Reprinted from

THEORETICAL AND APPLIED MECHANICS

Volume 43

Proceedings of the 43rd Japan National Congress for Applied Mechanics, 1994

Edited by Motoo HORI and Tsuneyoshi NAKAMURA
for Japan National Committee for Theoretical and Applied Mechanics, Science Council of Japan

Effects of String Pre-Tension on Impact between Ball and Racket in Tennis

Yoshihiko KAWAZOE

Department of Mechanical Engineering, Saitama Institute of Technology, Okabe, Saitama

The effects of string pre-tension on forces and duration of impact during the tennis stroke are investigated on the basis of a simple nonlinear impact model. It was found that a contradiction in many experiments of the conventional research work are due to the strong nonlinearity of the ball/string system. Although the impact force markedly increases with the impact velocity, it is not much affected by the string pre-tension. The contact time decreases with increasing the impact velocity, but the string pre-tension does not have a marked influence except at very low impact velocities. When the ball hits the off-center on the string face, the amplitude of a racket strung tightly is larger than that of a racket strung loosely.

I. INTRODUCTION

Much engineering research has been conducted to determine an optimal tennis racket, and numerous variables have been considered in order to assess the mechanical performance of the racket and string system along with their effect on the behavior of a ball after impact [1)]. However, there are a number of unclarified points concerning the impact phenomena and the optimum design of rackets in tennis. It is well accepted that the string is quite important. Nevertheless, regarding the effects of string pre-tension which means an initial tensile force, the conventional research works only demonstrate the complexity of the interaction of string and racket and suggest that more research should be conducted [2)].

The present paper investigates the effects of string pre-tension on forces and duration of impact during the tennis stroke based on the simple nonlinear mathematical model, in which the contact duration is determined by the natural period of a whole system composed of the mass of a ball, the stiffness of a ball and strings, and the reduced mass of a racket at the impact location on the racket face, considering the strong nonlinearity of the instantaneous deformations of a ball and strings. It also accounts for the contradiction in many experiments of the conventional research work concerning the effect of string pre-tension on the impact phenomena.

II. NONLINEAR MODELING OF IMPACT BETWEEN BALL AND TENNIS RACKET

2.1 Nonlinear Characteristics of a Ball and Strings

Figure 1 shows the test for obtaining the applied force-deformation curves schematically, where the ball was deformed between two flat surfaces as shown in (a) and the ball plus strings were deformed with a racket head clamped as shown in (b). The results for the ball and racket strung at a tension of 246 N (55 lb) are shown in Fig.2. Figure 3 indicates a racket being struck by a ball where the racket handle can be rotated. It seems that the ball deforms only at the side which contact to the strings.

Assuming that a ball with concentrated mass deforms only at the side in contact with the strings [3)], the curves of restoring force F_B vs. ball deformation, restoring force F_G vs. string deformation, and the restoring force F_{GB} vs. deformation of the composed ball/string system are obtained from Fig.2 as shown in Fig.4. These restoring characteristics are determined so as to satisfy a number

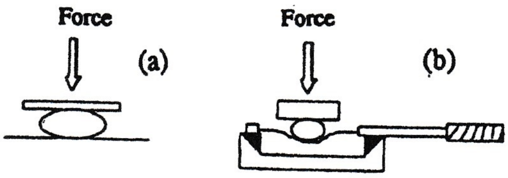

Fig.1 Illustrated applied force-deformation test.
(a) a ball,
(b) a composed ball/string system.

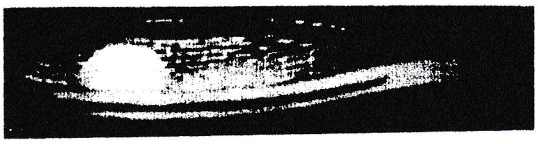

Fig.3 Racket being struck by a ball where the racket handle can be rotated.

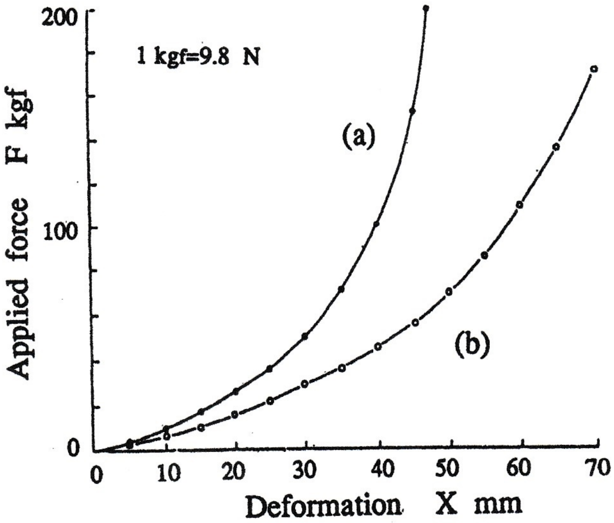

Fig.2 Results of a force-deformation test with pre-tension of strings 55 lb(246 N).

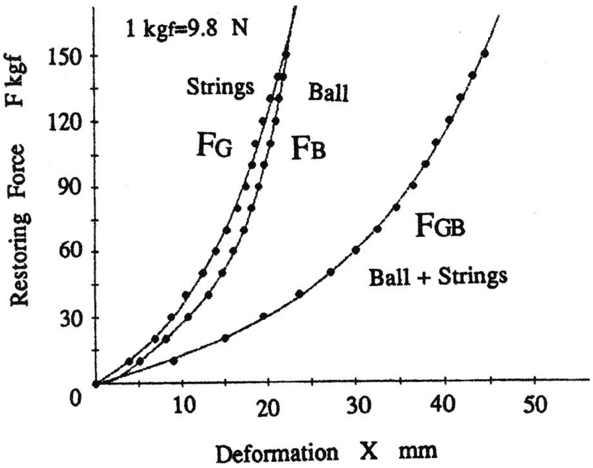

Fig.4 Restoring forces vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

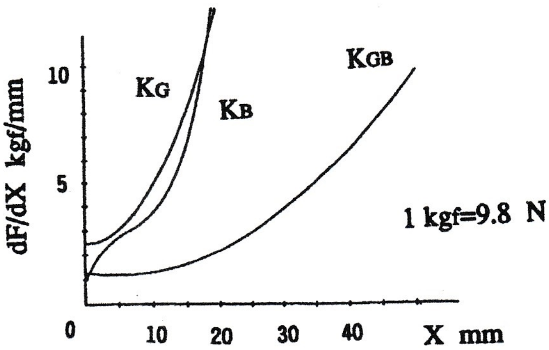

Fig.5 Stiffness vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

of experimental data using the least square method. The curves of the corresponding stiffness K_B, K_G and K_{GB} are derived as shown in Fig.5 by differentiation of the equations of restoring force with respect to deformation, respectively. The stiffness K_B of a ball, K_G of strings and K_{GB} of a composed ball/strings system exibit the strong nonlinearity.

2.2 Effects of String Pre-tension on the Restoring Force with Large Deformation

When the middle of a string of length $2L_S$, which is subjected to an initial tensile force denoted by the symbol S_o, is displaced laterally by a distance X from its equilibrium position shown in Fig.6, a restoring force F is developed by the string. We obtain the restoring force as Eq.(1) by inspection of the geometry [4].

$$F = \frac{2X[S_0 + AE\{(L_S^2 + X^2)^{1/2} - L_S\}/L_S]}{(L_S^2 + X^2)^{1/2}} \tag{1}$$

where the symbols A and E represent the cross-sectional area of the string and its modulus of elasticity. This exact nonlinear restoring force may be replaced with a simpler approximate representation shown as Eq.(2).

$$F = S_0(2/L_S)X + (AE/L_S^3)X^3 \tag{2}$$

This approximated restoring force against the string displacement contains a term with X raised to the third power and is, therfore, still nonlinear.

On the other hand, the mathematical expression for the curve of restoring force against strings displacement at a tension of 55 lb (Fig.4) can be written as

$$F_G = a_0X_G + b_0X_G^2 + c_0X_G^3 \tag{3}$$

where the constants a_0,b_0 and c_0 are determined so as to give a sufficiently close approximation to

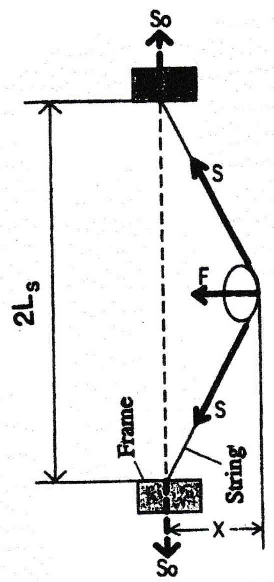

Fig.6 Deformation, tension, and restoring force of the String during impact with string pre-tension.

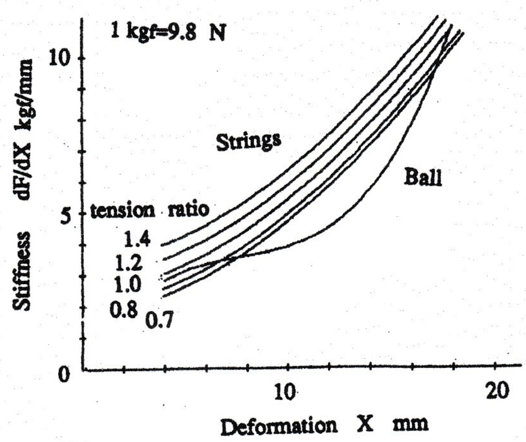

Fig.7 Stiffness vs. deformation of the strings strung with different pre-tension, being compared to the stiffnes-deformation curve of the ball.

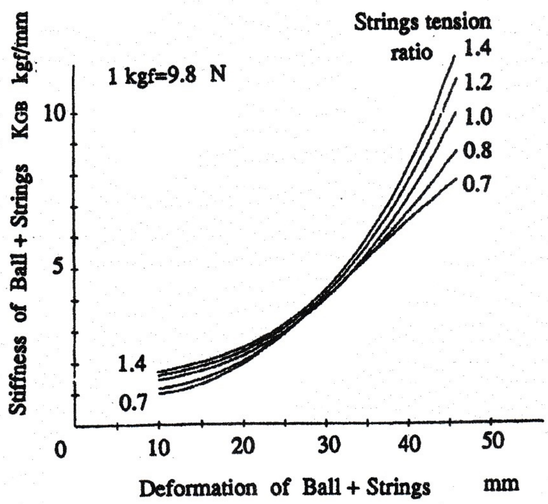

Fig.8 Stiffness vs. deformation of the composed ball/string system at different pre-tensions.

the large deformation of the experimental data. The term containing the pre-tension S_o in Eq.(2) corresponds to the first term in Eq.(3). Accordingly, we can regard the coefficient ao as being propotional to the pre-tension S_o.

Figure 7 is the obtained result of the stiffness-deformation curves of the strings for the pre-tension of the values from 0.7 times to 1.4 times that of the reference tension of 55 lb, being compared to the stiffness-deformation curve of the ball. Figure 8 shows the stiffness versus deformation of a composed ball-strings system for the various string tensions.

2.3 Modeling of Energy Loss of the Ball and Strings during Impact

The measured coefficient of restitution (COR) e_{BG} when a ball strikes the strings with a racket head clamped is shown in Fig.9. Although the COR when a ball strikes the rigid wall decreases with increasing the incident velocity [5], the coefficient e_{BG} is almost independent of the ball velocity and string tension, and this experimental value agrees well with that of the literature [6] being independent of the frame materials such as glassfibers, aluminum and wood. Thus, the coefficient e_{BG} can be inherent to the materials of ball and strings, and also be related to the energy loss ΔE_{BG} due to the instantaneous deformation of the ball and strings during impact.

If we assume a model composed of a concentrated mass m_B of the ball, a stiffness K_{GB} and a damping C_{GB} of the composed ball/strings system shown in Fig.10 as a model of the experiment in Fig.9, the stiffness K_{GB} and the damping coefficient C_{GB} have strong nonlinearity with respect to the impact velocity. Since the damping ratio of the composed ball/string system is related to the e_{BG} and the e_{BG} is regarded as being constant, the damping ratio of the composed system is be constant in spite of the nonlinearity of K_{GB} and C_{GB} [7].

In the present paper, we employ the coefficient e_{BG} in place of considering the energy loss ΔE_{BG} in order to make the analysis easier.

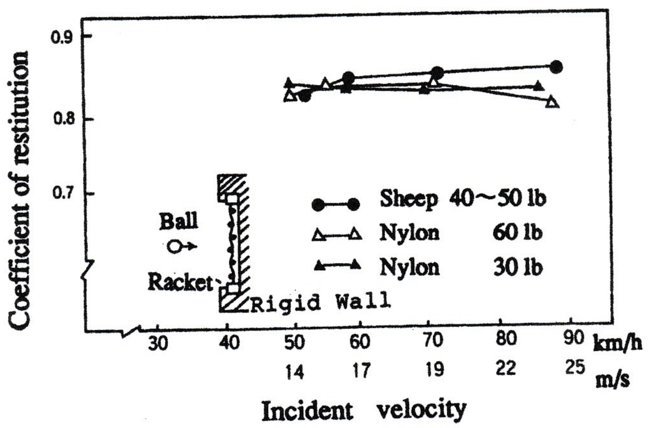

Fig.9 Measured coefficient of restitution between a ball and strings with racket head clamped.

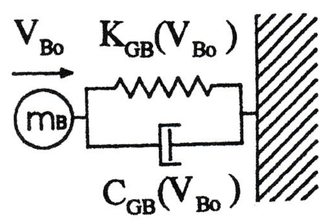

Fig.10 Approximate nonlinear impact model of a composed ball/string system.

2.4 Modeling with Contact Time between a Ball and a Racket

The result of measured contact duration, which means how long the ball stays on the strings, shows that the stiffness of the racket frame does not much affect the contact duration [8]. Therefore, the contact duration can be calculated using a model assuming the strings to be supported by a frame without vibration.

2.5 Introduction of the Reduced Mass of a Racket at the Impact Location

In case a racket rotates around the grip portion at the racket handle, the reduced mass M_r at the impact location on the string face can be derived as Eq.(4) from the principle of the conservation of angular momentum, being dependent on the moment of inertia I_A about the grip portion and the distance L_A between an impact location and a grip portion [9].

$$M_r = I_A/L_A^2 \tag{4}$$

III. APPROXIMATE NONLINEAR ANALYSIS OF IMPACT FORCE AND CONTACT TIME

The coefficient of restitution e_{BG} in the collision between a ball and strings with the racket head clamped could be regarded as the case in which the mass of racket frame is infinitely large. In case the vibration of racket frame is neglected, the momentum equation and the coefficient e_{BG} give the post-impact velocity V_B of a ball and V_R of a racket at the impact location. When a ball collides with a racket, post-impact velocities V_B and V_R are given as

$$V_B = -V_{B0}(e_{BG} - m_B/M_r)/(1 + m_B/M_r) + V_{R0}(1 + e_{BG})/(1+m_B/M_r) \quad (5)$$

$$VR = V_{R0}(M_r/m_B - e_{BG})/(1 + M_r/m_B) + V_{B0}(1 + e_{BG})/(1 + M_r/m_B) \quad (6)$$

where m_B is the mass of a ball, M_r is the reduced mass of a racket at the hitting location considering the rotation of the racket, and V_{B0} is the ball velocity before impact.

The impulse could be described as

$$\int F(t)dt = m_B V_{B0} - m_B V_B = (V_{B0} - V_{R0})\frac{(1 + e_{BG})m_B}{(1 + m_B/M_r)} \quad . \quad (7)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of the mass m_B of a ball, the stiffness K_{GB} of ball/strings, and the reduced mass M_r of the racket, it could be obtained as

$$T_c = \pi\sqrt{m_B}/\sqrt{K_{GB}(1 + m_B/M_r)} \quad (8)$$

Actually, the stiffness K_{GB} has a strong nonlinearity and its value changes during impact also depending on the impact velocity. In order to make the analysis simpler, the equivalent force F_{mean} can be introduced during impact time T_C, which is described as

$$\int F(t)dt = F_{mean} \cdot T_C \quad . \quad (9)$$

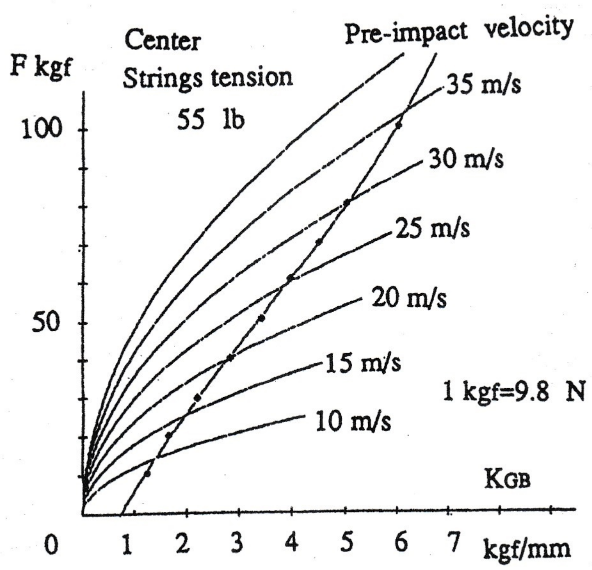

Fig.11 Graphical description of the derivation of equivalent impact force and the equivalent stiffness of the composed ball/string system against the impact velocity.

Thus, from Eq.(7), Eq.(8) and Eq.(9), the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity is given by

$$F_{mean} = (V_{Bo} - V_{Ro})(1 + e_{BG}) \cdot \sqrt{m_B}\sqrt{K_{GB}}/(\pi\sqrt{1 + m_B/M_r}) \tag{10}$$

On the other hand, from the approximated curves shown in Fig.4 and Fig.5, the relationship between F_{GB} and K_{GB} can be expressed in the form

$$F_{mean} = func.(K_{GB}) \ . \tag{11}$$

From Eq.(10) and Eq.(11), the parameters K_{GB} and F_{mean} against the impact velocity can be obtained as shown graphically in Fig.11, accordingly the contact duration T_C can also be determined against the pre-impact velocity from Eq.(8).

The measured contact times during actual forehand strokes [10] agrees well with the calculated ones [5] when a ball hits the center of the strings face.

IV. EFFECTS OF STRING PRE-TENSION ON IMPACT FORCE AND CONTACT TIME

The calculated results of equivalent impact force F_{mean} against the impact velocity with the rackets strung in different tensions are shown in Fig.12. Although the impact force markedly increases with the impact velocity, it is not much affected by the string tension. The calculated equivalent stiffness K_{GB} of the composed ball/strings system is shown in Fig.13. Also, the calculated contact time T_C against the impact velocity is shown in Fig.14. It is seen that the contact time decreases with increasing the impact velocity, and that the string tension does not have a marked influence on the contact time except at very low impact velocities.

The equivalent impact force F_{mean}, the equivalent stiffness K_{GB} of composed ball/string system, and the contact time T_C are shown in Fig.15, Fig.16, and Fig.17 against the string tension ratio, respectively, relative to the impact velocity as a parameter.

Figure 18 is the measured accelerations of the racket handle during actual forehand drive with the rackets strung in different tensions [11)12)]. The first largest peaks in the figures are mainly caused by the shock during impact, because the vibrations of the racket frame are very small when a ball hits the center which is near the node of the first vibration mode. It is seen that there is almost no differnce between the loosely strung racket (35 lb) and the tightly strung one (60 lb). Figure 12 gives a clear theoretical explanation for the first time about the reason why the string tension does not have an influence on the impact force.

The measured contact times vs. the impact velocity with the rackets strung in different tensions (55 lb and 75 lb) are shown in Fig.19. Although ithas been left unclarified why the string tension does not have a significant effect on the the contact time in the actual impact, the calculated results in Fig.14 explain the experimental results in Fig.19 fairly well.

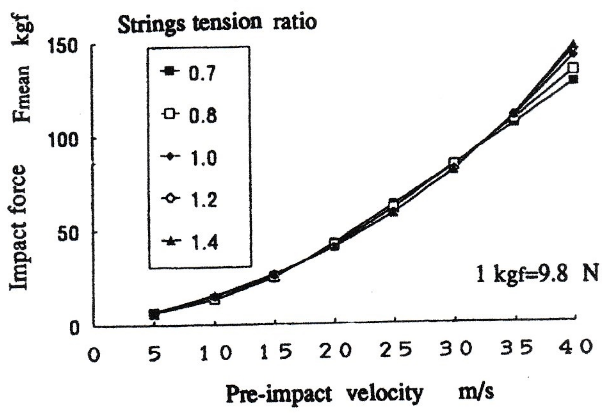

Fig.12 Calculated impact force vs. impact velocity with the rackets strung at different tensions.

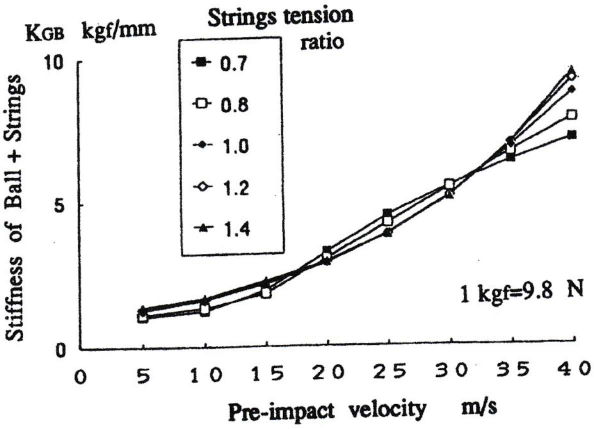

Fig.13 Equivalent stiffness vs. impact velocity with the rackets strung at different tensions.

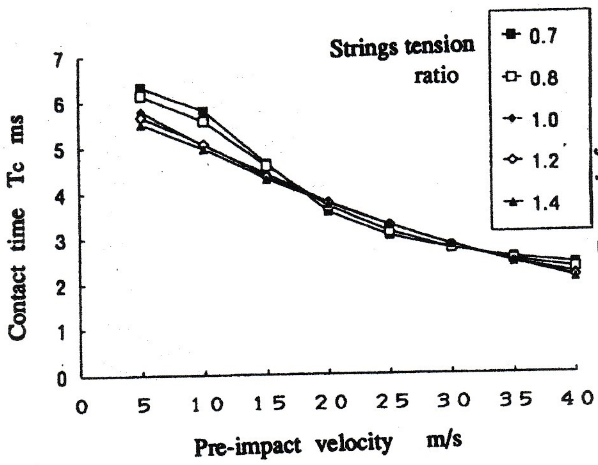

Fig.14 Calculated contact time vs. impact velocity with the rackets strung at different tensions.

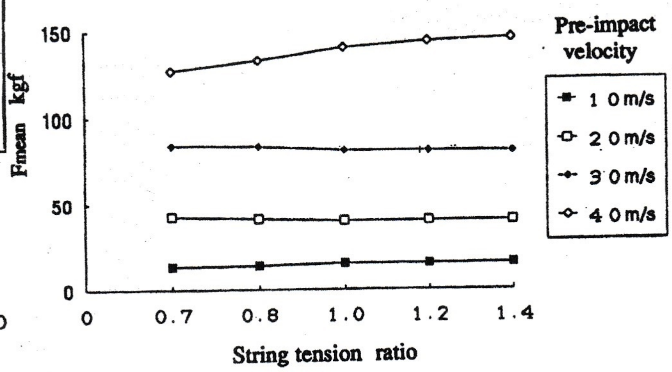

Fig.15 Equivalent impact force vs. string tension ratio relative to the impact velocity as a parameter.

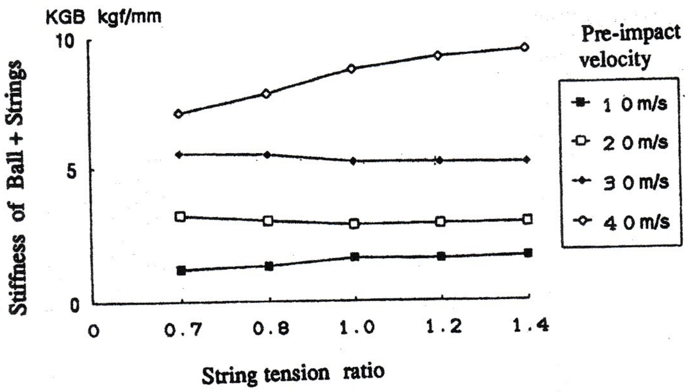

Fig.16 Equivalent stiffness vs. string tension ratio relative to the impact velocity as a parameter.

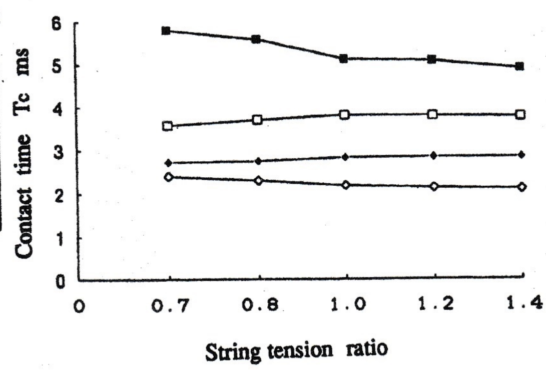

Fig.17 Contact time vs. string tension ratio relative to the impact velocity as a parameter.

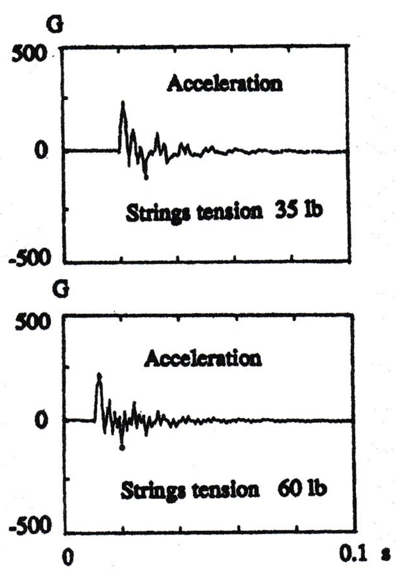

Fig.18 Measured acceleration of the racket handle during actual forehand drive with the rackets strung at different tensions.

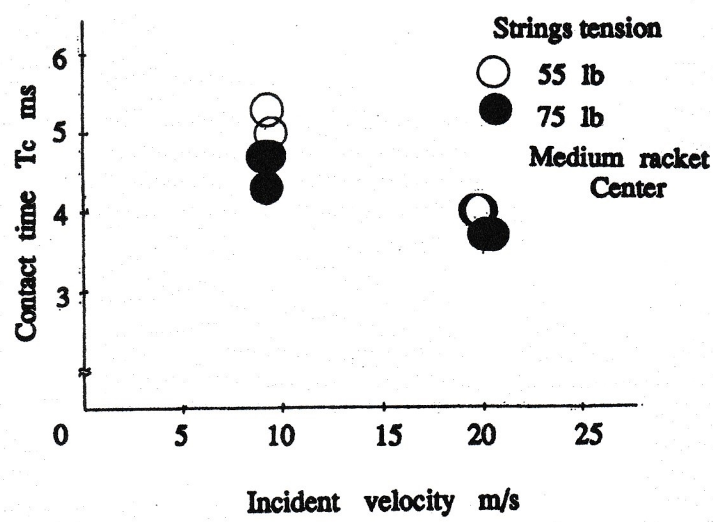

Fig.19 Measured contact time vs. impact velocity with the rackets strung at different tensions.

V. EFFECTS OF STRING PRE-TENSION ON THE RACKET VIBRATIONS

5.1 Approximation of Impact Force-Time Curve

Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse shownon the left in Fig.20, which is similar in shape to the actual impact force. The mathematical expession is

$$F(t) = F_{max} sin(\pi t/T_c) \ ; \quad (0 \leq t \leq T_c) \tag{12}$$

where

$$F_{max} = \pi F_{mean}/2 \tag{13}$$

The Fourier spectrum of Eq.(12) is shown graphically on the right in Fig.20 and represented as

$$S(f) = 2F_{max}T_c \frac{|\ cos(\pi f T_c)\ |}{\pi\ |\ 1-(2fT_c)^2\ |} \tag{14}$$

where f is the frequency.

Figure 21 shows the examples of the calculated shock shape during impact, where the ball strikes the center on the string face at a velocity of (a) 20 m/s and (b) 30 m/s with the racket strung at 55 lb, respectively.

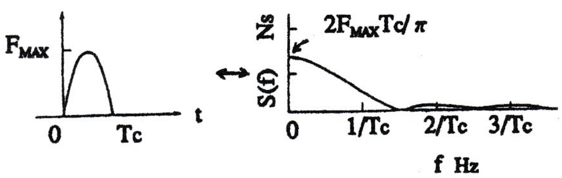

Fig.20 Assumed shock shape and its spectrum during ball/racket impact.

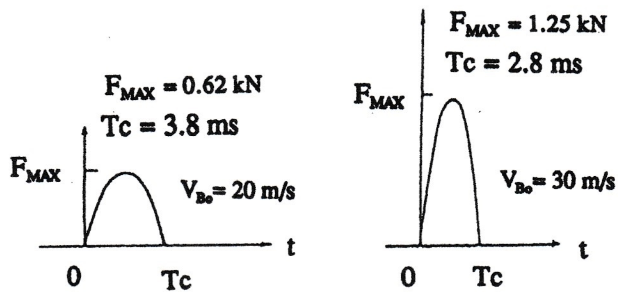

Fig.21 Calculated shock shape when a ball strikes the center on the string face of the racket at velocities of 20 m/s and 30 m/s.

5.2 Prediction of the Racket Vibration Induced by Impact

The vibration characteristics of a racket can be identified using the experimental modal analysis [13)14)] and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of the racket.

When the impact force $S_j(\omega_k)$ applies to the point j on the racket face, the amplitudes X_{ij} at point i is expressed as

$$X_{ij} = r_{ijk} S_j(\omega_k) \tag{15}$$

where r_{ijk} denotes the residue of k-th mode between arbitrary point i and j derived on the basis of the experimental modal analysis, and $S_j(\omega_k)$ is the impact force component of k-th frequency ω_k in the frequency region [9)]. When the hitting location and the pre-impact velocity are given, the amplitude of racket vibrations can be simulated by using Eq.(14) and Eq.(15).

Figure 22 is the result of the predicted amplitude of vibration of rackets with different string tensions (standard and a 1.2 times the standard) when the ball hits the the racket at a velocity of 30 m/s. It is seen that the predicted amplitude of racket vibrations are markedly small when a ball hits the center, whereas it is large when a ball hits the off-center(the top or the place close to the throat of the racket) on the string face, and is also seen that the amplitude of a racket strung at higher tension is larger than that of a racket strung at lower tension.

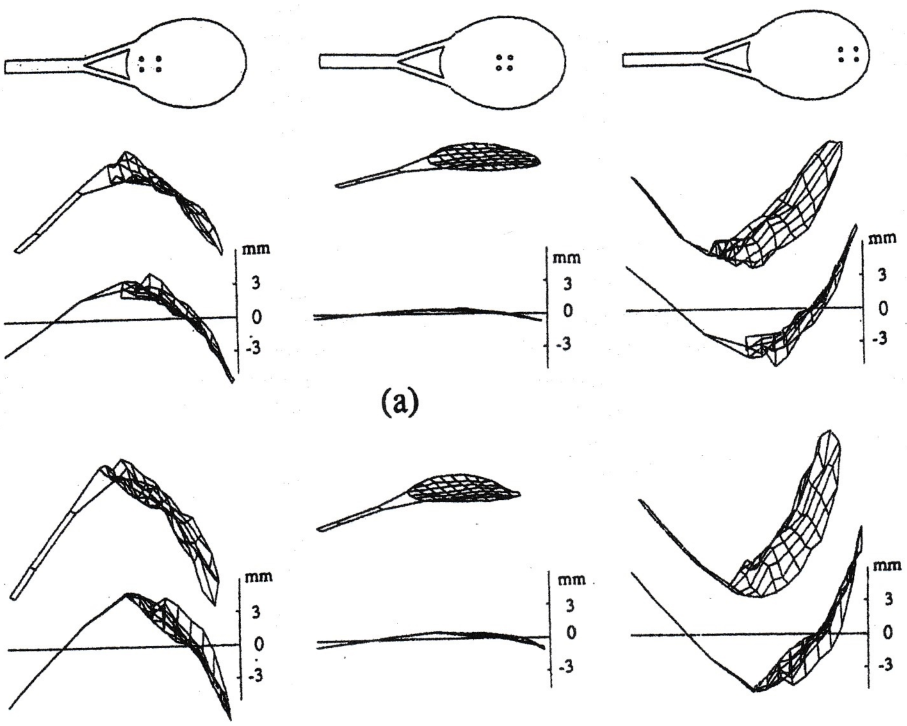

Fig.22 Predicted amplitude of the racket frame vibrations immediately after impact at a velocity of 30 m/s with the rackets strung at different tensions.
(a) standard tension (b) 1.2 times the standard

VI. CONCLUSIONS

The effects of string pre-tension on forces and duration of impact during the tennis stroke have been investigated using a simple nonlinear impact model. With this model, it is found that the contradiction in many experiments of the conventional research work are due to the strong nonlinearity of the ball/string system. The main results are summarized as follows:

(1) Although the impact force markedly increases with the impact velocity, it is not much affected by the string pre-tension.

(2) The contact time decreases with increasing the impact velocity, but the string pre-tension does not have a marked influence on the contact time except at very low impact velocities.

(3) When a ball hits the center on the string face, the predicted amplitude of vibration of rackets with different string pre-tensions is very small.

(4) When the ball hits the off-center, the amplitude of a racket strung at higher tension is larger than that of a racket strung at lower tension.

Acknowledgments

The author is grateful to Mr.A.Miura of Tohoku University, Mr. R.Tomosue of the University of Tokyo, Dr.D.Y.JU of Saitama Institute of Technology, and Yamaha company for their help in the present study, and also to Messers E.Yamamoto and K.Fukuda for their help in carrying out the study as senior students during the 1992 academic year.

REFERENCES

1) Groppel,J.L.,Shin,I.S.,Spotts,J. and Hill,B., Effects of different string tension patterns and racket motion on tennis racket-ball impact, Int.J.Sport Biomech., Vol.3(1987),pp.142-158.

2) Groppel,J.L.,Shin,I.S.,Thomas,J.A. and Welk,G.J., The effects of string type and tension on impact in midsized and oversized tennis racquets, Int.J.Sport Biomech., Vol.3(1987), pp.40-46.
3) Kawazoe,Y., CAE of tennis rackets with impact phenomena (Prediction of racket response and a view of restitution in racket-ball impact),Trans.JSME, 58-552(1992),pp.143-150. (in Japanese)
4) Timoshenko,S.,Young,D.H.,and Weaver,W., Vibration Problems in Engineering,John Wiley & Sons, Inc.,(1974),p.146.
5) Kawazoe,Y., Coefficient of restitution between a ball and a tennis racket, Theoretical and Applied Mechanics, Vol.42(1992),pp.197-208.
6) Asami,T.,Miyasita,M. and Watanabe,T., as editors, Handbook of physical education sports in the future,Vol.7,(Kohdansha, 1984),p.277.(in Japanese)
7) Kawazoe,Y. and Kanda,Y., Analysis of impact phenomena with a ball-racket system in tennis (Effects of frame vibrations and optimum design of a racket), Trans.JSME,59-563 (1993), pp.2116-2123.(in Japanese)
8) Kawazoe,Y., Impact phenomena between racket and ball during tennis stroke, Theoretical and Applied Mechanics,Vol.41(1992),pp.3-13.
9) Kawazoe,Y., Analysis of coefficient of restitution during a nonlinear impact between a ball and strings considering vibration modes of racket frame, Trans.JSME, 59-562(1993), pp.1678-1685.(in Japanese)
10) Nagata,A., Analysis of tennis movement, J.J.Sports Sci.,2-4 (1983),pp.245-259.(in Japanese)
11) Kawazoe,Y. and Tomosue,R., Impact vibrations of a racket handle and the wrist joint in tennis, Trans.JSME,59-560(1993), pp.1001-1008.(in Japanese)
12) Kawazoe,Y., CAE of tennis rackets with impact phenomena (The effect of strings tension on a ball-racket impact), Trans.JSME,59-558(1993), pp.521-528.(in Japanese)
13) Kawazoe,Y.,Dynamics and computer aided design of tennis racket, Proc.Int. Sympo. on Advanced Computer for Dynamics and Design'89(1989),pp.243-248.
14) Kawazoe,Y., Dynamics and computer aided design of tennis racket (The influence of some factors on the mode characteristics and a view of optimum design), Trans. JSME,56-526(1990),pp.1511-1517.(in Japanese)