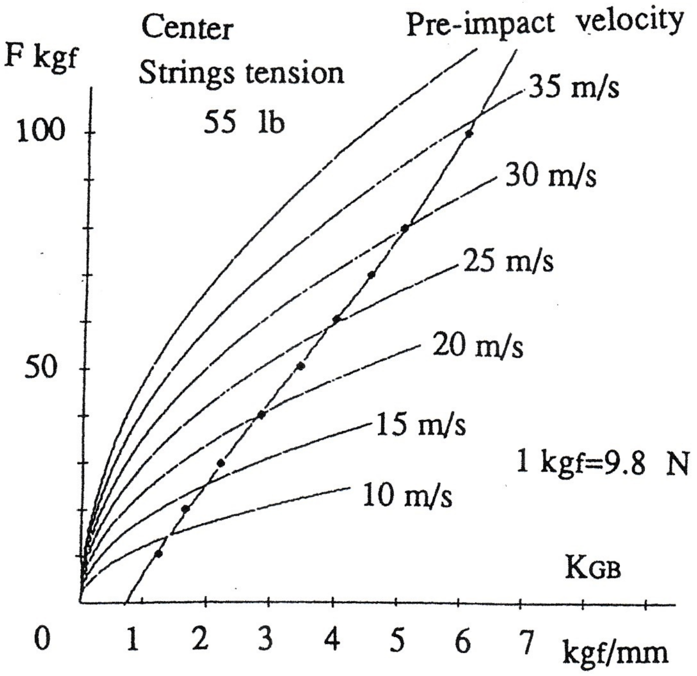


Fig.9. Graphical description of the derivation of impact force and the stiffness of the composed ball/strings system against the impact velocity

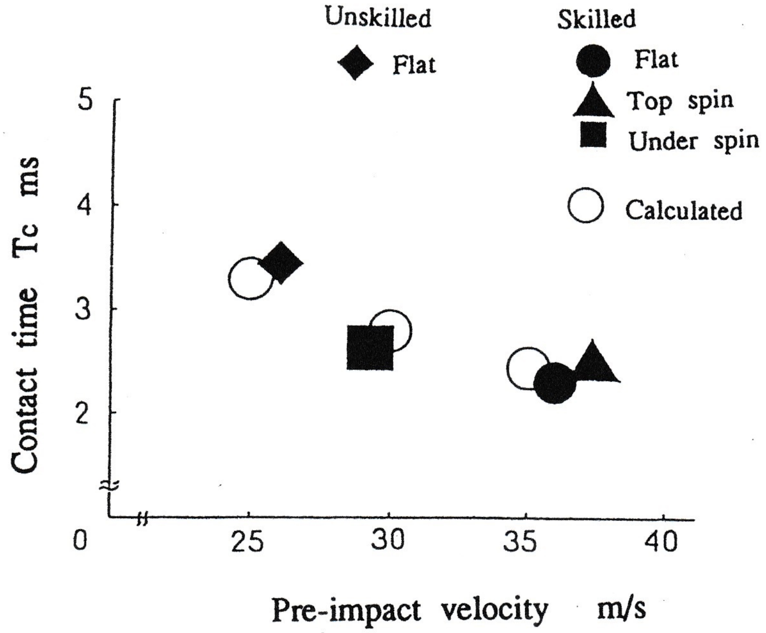


Fig.10. Comparison between the measured contact times during strokes and the calculated results.

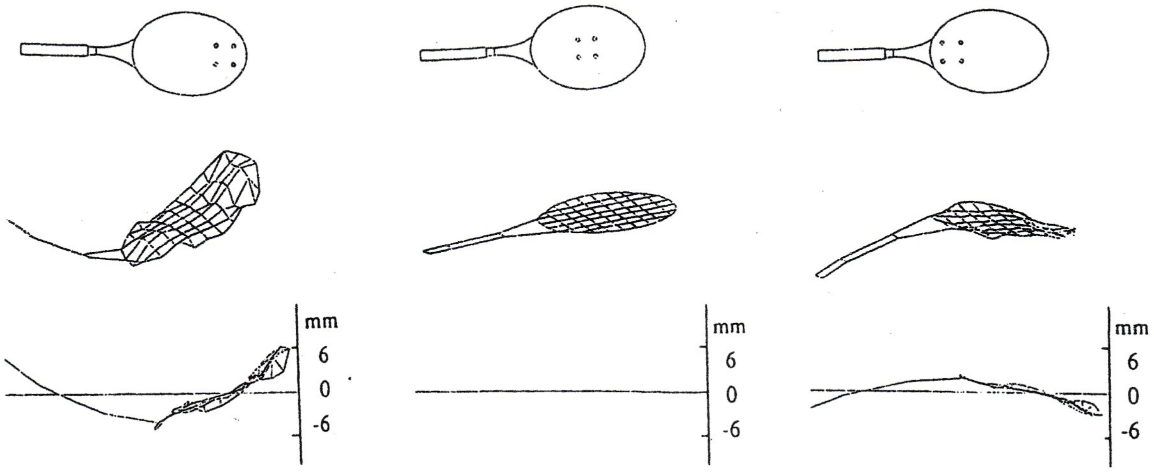
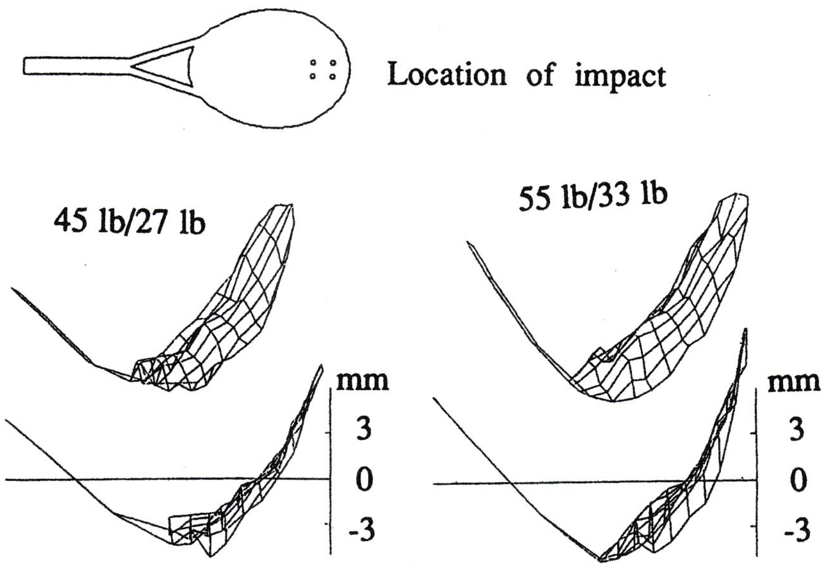


Fig.12. Prediction of amplitude of racket vibration when a ball hits the racket at a velocity of 30 m/s.



(a) (b)

Fig.13. Prediction of amplitude of racket vibration when a ball hits the top (off-center) on the strings at a velocity of 30 m/s. Racket: Yamaha PROTO-EX110(366 g, 685 mm, 110 in^2, center of gravity: 325 mm)
(a) strung with lower tension (b) strung with higher tension

If the longitudinal mass distribution of racket frame is assumed to be uniform, the energy loss ΔE_1 due to racket vibrations can be derived as

$$\Delta E_1 = M_R V_m^2/2 \tag{12}$$

where V_m is the mean velocity amplitude calculated from the amplitude distribution of the post-impact vibrations of a racket frame, and M_R is the total mass of a racket.

V. DERIVATION OF THE COEFFICIENT OF RESTITUTION CONSIDERING THE ENERGY LOSS DURING IMPACT

5.1 Energy Loss during Impact

The coefficient of restitution (COR) might be related to the energy loss during impact.

The COR can be derived considering the main sources of energy loss such as the impact between the ball and strings, rotation of the racket and vibration of racket frame.

Since the measured coefficient of restitution e_{BG} inherent to the materials of ball-string system is considered as one of the sources of energy loss, another energy loss besides ΔE_1 is the energy loss ΔE_2 due to the rotational motion of the racket as a rigid body. If a ball collides with a racket at rest ($V_{Ro} \doteq 0$), the energy loss ΔE_2 could be obtained as

$$\begin{aligned} \Delta E_2 &= [m_B V_{Bo}^2 - (m_B V_B^2 + M_r V_R^2)]/2 \\ &= [m_B M_r/(m_B + M_r)](1 - e_{BG}^2) V_{Bo}^2/2 \qquad (13) \end{aligned}$$

5.2 Derivation of the Coefficient of Restitution

When a ball strikes a racket at rest ($V_{Ro} = 0$), the coefficient of restitution $e_r = (V_R - V_B)/V_{Bo}$ with respect to the relative velocities corresponds to the total energy loss ΔE ($= \Delta E_1 + \Delta E_2$) being expressed as

$$\begin{aligned} \Delta E &= [m_B V_{Bo}^2 - (m_B V_B^2 + M_r V_R^2)]/2 \\ &= [m_B M_r/(m_B + M_r)](1 - e_r^2) V_{Bo}^2/2 \qquad (14) \end{aligned}$$

From the above equation, the coefficient of restitution e_r is obtained as

$$\begin{aligned} e_r &= (V_R - V_B)/V_{Bo} \\ &= \sqrt{1 - 2\Delta E(m_B + M_r)/(m_B M_r V_{Bo}^2)} \qquad (15) \end{aligned}$$

If the rebound velocity relative to the incident velocity of a ball is defined as the coefficient of restitution $e = -V_B/V_{Bo}$, which is often used for the comparison to the experiments, it is written as Eq.(16) using Eq.(3) and Eq.(15).

$$\begin{aligned} e &= -V_B/V_{Bo} \\ &= \sqrt{1 - 2\Delta E(m_B + M_r)/(m_B M_r V_{Bo}^2)} - (1 + e_{BG})/(1 + M_r/m_B) \qquad (16) \end{aligned}$$

VI. ANALYTICAL RESULTS AND THE EXPERIMENTAL RESULTS

The energy loss due to the vibrations is very small when a ball hits the center, whereas it is large when a ball hits the off-center (the top or the near of the racket face). Furthermore, the energy loss due to vibrations of a racket with higher string tension is larger than that of a racket with lower tension.

Figure 14 is a comparison between the measured distribution of the COR (impact velocity: 26.4 m/s) and the calculated one(30.0 m/s) when a ball hits the string face of a freely-supported racket, showing a good agreement between them. The COR tends to be maximized along the longitudinal axis of the racket and peaks close to the throat due to the mass distribution of a racket.

Figure 15 is the predicted distribution of the COR of a racket (mass:366 g, hitting area:110 in^2) made of carbon graphite, showing that the COR tends to become lower at the tip and at the near on the string face with an increase of impact velocity due to the vibration modes of racket frame.

Figure 16 is the predicted coefficient of restitution when a ball strikes rackets with various physical properties at the top on the string face (off-center impact) at different velocities. The rackets in Fig.14 are a Yamaha racket EX-II (standard, 360 g, length 680 mm, area 100 in^2, center of gravity: 308 mm from grip end, 1st vibration mode: 122 Hz), a Yamaha PROTO-02 (very stiff frame called wide body, 370 g, 680 mm, 100 in^2, center of gravity:317 mm, 215 Hz), a Yamaha PROTO-EX110 (366 g, 685 mm, 110 in2, center of gravity:325 mm, 132 Hz), and a Yamaha EOS100 (super light weight, 290 g, 680 mm, 100 in^2, center of gravity:350 mm, 171 Hz). It is seen that each racket has its own characteristics with respect to the coefficient of restitution. The coefficients of restitution with PROTO-02 (wide body racket) and EOS100 (super light and top-heavy) are not affected by an increase of

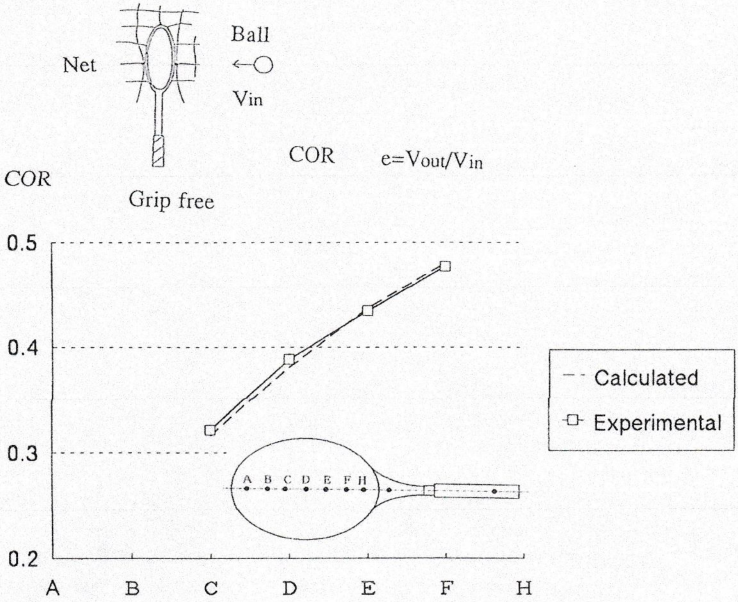


Fig.14. Comparison between the measured distribution of the COR (impact velocity 26.4 m/s) and the calculated results (30.0 m/s).

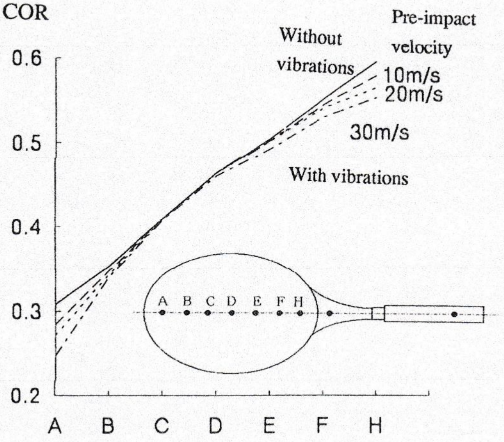


Fig.15. Predicted distribution of the COR vs. impact velocity

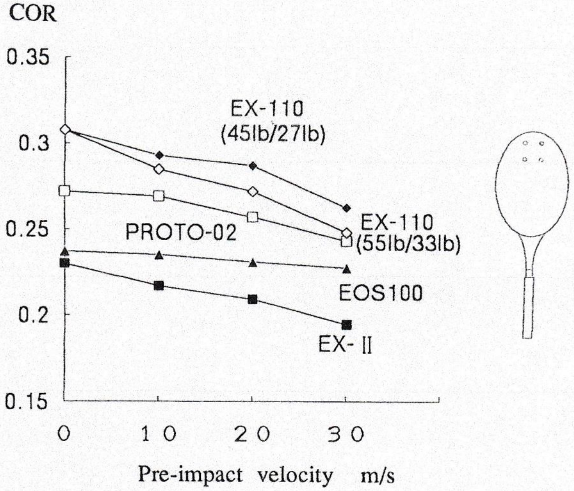


Fig.16. Predicted coefficient of restitution when a ball strikes rackets with various physical properties at the top on the string face (off-center impact) at different velocities.

impact velocity at the top and higher than the standard EX-II. Although the coefficient with PROTO-EX110 strung loosely is the highest, it decreases with increasing impact velocity. Particularly with PROTO-EX110 strung tightly it remarkably decreases in the off-center impact.

VII. CONCLUSIONS

The distribution of the coefficient of restitution (COR) of a tennis racket is predicted using a simple nonlinear impact model and an experimentally identified vibration model of tennis racket, considering the main sources of energy loss such as the impact between the ball and strings, rotation of the racket and vibration of racket-frame. The calculated COR can explain the experimental results very well. It was found that each racket having different physical property has its own characteristics with respect to the coefficient of restitution. The main results are summarized as follows:

(1) The calculated contact time between a ball and a racket agrees well with the measured one during actual forehand stroke by a player.

(2) The calculated COR on the racket face agrees well with the measured one.

(3) The COR tends to be maximized along the longitudinal axis of the racket and peaks close to the throat (very near on the string face) due to the mass distribution of a racket.

(4) The COR tends to become lower at the tip and at the near of the racket face with increasing impact velocity due to the vibration modes of racket frame.

(5) When the center of the racket face is struck, the value of COR is almost independent of the impact velocity, accordingly it is very close to that of racket without frame vibrations.

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