Analysis of Impact Phenomena in a Tennis Ball-Racket System*
(Effects of Frame Vibrations and Optimum Racket Design)

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The performance of a tennis racket in terms of the coefficient of restitution (COR) is closely related to impact phenomena. This paper investigates the effects of frame vibrations on the coefficient of restitution and the contact time during impact of a ball/string system and a simulated frame model, using FEM simulation and modal analysis. The results show that the COR is mainly affected by a rigid motion and a bending vibration with two nodes of racket frame. In addition, the COR increases with an increase of frame rigidity, but then saturates at a certain rigidity depending on the impact velocity. Furthermore, the COR increases as the impact point approaches the center of rotation and the node of racket frame vibration.

Key Words: Dynamics of Machinery, Vibration of Moving Body, Transient Response, Finite-Element Method, Modal Analysis, Impact, Coefficient of Restitution, Tennis Racket, Optimum Design

1. Introduction

1.1 Background and current research themes

Material composites have increased the degree of freedom of design and manufacturing for sports products. At the current stage, very specific designs are targeted to match the physical and technical levels of each user.

The performance of a racket can be evaluated with regard to physical characteristics such as weight distribution, rigidity distribution, face size, and string tension, if the behavior of the racket from the time of ball impact at a certain speed and angle to the time of ball release (contact time) is clarified and the resulting ball speed and spin become known.

However, ball and racket impact is an instantaneous non-linear phenomenon (contact time is 6 - 3 ms, with shorter times at higher impact speeds) creating large deformations in the ball/string and vibrations in the racket. The problem is further complicated by the involvement of humans in the actual strokes. These problems make analysis extremely difficult. Therefore, there are many unknown factors involved in the mechanisms explaining how the interrelated actions of the strings, frame, and ball influence the racket capabilities[1].

Matuhisa et al.[2] investigated the restitution characteristics for a frontal impact between a racket and ball using a model with the tennis ball/string approximated by a 1 degree of freedom vibration system, the frame approximated by a uniform cross-sectional beam, and the arm approximated by a limited degree of freedom. Their results showed an influence of frame bending rigidity on restitution characteristics that is almost nonexistent in actual rackets with looser strings resulting in better recovery. Although the model is simple and is able to determine the impact characteristics, the strong non-linearity of string restoring characteristics is not considered and the correlation between the impact point and the frame vibration modes are not mentioned.

Yamaguchi et al.[3] used a limited degree of freedom linear spring and mass system to approximate the ball/string and a uniform cross-sectional beam to

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approximate the frame in order to investigate the restitution characteristics of a frontal impact between a racket and ball using a calculation by the finite-element method. Although this research is very interesting, the following problems still remain: (1) Although the influence of the spring constant of the strings at a fixed impact speed (30 m/s) is investigated, when the extremely low impact speed range is excluded, the equivalent spring constants of the ball and strings are determined primarily by the impact speed and does not change appreciably due to the non-linear restoring characteristics, even when the initial tension of the strings is changed[4]. When the impact speed is approximately 20 m/s, actual measured results have shown almost unchanged contact times and impact forces, even when the initial string tension is different[5]. (2) The restitution characteristics are discussed from the standpoint of matching the natural frequencies of the beam and ball. However, since the spring rigidity of the strings is similar to that of the ball[5], the natural frequencies of the frame should probably be compared to the frequency of the compound ball/string system. (3) Impact is an instantaneous transient phenomenon. The frequency of the ball/string system changes with increases in deformation caused by the non-linear restoring characteristics. The impact force spectrum also includes components from the zero frequency[9]. Since the equivalent spring constant increases with increases in the impact speed, the frequency of the ball/string system also varies greatly depending on the impact speed. With these types of phenomena, the matching principles for frequencies are difficult to apply, and therefore require further investigation. (4) The reason for good rebound when impact is at a position near the center of gravity of the beam was presumed to be the support from the first order vibration modes of a beam with free ends. However, calculations showed that when the rigidity of the beam is low, restitution rapidly deteriorates for an impact at the position at the antinode of the vibration while remaining nearly constant for an impact at the nodes. This contradicts the above hypothesis. (5) Since the ball/string energy loss at impact are ignored, further investigation is required in this area.

Assuming impact at the near (off center) of the string face near the center of gravity of the racket and based on measured results, Takatuka[6] approximated the ball to a concentrated mass and spring, the strings to a spring, and the frame to 2 equivalent masses and a single spring system. Using these approximations, a 3 mass-2 spring model with no energy loss was used to investigate the relationship between restitution characteristics and frame rigidity. Although this model simplifies the impact system, it provides a qualitative explanation of the increase in restitution characteristics accompanying increased frame bending rigidity (increased bending frequency) in the experiments and by experience. However, the calculations using this model use many measured values found from impact experiments. Therefore it is not possible to analyze impact at any given impact speed and point using this method.

On the other hand, Kawazoe[7]-[9] determined impact forces and contact times using impact analysis on a rigid frame and a one degree of freedom model for a compound ball/string system, considering the strong non-linearity of ball/string restoring characteristics and the energy loss. By applying the results to a vibrational model for a racket identified experimentally, the racket response was determined and the distribution of the coefficient of restitution was predicted. The predicted values of the coefficient for restitution at any given impact speed and position agree well with experimental results. The model provides an explanation for the mechanism of impact phenomena related to restitution characteristics. The model can also provide a physical explanation of the vibration acceleration waveform of the racket handle and wrist joint in actual impact experiments[10]. However, there is no theoretical proof for the basic assumption of a rigid frame when elucidating contact time and impact force between the racket and ball.

1.2 Purpose of this research

In the present study, a previously reported compound ball/string system with a single degree of freedom was used and the frame was approximated by a beam with steps. The finite-element method was then used to simulate behavior during impact. The finite-element model of a beam with steps brings the center of gravity position and the frequencies of first and second order into agreement with the actual system. Moreover, the energy loss at the time of impact between the ball/string was investigated and the influence of frame vibration on contact time and restitution characteristics for the ball and racket was analytically clarified. However, this analysis assumed a frontal impact between the ball and racket with no ball rotation (spin).

2. Impact Model and Simulation Calculation Method

2.1 Single degree of freedom model for compound ball/string system

2.1.1 Equivalent spring constant of compound ball/string system Figure 1(a) shows a compound ball/string system with a single degree of freedom. In this case, a ball of velocity V_{Bo} is assumed to strike a racket with the frame around the strings (racket head) clamped. The ball at the time of impact is assumed to deform only on the side in contact with the strings. A ball with the mass concentrated at the center of the actual ball is assumed to impact the clamped frame through the compound ball/string spring and the compound damping. Figure 1(b) shows the model for impact of the compound ball/string system of Fig. 1(a) on a beam with steps imitating the racket frame.

The spring rigidity K_{GB} of the compound ball/string system in Figure 1(a) is largely dependent on impact velocity and the spring becomes harder as the deformation increases[5]. Therefore, the spring rigidity also changes moment by moment during the impact. Based on the experimental results showing that contact time is nearly unaffected by frame rigidity[8], the frame is assumed to be a rigid body over the contact time and the contact time itself is assumed to be 1/2 of the period of the system consisting of the compound ball/string and the rigid frame. By using these assumptions and introducing the constant equivalent force $F_{\mathrm{MEAN}}(\int F(t)dt = F_{\mathrm{MEAN}} \cdot T_c)$ during the contact time T_c, the equivalent spring constant K_{GB} for the compound ball/string system can be determined at any given impact velocity[8]. Figure 2 shows the calculation results for K_{GB} for the compound ball/string system when the ball impacts on the racket strings at the top, the center, and the near. The horizontal axis is the impact velocity. The racket was a normal conventional racket (standard frame rigidity) made from fiberglass, graphite and Kevlar with a mass of 360 g and an initial string tension of 55 lb (246 N).

Although the equivalent spring constant of the compound ball/string system is somewhat different, depending on the impact location on the racket surface, the purpose of the present study was to determine how ball rebound velocity in particular is affected by frame vibration. Therefore, an average value was used for the the racket face and the impact calculations were carried out. In other words, $K_{GB}=2.74\times10^4$ N/m was used for an impact velocity of 20 m/s, $K_{GB}=5\times10^4$ N/m was used for an impact velocity of 30 m/s, and $K_{GB}=8.33\times10^4$ N/m was used for an impact velocity of 40 m/s.

2. 1. 2 Equivalent damping coefficient of compound ball/string system Ball displacement x_1 and velocity $dx_1/dt(=V_B)$ are derived in the compound ball/string system shown in Fig. 1(a). Since the ball separates from the strings when the ball/string deformation restores, if displacement x_1 is assumed to be 0 when $t=T_c$ (T_c: Contact time when the frame is clamped), contact time T_c can be obtained. In addition, when contact time T_c is substituted into the equation of dx_1/dt, the velocity V_B of the ball when separating from the strings can be determined. That is, the contact time T_c and the velocity V_B of the ball when separating from the strings can be expressed by the equations below.

$$T_c=\pi/\omega_d \tag{1}$$

$$V_B=-V_{Bo}\exp(-\Omega\zeta T_c) \tag{2}$$

Where:

$$\Omega=\sqrt{K_{GB}/m_B},\quad \zeta=C_{GB}/2\sqrt{m_B K_{GB}},$$
$$\omega_d=\Omega\sqrt{1-\zeta^2} \tag{3}$$

When a ball strikes the strings with the frame around the strings (racket head) clamped with an impact velocity in the range of a normal swing, the coefficient of restitution (ratio of ball rebound velocity to incident velocity) is almost independent of the tension and impact velocity, and is nearly constant, as shown in Fig. 3[8]. When the restitution coefficient value of 0.83 is assumed to be characteristic of impacts between a ball and strings, the ball rebound velocity for Fig. 1(a) can be expressed as follows.

$$V_B=-0.83\,V_{Bo} \tag{4}$$

From Eq.(2) and Eq.(4), the equivalent damping coefficient ratio of the single degree of freedom

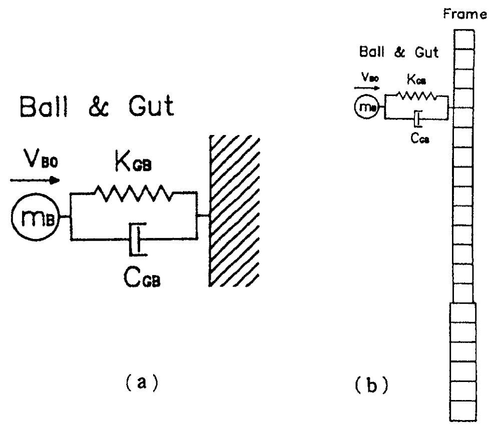


Fig. 1 Compound ball/string system and racket frame model

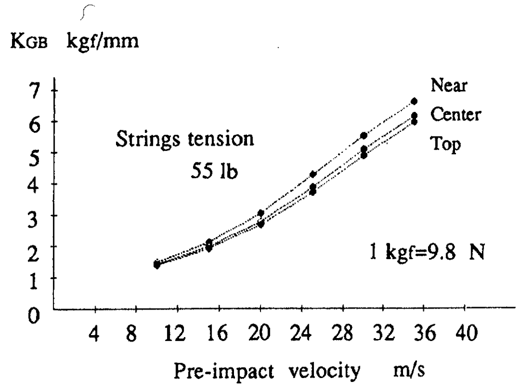


Fig. 2 Relationship between equivalent spring constant and impact velocity in a compound ball/string system

compound ball/string system model is approximately 0.059 and it is not dependent on impact velocity. Since equivalent spring rigidity K_{GB} of the compound system increases as the impact velocity increases, the independence of the damping coefficient ratio with respect to impact velocity means that damping coefficient C_{GB} is proportional to $K_{GB}^{1/2}$ and increases with increases in impact velocity.

2.2 Frame model

Figure 1(b) shows the model for impact of the compound ball/string system of Fig. 1(a) on a beam with steps and no restrictions (free) imitating the racket frame. In this model, the overall length (L= 680 mm), weight (360 g, including the strings), center of gravity position (308 mm from the end of the grip) and first order (2 node bending) and second order (3 node bending) natural frequencies (122 Hz and 337 Hz) become in agreement with an actual racket system by creating a beam with steps from a uniform cross-sectional beam at 204 mm from the grip side and 476 mm from the top side. The equivalent bending rigidity EI and equivalent mass ρA per unit length are EI_1=146 Nm^2 and ρA_1=0.696 kg/m on the grip side and EI_2=128.5 Nm^2 (=0.88 EI_1) and ρA_2=0.458 kg/m (=0.659ρA_1) on the top side. The effect of frame vibration on contact time was investigated by simulation using this model. Figure 4 shows the vibration modes of the beam with steps. The node positions for the first mode are 0.200L and 0.769L from the end of the grip. The node locations of the second mode are 0.125L, 0.484L, and 0.864L. Third mode frequency also approached the actual measured values.

2.3 Simulation calculation method

In the model from Fig.1(b), the ball is assumed to be a mass and the frame (beam with steps) is divided into 20 beam elements and discretized using the finite-element method. Although the problem of setting boundary conditions for a racket in actual use has yet to be solved, vibration modes found by experimental modal analyses with the racket handles supported by hand are very similar to vibration modes with the racket unrestricted (racket laying on a soft sponge during the experiment)[10]. Therefore, the boundary conditions were assumed to be free in all areas. The degree of freedom for the entire system is 43. The simulation was executed by direct integration based on the Runge-Kutta method. Time interval Δt was 10^{-7} to 10^{-6} seconds. Impact completion was assumed to be the point at which K_{GB} of the compound ball/string system changed from compression to tension.

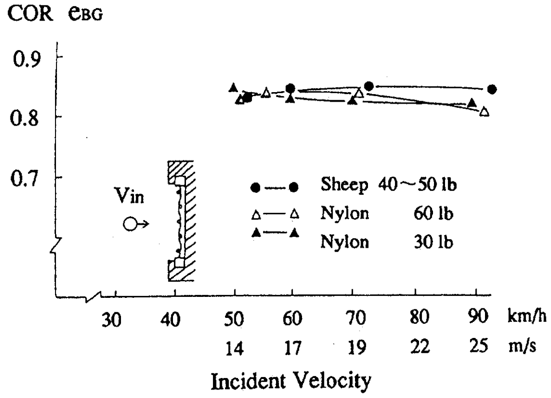


Fig. 3 Coefficient of restitution (ratio of ball rebound velocity to incident velocity) when a ball strikes strings in a clamped frame (racket head)

3. Simulation Calculation Results and Discussion

3.1 Impact behavior and ball rebound velocity

The calculation results for impact behavior when a ball with a velocity of 20 m/s strikes a stationary racket are shown in Figs. 5, 6, and 7. In each of the figures (a) is the standard racket (EI_1=146 Nm^2) and (b) is the case in which frame rigidity is increased about 68-fold (EI_1=9999 Nm^2). The figures show the results when the impact point is on the grip side (0.60L), middle (0.75L), and top side (0.90L) in the impact zone. The impact point at 0.75L is near the node of the first vibration mode (2 node) of the frame. The upper areas of the figures show time histories for ball displacement and displacement at the impact point and both ends of the beam model. The middle areas of the figures show the velocities of each point and the lower areas show the deformation changes over time for the ball and frame (beam with steps). When the ball deformation and displacement at the impact point on the beam with steps are in agreement, the ball separates from the racket. The drawings in (c) show the deformations over time for EI_1=9999 Nm^2 as drawn from the deformations when EI_1=146 Nm^2. The following points can be derived from these figures: (1) The EI_1=9999 Nm^2 frame can be assumed to be nearly a rigid body. (2) At the 0.60L and 0.90L impact points, substantial deformation due to vibration remains in the frame at the completion

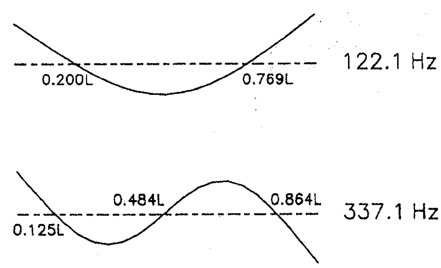
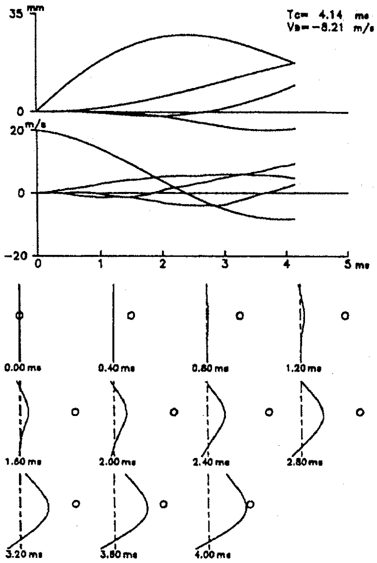
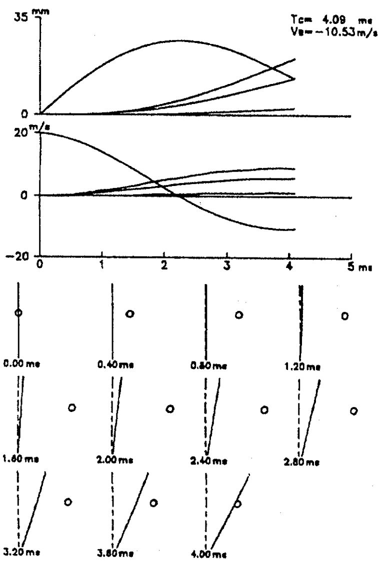


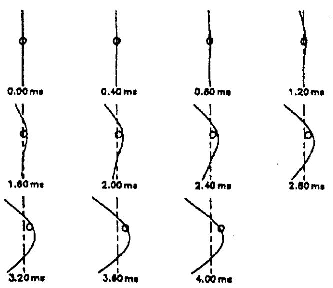
Fig. 4 Vibration modes of a beam with steps imitating a racket frame



(a) EI_1=146 Nm^2(EI_2=0.88 EI_1)

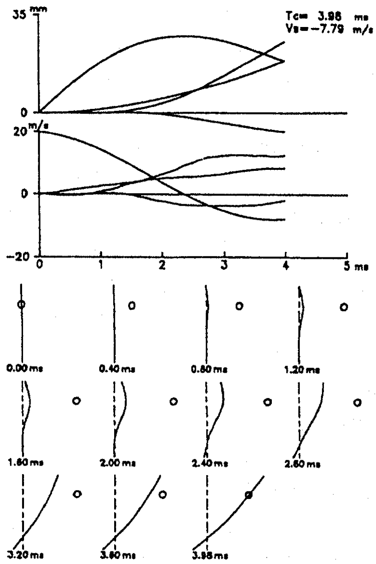


(b) EI_1=9 999 Nm^2(EI_2=0.88 EI_1)

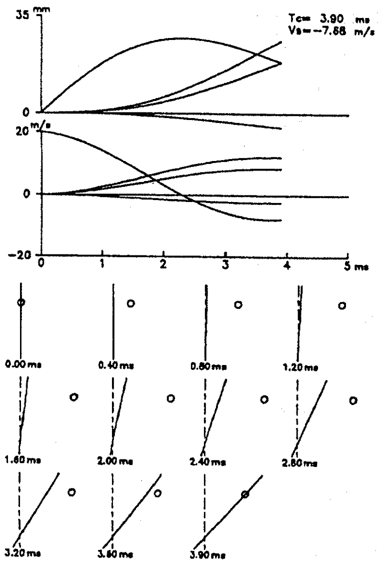


(c) [(a)-(b)]

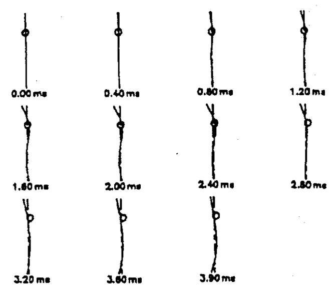
Fig. 5 Calculated results for impact behavior in the impact on grip side (0.60L)



(a) EI_1=146 Nm^2(EI_2=0.88EI_1)

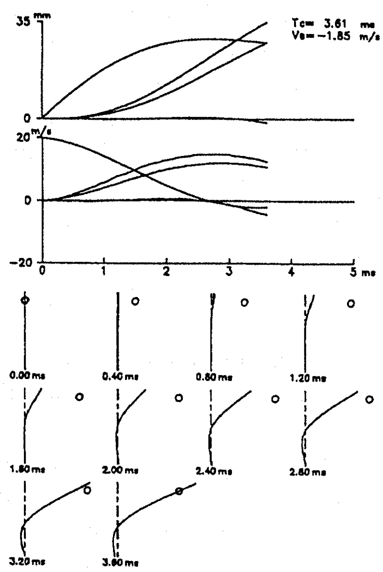


(b) EI_1=9 999 Nm^2(EI_2=0.88EI_1)

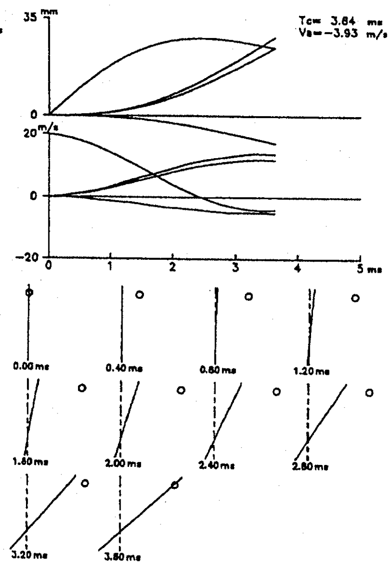


(c) [(a)-(b)]

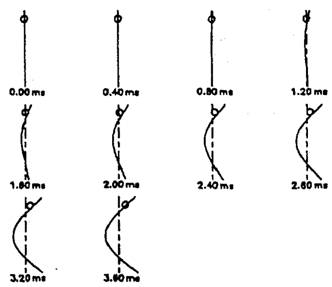
Fig. 6 Calculated results for impact behavior in the impact on the center (0.75L)



(a) EI_1=146 Nm^2(EI_2=0.88EI_1)



(b) EI_1=9 999 Nm^2(EI_2=0.88EI_1)



(c) [(a)-(b)]

Fig. 7 Calculated results for impact behavior in the impact on top side (0.90L)

time of impact. The deformation shape is similar to the first order vibration mode of the frame. (3) At the $0.75L$ impact point, there is little deformation due to frame vibration. (4) Frame rotation increases as the impact point moves away from the center of gravity ($0.453L$).

3.2 Effect of frame vibration on coefficient of restitution

Figure 8 shows the changes in coefficient of restitution (ratio of ball rebound velocity to incident velocity) caused by different impact points for the standard frame ($EI_1=146\ \mathrm{Nm^2}$) and the rigid frame ($EI_1=9999\ \mathrm{Nm^2}$). The coefficient of restitution for the rigid frame decreases almost linearly as the impact point moves away from the center of gravity. However, for the standard frame, in addition to the changes seen with the rigid frame, the coefficient of restitution decreases as the impact point moves away from the node position of the first order vibration of the frame ($0.769L$).

3.3 Effect of frame vibration on contact time

Figure 9 shows the changes in contact time at the impact point for the standard and rigid frames when damping of the compound ball/string system is taken into consideration ($\zeta=0.059$) and when there is no damping. From this figure, the contact time was found to decrease as the impact point moves away from the center of gravity (to the outside). In addition, differences in frame rigidity and damping of the compound ball/string system resulted in almost no difference in the contact time.

4. Impact Analysis of Ball and Racket System and Discussion of Racket Restitution Capabilities

4.1 Impact analysis of compound ball/string system and rigid frame

For simplicity in the discussion, an impact model shown in Fig. 10 is analyzed wherein the frame is taken to be a rigid body and the damping from the compound ball/string system is ignored. The frame mass is assumed to be m_R, the moment of inertia around the center of gravity is assumed to be I_R, and the distance between the center of gravity of the frame and the impact point is assumed to be d. Based on these definitions, the angular frequency ω for the model, excluding rigid movement (since the center of gravity for the frame and the impact position are different, there is some translation and rotation movement of the frame), can be expressed as follows.

$$\omega=\sqrt{\{(1+\alpha)m_B+m_R\}K_{GB}/(m_B m_R)} \qquad (5)$$

where

$$\alpha=d^2/(I_R/m_R) \qquad (6)$$

After impact on the frame through compound spring K_{GB} at impact velocity V_{Bo}, the ball velocity dX_1/dt becomes the following.

$$dX_1/dt=V_T+V_R+V_I\cos\omega t \qquad (7)$$

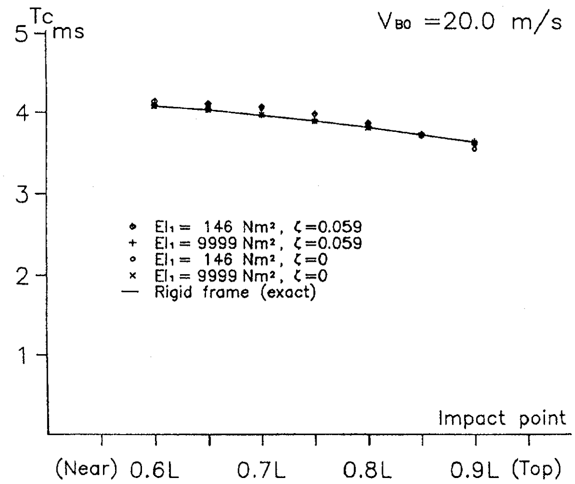


Fig. 9 Contact time at impact points for standard and rigid frames (when consideration of damping of ball/string system is included and not included)

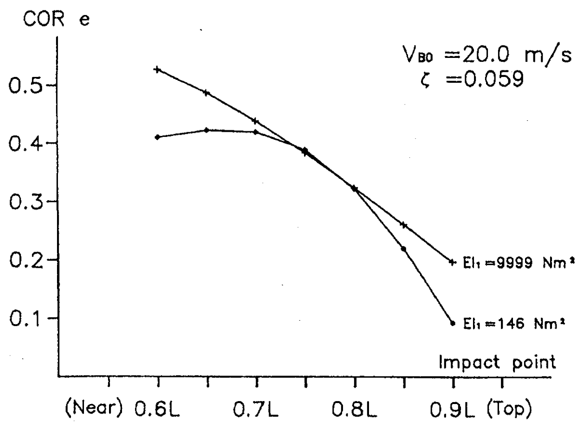


Fig. 8 Coefficient of restitution for standard and rigid frames (ratio of ball rebound velocity to incident velocity)

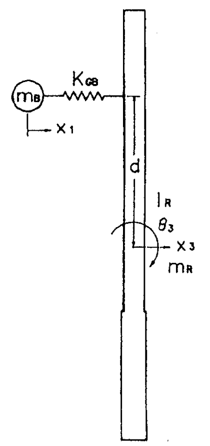


Fig. 10 Analysis model when frame is taken as rigid body and damping of compound ball/string system is ignored

Where V_T, V_R, and V_I are the velocity components of translation and rotation for the rigid body, and of characteristic vibration and expressed as follows:

$$V_T = V_{Bo} m_B/(m_B + m_R) \quad (8)$$

$$V_R = V_{Bo} \alpha m_B m_R / [\{m_B + m_R\}\{(1+\alpha)m_B + m_R\}] \quad (9)$$

$$V_I = V_{Bo} m_R / \{(1+\alpha)m_B + m_R\} \quad (10)$$

The contact time T_c and ball rebound velocity V_B then become the following:

$$T_c = \pi/\omega = \pi\sqrt{m_B m_R} / \sqrt{\{(1+\alpha)m_B + m_R\}K_{GB}} \quad (11)$$

$$V_B = V_T + V_R - V_I = V_{Bo}\{(1+\alpha)m_B - m_R\} / \{(1+\alpha)m_B + m_R\} \quad (12)$$

When the m_R, I_R, and d values of the standard frame are used, the results of contact time T_c from Eq. (11) becomes a solid line in Fig. 9. Contact time T_c is clearly controlled primarily by Eq. (11).

4.2 Modal Analysis of System Containing Compound Ball/string and Elastic Frame

For simplicity, when ζ is assumed to be 0 in Fig. 1(b) and modal analysis is executed, ball velocity dX_1/dt can be written as follows:

$$dX_1/dt = V_T + V_R + V_1 \cos\omega_1 t + V_2 \cos\omega_2 t + V_3 \cos\omega_3 t + \cdots \quad (13)$$

where V_T and V_R represent the velocity components of the translation and rotation movement of the rigid body and are the same as above for the rigid body frame. ω_i and V_i $(i=1, 2, 3, \cdots)$ represent the ith angular frequency and its velocity component.

Now, the ball impact velocity V_{Bo} $(t=0)$ can be expressed by the following equation:

$$V_{Bo} = V_T + V_R + V_1 + V_2 + V_3 + \cdots \quad (14)$$

If $t=0$ in Eq. (7) then $V_{Bo} = V_T + V_R + V_I$ and, by comparison with Eq. (14), the following equation is developed:

$$V_I = V_1 + V_2 + V_3 + \cdots \quad (15)$$

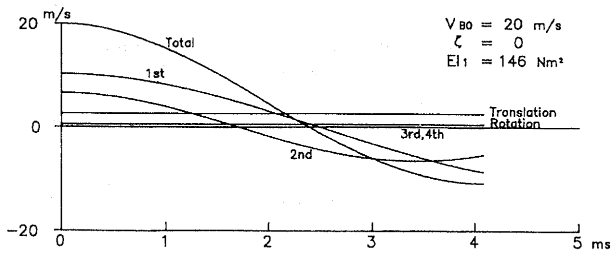
From Eq. (13) and Eq. (15), the following inequality can be derived for ball rebound velocity V_B.

$$|V_B| \leqq |V_T + V_R - V_I| \quad (16)$$

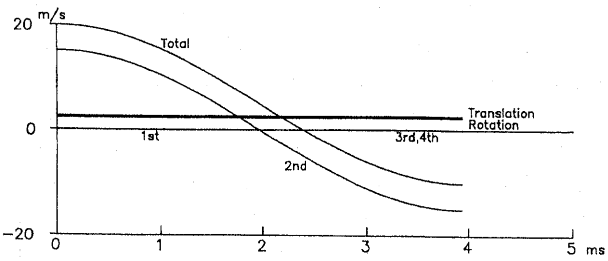
Therefore, when the beam is elastic the rebound velocity cannot exceed the rebound velocity obtained for the rigid body.

Figure 11 shows the changes in each term of Eq. (13) when an impact velocity of 20 m/s occurs at impact points 0.60L and 0.769L (node of first mode of the frame). For reference, the frequency and mode shape of the ball and racket system at this time are shown in Fig. 12.

From Fig. 11 and Fig. 12, the third order (second order of frame) and higher order vibrations of the ball and racket system appear to have little influence. When the ball impacts at the node positions of the frame, the natural frequency of the frame itself appears as natural frequency of the system and the V_i of the corresponding mode is 0. For impact at the node position of the first mode vibration of the frame, $V_1=0$. If V_3 and higher order terms in Eq. (15) are then ignored, $V_1 \fallingdotseq V_2$ and, from the relationship in Eq. (16), the restitution velocity increases. Rigid body movement at impact only serves to remove momentum and adversely affect the restitution. In addition, rigid body movement is completely unrelated to frame rigidity and translation movement is unrelated to impact position. However, from Eq. (9), rotational movement adversely affects restitution as α increases thereby having a large effect on the rebound velocity of the ball.

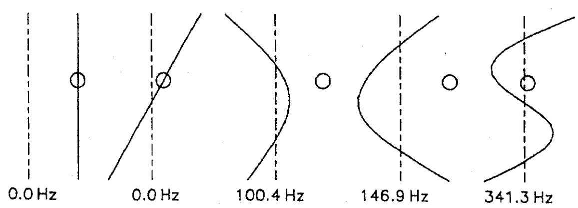


(a) Impact point at 0.60L

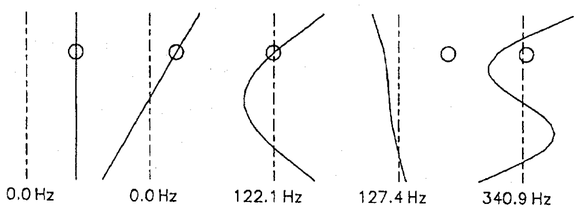


(b) Impact point at 0.769L (Node of first mode of frame)

Fig. 11 Changes in each term of Eq. (13) for impact points at 0.60L and 0.769L



(a) 0.60 L



(b) 0.769 L

Fig. 12 Natural frequency and mode shape of ball and racket system when impact points are 0.60L and 0.769L

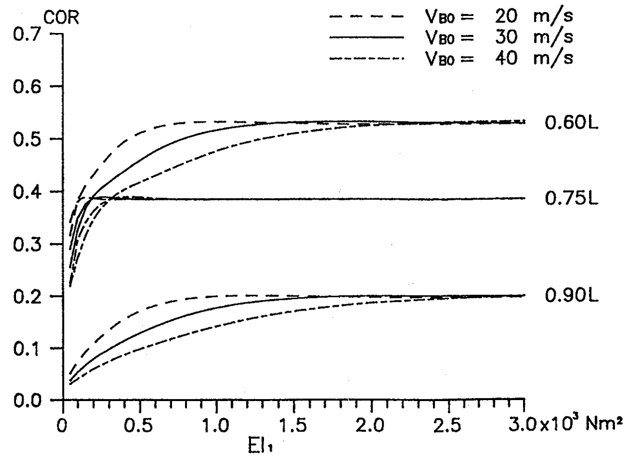


Fig. 13 Effect of frame rigidity on coefficient of restitution (impact velocities: 20 m/s, 30m/s, and 40m/s)

5. Effect of Frame Rigidity on Coefficient of Restitution

Figure 13 shows the changes in the coefficient of restitution occurring when frame rigidity EI_1 (EI_2= 0.88EI_1) is varied from 50 to 3 000 Nm^2. From the figure, increases in the coefficient are initially rapid, and then taper off to a saturated condition as frame rigidity is increased further. The frame rigidity at which the coefficient begins saturation is higher at greater impact velocities. The results shown in Fig. 13 are essentially simple with small maxima and minima and no unevenness whereby racket design can have an influence. When the damping of the ball/string system is ignored, equal coefficients of restitution converge at each impact point regardless of impact velocity.

6. Conclusion

The major results of the present study are summarized below.

(1) Impact in a ball and racket system is controlled by the rigid body movement and the first order (2 node bending) mode vibration of the frame. The contribution of second order (3 node bending) and higher mode vibrations of the frame is small.

(2) Although the coefficient of restitution increases with increases in frame rigidity, this increase saturates at a certain rigidity. The frame rigidity at which saturation is reached is higher for faster impact velocities and the changes are essentially simple.

(3) The coefficient of restitution is greatly influenced by the rigid body movement of the ball and racket system. As impact positions move away from the center of rotation (nearly the center of gravity of the racket) the coefficient decreases. As impact approaches the positions of the nodes of the frame vibration the coefficient increases.

(4) When the impact velocity of the ball and racket are fixed, the contact time is affected little by frame rigidity and is nearly the same as for a rigid body frame. In addition, contact time changed little at different impact positions.

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