720 テニスラケットの打球感に関する研究

（グリップおよび手首関節の衝撃振動のメカニズム）

Study on the Feel of Tennis Racket at Impact

: Mechanism of Impact Shock Vibrations of a Racket Grip and a Player's Wrist Joint

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At the current stage, the performance of tennis racket is based on the feeling of an experienced tester or a player. However, the optimum racket depends on the physical and technical levels of each user. Accordingly, there are a number of unclear points regarding the relationship between the performance estimated by a player and the physical properties of a tennis racket.

This paper investigated the mechanism of racket performance in terms of the feel or comfort of the arm or hand in an impact. It deals with the shock vibrations of the wrist joint caused by the impact when a male tournament player hits flat forehand drive. It is based on the identification of the racket-arm system and the predicted coefficient of restitution between a racket and a ball.

The predicted waveform of the shock vibrations at the wrist joint agrees fairly well with the measured ones during actual forehand stroke by a player, showing that the shock vibrations of the wrist joint transmit from the racket with an impulse at the impact location and the several vibrations mode components of a racket frame and strings. The predicted results could also explain the difference in racket performance in terms of feel or comfort between the rackets with different physical properties.

Figure A1 shows the locations of attached accelerometers at the wrist joint and the elbow joint in the experiment, where a male tournament player hits flat forehand drive. Figure A2 shows an impact model for the prediction of shock forces. The impact force S_0 at P_0 causes a shock force S_1 on the player's hand P_1, a shock force S_2 on his elbow P_2, and finally a shock force S_3 on the player's shoulder P_3 during the impact at which the player hits the ball with his racket. Figure A3 shows the predicted shock vibrations of a wrist joint compared with the experimental ones when a ball is struck at the topside of the racket face.

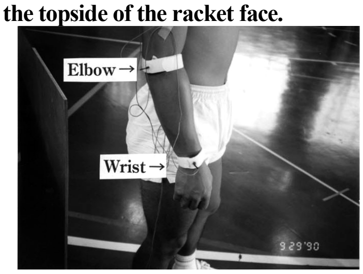

Fig.A1 Accelerometers attached at the wrist and the elbow.

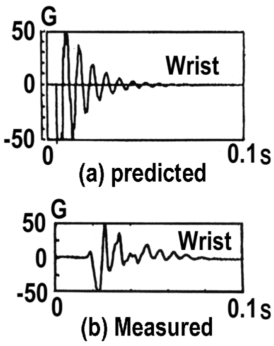

Fig.A3 Predicted shock vibrations of a wrist joint compared with the experimental.

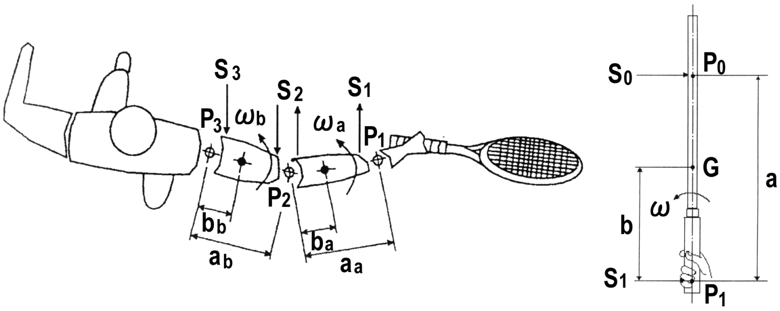

Fig.A2 Impact model for the prediction of the shock force transmitted to the arm joints from a racket.

1. INTRODUCTION

With sport equipment, engineering technology has advanced to enable manufacturers to discover and synthesize new materials and new design. There are rackets of all compositions, sizes, weights, shapes and string tension. At the current stage, very specific designs are targeted to match the physical and technical levels of each user. However, ball and racket impact in tennis is an instantaneous phenomenon creating vibrations and large deformations of ball/strings in the racket. The problem is further complicated by the involvement of humans in the actual strokes. Therefore, there are many unknown factors involved in the mechanisms explaining how the specifications and physical properties of the racket frame influence the racket capabilities.

This paper investigates the mechanism of racket performance in terms of the feel or comfort of the arm or hand in an impact. It deals with the shock vibrations of the wrist joint caused by the impact when a male tournament player hits flat forehand drive. It presents a new approach and a method for the prediction of the impact shock vibrations of a racket grip and a player's wrist joint in order to give the physical explanation for the measured waveforms. It is based on the identification of the racket characteristics, the damping of the racket-arm system, the equivalent mass of the player's arm system and the approximate nonlinear analysis of the impact in tennis.

2. PREDICTION OF SHOCK FORCES TRANSMITTED TO THE ARM JOINT FROM A RACKET IN THE IMPACT OF TENNIS

2.1 IMPACT SHOCK FORCES OF AN ARM JOINT SYSTEM

Figure 1 shows the situation of experiment where a male tournament player hits flat forehand drive and Fig.2 shows the locations of attached accelerometers at the wrist joint and the elbow joint in the experiment. An accelerometer was also attached at 210 mm distance from the grip end on the racket handle. Figure 3 shows an impact model for the prediction of shock force transmitted to the arm joint from a racket. The impact force S_0 at P_0 causes a shock force S_1 on the player's hand P_1, a shock force S_2 on his elbow P_2, and finally a shock force S_3 on the player's shoulder P_3 during the impact at which the player hits the ball with his racket.

Obviously , the velocity variations of the body segments are proportional to the respective impulses $I=\int S\,dt$ and inversely proportional to the mass to which they are applied. Since the intensity of the impulse decreases with the distance from the point of impact with the ball, it can be assumed that the shoulder does not basically alter its velocity, despite the presence of the shock force S_3. Generally speaking, the shock forces S_0 ,S_1 ,S_2 , and S_3 , which is mainly responsible for the sudden changes in velocity that take place in the brief interval of time considered, is one order of magnitude higher than the other forces in play during the same interval; consequently the gravity force and muscular action are not taken into account in this paper. In other words, we consider the racket to be freely hinged to the forearm of the player, the forearm being freely hinged to the arm and the arm freely hinged to the player's body. Clearly, this schematization only refers to the interval lasting no longer than one hundredth of a second: indeed, both before and afterwards, in the absence of shock forces S_0 , S_1 ,S_2 , and S_3, all the movements depend on the intensity of the muscular forces and gravity forces in play. Therefore, the intensity with which the player grips the racket at the moment of impact with the ball has no effect on the phenomenon itself.

If we indicate the articulation of the hand by P_1 , the articulation of the elbow by P_2 and the articulation of the shoulder by P_3, the limb is schematized by two straight line

Fig.1 Experiment where a male player hits flat forehand drive.

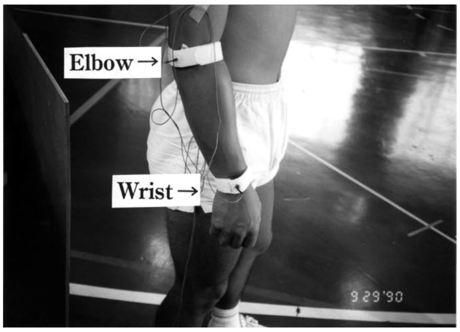

Fig.2Accelerometers attached at the wrist and the elbow.

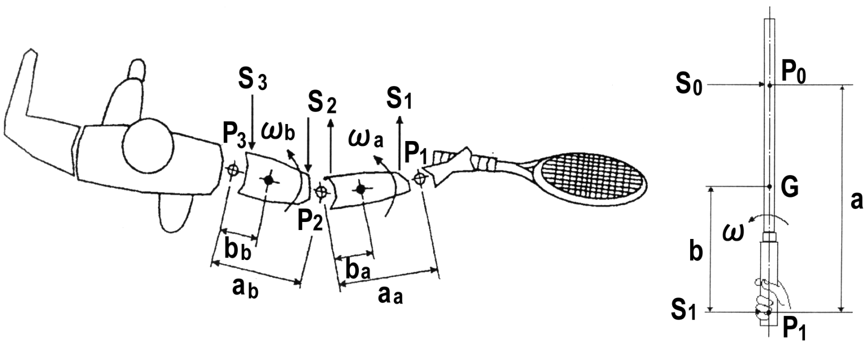

Fig.3 Impact model for the prediction of the shock force transmitted to the arm joints from a racket.

segments P_1 P_2 and P_2P_3. The forearm length $a_a = P_1P_2$, with a mass m' to which the mass m'' of the hand is added, concentrated at P_1 : consequently, the total mass of the forearm is equal to $m_a = m' + m''$ and the distance b_a of the center of mass from elbow is

$$b_a = G_1P_2 = [\, m'(a_a/2) + m''a_a]/m_a \qquad (1)$$

Moreover, if we indicate the moment of inertia around the elbow P_2 by J_a, the mass of the arm, with a length of $a_b = P_2P_3$, by m_b, the distance of the center of mass from the shoulder P_3 by $b_b = G_3P_3$, while the moment of inertia with respect to the shoulder P_3 by J_b, we obtain the following relationship from the equations of motion with respect to the arm P_2P_3.

$$S_3 = S_2\,(m_b a_b b_b / J_b - 1) \qquad (2)$$

We can also derive the following relationship from the equations of motion for the forearm P_1P_2 and a few calculation steps.

$$S_2 = S_1\,(m_a a_a b_a / J_a - 1) / [\,1 + (m_a a_b^2 / J_b)(1 - m_b b_a^2 / J_a)] \qquad (3)$$

$$\mathrm{d}V_1/\mathrm{d}t = \mathrm{d}(\omega_a a_a + \omega_b a_b + V_3)/\mathrm{d}t = S_1\,[\,\mu_a a_a^2 / J_a - \chi_a a_b^2 / J_b] \qquad (4)$$

where

$$\mu_a = [1 + (m_a a_b^2 / J_b)(1 - b_a / a_a)] / [1 + (m_a a_b^2 / J_b)(1 - m_a b_a^2 / J_a)] \qquad (5)$$

$$\chi_a = (m_a a_a b_a / J_a - 1) / [\,1 + (m_a a_b^2 / J_b)(1 - m_b b_a^2 / J_a)\,] \qquad (6)$$

i.e. by assuming

$$M_H = 1 / [\,\mu_a a_a^2 / J_a - \chi_a a_b^2 / J_b\,] \qquad (7)$$

finally we have the acceleration A_{nv} at the grip portion and the wrist joint as

$$A_{nv} = \mathrm{d}V_1 / \mathrm{d}t = S_1 / M_H \qquad (8)$$

From Eq.(8) we can deduce that the inertia effect of the arm and the forearm can be attributed to a mass M_H concentrated in the hand; therefore the analysis of impact between ball and racket can be carried out by assuming that the racket is free in space, as long as the mass M_H is applied at point P_1 of the hand grip.

If the impact force S_0 between a ball and the racket is given when the ball hits the racket, the shock force S_1 can be obtained with a few steps as

$$\begin{aligned} S_1 &= M_H \mathrm{d}V_1 / \mathrm{d}t = M_H A_{nv} \\ &= S_0 (M_R ab/J - 1) / [\,1 + (M_R / M_H)(1 - M_R b^2 / J)] \end{aligned} \qquad (9),$$

where we indicate the mass of the racket by M_R, the distance between the grip location on the handle and the impact location on the string face by a, the distance between the grip location on the handle and the center of mass of the racket by b, and the moment of inertia with respect to the articulation P_1 of the hand by J. Thus, the shock forces S_2 and S_3 also can be obtained from Eq.(3) and Eq.(2), respectively.

2.2 DERIVATION OF THE IMPACT FORCE AND THE CONTACT TIME AT THE INSTANT WHEN THE PLAYER HITS THE BALL WITH HIS RACKET

The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum when the moment of inertia and the distance between an impact location are given. The reduced mass M_r at the impact location with a racket-arm system can be derived as

$$\begin{aligned} M_r &= 1/[\,1/(M_R + M_H) + c^2/I_G\,] \\ &= (M_R + M_H) I_G / [\,I_G + (M_R + M_H) c^2\,] \end{aligned} \qquad (10)$$

where

$$c = c_o + (L_{Go} - L_H) M_H / (M_R + M_H) \qquad (11)$$

$$I_G = I_{Go} + M_R \triangle G^2 + M_H (L_{Go} - L_H - \triangle G)^2 \qquad (12)$$

$$\triangle G = (L_{Go} - L_H) M_H / (M_R + M_H) \qquad (13)$$

and L_{Go} denotes the distance between the center of mass and the grip end of the racket, I_{Go} the moment of inertia with respect to the center of gravity of the racket, c_o the distance between the center of gravity and the impact location of the racket, and L_H the distance of the point P_1 of the hand grip from the grip end The moment of inertia with respect to the center of gravity and the distance of the center of gravity from the impact location of the racket-arm system are indicated by I_G and c, respectively.

In case the vibration of the racket frame is neglected, the post-impact velocity V_B of a ball and V_R of a racket head at the impact location are derived using the momentum equation and the measured coefficient restitution e_{BG} with a ball striking the a racket head clamped. The impulse at impact between ball and racket could also be obtained.

It is assumed that the contact time T_c during impact is half the natural period of a whole system composed of the mass m_B of a ball, the equivalent stiffness K_{GB} of ball/strings, and the reduced mass M_r of the racket. If we introduce the equivalent force F_{mean} during contact time T_c, the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity is derived. On the other hand, from the measured restoring force characteristics of a ball and strings, the restoring force can be expressed as a function of K_{GB}. Thus, the parameters K_{GB} and F_{mean} against the pre-impact velocity can be obtained. Accordingly the contact time T_C can also be determined against the pre-impact velocity. Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is the more likely impulse waveform.

The vibration characteristics of a racket can be identified using the experimental modal analysis (Kawazoe 1989, 1990) and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of the racket. When the impact force component of k-th mode frequency f_k in the frequency region applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i can be derived using the residue r_{ijk} of k-th mode between arbitrary point i and j (Kawazoe, 1993).

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along a racket frame when an impact location on the string face and the impact velocity are given.

The coefficient of restitution e_r (COR) can be derived considering the energy loss E during impact. The main source of energy loss is E_1 due to racket vibrations as well as E_2 due to the instantaneous large deformation of a ball and strings corresponding to the coefficient e_{BG}.

The post-impact ball velocity V_B is represented as

$$V_B = -V_{Bo}(e_r - m_B/M_r)/(1 + m_B/M_r) + V_{Ro}(1 + e_r)/(1 + m_B/M_r) \qquad (14)$$

Furthermore, the force-time curve of impact between a ball and a racket considering the vibrations of a racket frame can be derived as

$$S_0(t) = S_{0max}\ \sin(\pi t / T_c) \quad (0 \leqq t \leqq T_c) \qquad (15)$$

where

$$S_{0max} = \pi F_{mean} / 2 = (\pi / (2T_c))\,(V_{BO} - V_{Ro})(1+e_r)\, m_B / (1 + m_B / M_r) \qquad (16)$$

2.3 PREDICTION OF SHOCK FORCES AND SHOCK ACCELERATIONS TRANSMITTED TO THE ARM JOINT FROM A RACKET IN THE IMPACT OF TENNIS

The shock acceleration at the hand grip considering the equivalent mass M_H of the arm system can be represented as

$$A_{nv}(t) = S_0(t)\,[1/(M_R + M_H) - (a/I_G)X] \qquad (17)$$

where X denotes the distance between the center of mass of racket-arm system and the location of hand grip, a the distance between the center of mass of racket-arm system and the impact location of the racket, I_G the moment of inertia with respect to the center of mass of racket-arm system, respectively. The maximum shock force $S_{1\,max}$ transmitted to a wrist joint corresponds to the maximum impact force $S_{0\,max}$.

3. PREDICTION OF THE WAVEFORMS OF SHOCK VIBRATIONS AT THE GRIP

Although the frequency drops slightly for the hand-held racket compared to the freely suspended racket, the positions of nodes on the string face are nearly identical. With a primary vibration mode, the position of the node on the handle for the hand-held racket shifts somewhat to the held position. Although the damping of frame vibrations is remarkably larger for the hand-held racket compared to the freely suspended racket, there is no big difference in the initial amplitude distributions of a racket frame between them.

The vibration acceleration component of k-th mode at the location i of hand grip is represented as

$$A_{ijk}(t) = -(2\pi f_k)^2\, r_{ijk}\, S_{0j}(2\pi f_k)\, exp(-2\pi f_k \zeta_k t)\, sin(2\pi f_k t) \qquad (18)$$

where j denotes the impact location between ball and racket on the string face, ζ_k the damping ratio of k-th mode, $S_{0j}(2\pi f_k)$ the fourier spectrum of Eq.(15).

The summation of Eq.(17) and Eq.(18) represents the shock vibrations at the hand grip.

4. PREDICTION OF THE WAVEFORMS OF SHOCK VIBRATIONS AT THE WRIST JOINT

Figure 4 shows the predicted shock vibrations of a wrist joint compared with the experimental ones when a ball is struck at the topside of the racket face. This racket is made of 75 % graphite, 20 % fiberglass and 5% others, with 685 mm of total length, 100 in^2 of face area, 342 g of mass including string mass, 310 mm of the center of mass from grip end, 14.2 gm^2 of moment of inertia about the center of racket mass, 60 lb. of string tension. The center of mass of racket-arm system shifts to the location of 131 mm from the grip end. In the figure, the first largest peak

During the impact was caused by the shock and vibrations of a racket frame, followed by the residual vibrations. The shock vibrations are composed of the shock acceleration and the racket vibration components, and each component has its own time history and magnitude depending on the impact velocity, impact location, grip location of racket handle and the physical properties of a racket. The damping ratio of a hand-held racket during actual impact is estimated as about 2.5 times that of the one identified by the experimental modal analysis with small vibration amplitude. Furthermore, the damping of the waveform at the wrist joint was 3 times that at the grip portion of the racket handle. The predicted waveforms of the shock vibrations with the racket handle and the wrist joint agrees fairly well with the measured ones during actual forehand stroke by a player. The mechanism of the shock vibrations of the elbow joint, however, is left unsolved.

Figure 5 shows the waveforms of predicted shock vibrations at the grip of various types of tennis rackets available in the market shown in Table 1, where the ball hits the top side off the center line on the string face of freely suspended rackets. In Table 1, the sign I_{GY} denotes the moment of inertia about the center of mass, the I_{GR} the moment of inertia about the grip portion 70 mm from the grip end, the I_{GX} the moment of inertia about the longitudinal axis of racket head.

Figure 6 shows the estimation of the feel or comfort of various types of tennis rackets by the initial peak-peak value of acceleration waveform, where the ball hits the top side and the near side on the string face of freely suspended rackets and the hand-held racket shown in Table 1.

The predicted results could explain the difference in racket performance in terms of feel or comfort between the rackets with different physical properties.

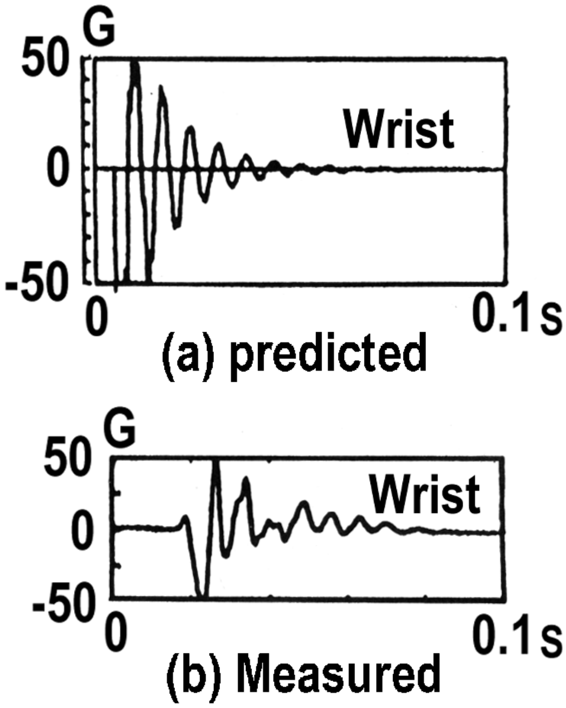

Fig. 4 Predicted shock vibrations of a wrist joint compared with the experimental.

5. CONCLUSIONS

The predicted waveform of the shock vibrations at the wrist joint agrees fairly well with the measured ones during actual forehand stroke by a player.

It shows that the shock vibrations of the wrist joint transmit from the racket with an impulse at the impact location and the several vibrations mode components of a racket frame and strings.

The predicted results could also explain the difference in racket performance in terms of feel or comfort between the rackets with different physical properties.

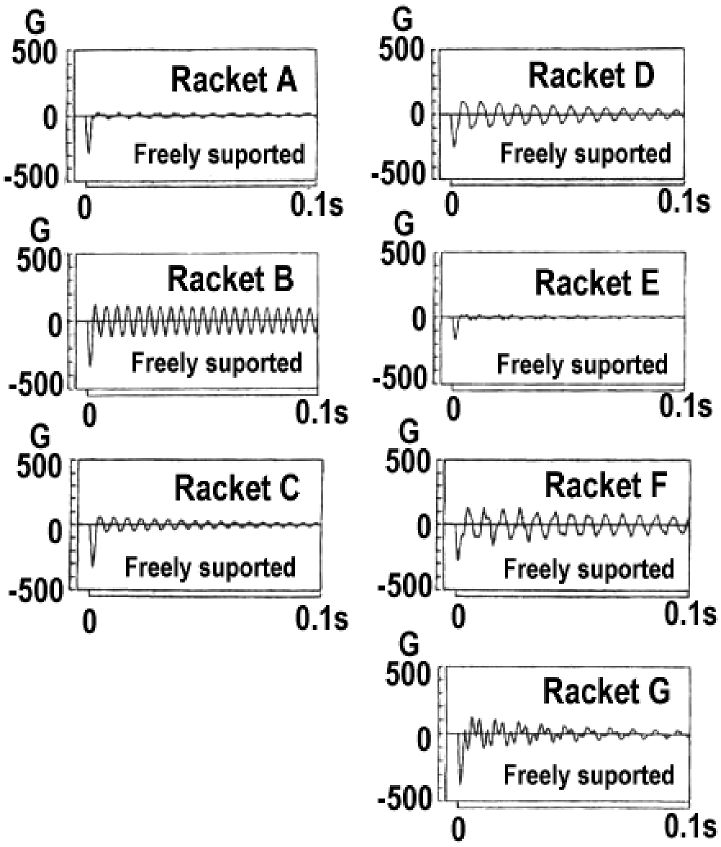

Fig.5 Waveforms of predicted shock vibrations at the grip of various types of tennis rackets, where the ball with 30 m/s hits the topside off the centerline of freely suspended rackets.

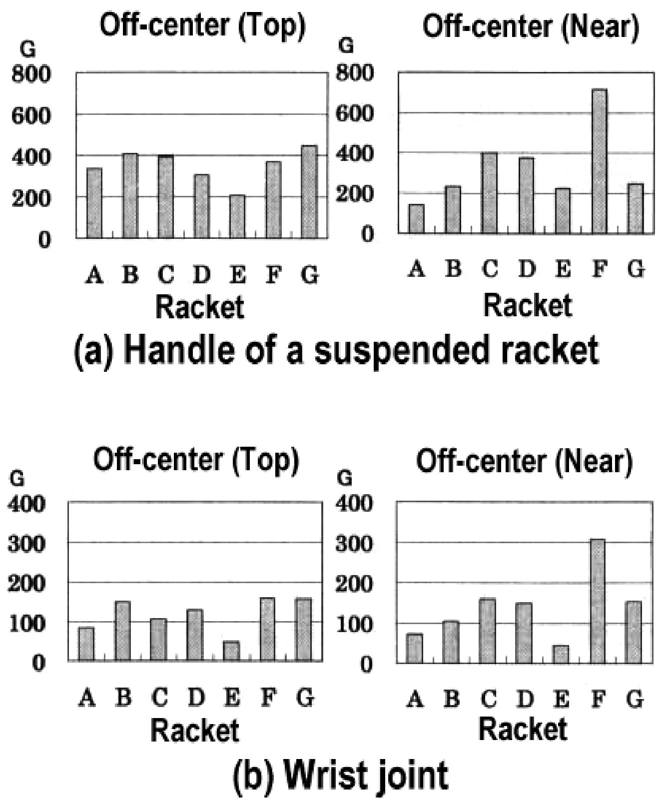

Fig.6 Estimation of the feel or comfort of various types of tennis rackets by the initial peak-peak value of acceleration waveform, where the ball hits the top side and the near side on the string face of (a) freely suspended rackets and (b) the hand-held rackets.

Table 1 Physical properties of different type of tennis rackets

Racket	A	B	C	D	E	F	G
Face area	100 in^2	100 in^2	100 in^2	110 in^2	120 in^2	120 in^2	68 in^2
Total length	27 in 680 mm	27 in 680 mm	27 in 680 mm	27 in 685 mm	27 in 685 mm	27 in 690 mm	27 in 685 mm
Mass (+Strings)	360 g	370 g	290 g	366 g	349 g	292 g	375 g
Center of Gravity	308 mm	317 mm	350 mm	325 mm	323 mm	363 mm	335 mm
I_{GY}	13.1 $g \cdot m^2$	14.0 $g \cdot m^2$	11.4 $g \cdot m^2$	16.9 $g \cdot m^2$	16.0 $g \cdot m^2$	14.0 $g \cdot m^2$	14.8 $g \cdot m^2$
I_{GR}	33.5 $g \cdot m^2$	36.6 $g \cdot m^2$	34.1 $g \cdot m^2$	40.7 $g \cdot m^2$	38.0 $g \cdot m^2$	39.0 $g \cdot m^2$	41.2 $g \cdot m^2$
I_{GX}	1.29 $g \cdot m^2$	1.62 $g \cdot m^2$	1.12 $g \cdot m^2$	1.68 $g \cdot m^2$	2.21 $g \cdot m^2$	1.78 $g \cdot m^2$	0.94 $g \cdot m^2$
1st freq	122 Hz	215 Hz	171 Hz	132 Hz	142 Hz	137 Hz	103 Hz
Strings tension	55 lbs	55 lbs	55 lbs	63 lbs	79 lbs	79 lbs	50 lbs
Reduced mass	170 g	196 g	175 g	220 g	205 g	206 g	188 g

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REFERENCES

(1)Casolo,F. & Ruggieri,G. (1991), Dynamic analysis of the ball-racket impact in the game of tennis, *Meccanica*,Vol.24, pp.501-504.

(2) Kawazoe,Y. (1989), Dynamics and computer aided design of tennis racket. *Proc. Int. Sympo. on Advanced Computers for Dynamics and Design'89*, pp.243-248.

(3) Kawazoe,Y. (1992), Impact phenomena between racket and ball during tennis stroke, *Theoretical and Applied Mechanics*, Vol.41, pp.3-13.

(4) Kawazoe,Y. (1993), Coefficient of restitution between a ball and a tennis racket, *Theoretical and Applied Mechanics*, Vol.42, pp.197-208.

(5) Kawazoe,Y. (1994), Effects of String Pre-tension on Impact between Ball and Racket in Tennis, *Theoretical and Applied Mechanics*, Vol.43, pp.223-232.

(6) Kawazoe,Y. (1994), Computer Aided Prediction of the Vibration and Rebound Velocity Characteristics of Tennis Rackets with Various Physical Properties, *Science and Racket Sports*,pp.134-139.E & FN SPON.

(7) Kawazoe,Y. (1997), Experimental Identification of Hand-held Tennis Racket Characteristics and Prediction of Rebound Ball Velocity at Impact, *Theoretical and Applied Mechanics*, Vol.46, 165-176.

(8) Kawazoe,Y., Tomosue,R. & Miura,A. (1997), Impact shock vibrations of the wrist and the elbow in the tennis forehand drive: remarks on the measured wave forms cosidering the racket physical properties, *Proc. of Int. Conf. on New Frontiers in Biomechanical Engineering*,pp.285-288.

(9) Kawazoe,Y. (1992), Ball/racket impact and computer aided design of rackets. *International J. Table Tennis Science, No.*1, pp.9-18.

(10) Kawazoe,Y. (1997), Performance Prediction of Tennis Rackets with Materials of the Wood and the Modern Composites, *5th Japan Int. SAMPE Sympo.& Exhibition,* pp.1323-1328.

(11) Kawazoe,Y. & Kanda,Y. (1997), Analysis of impact phenomena in a tennis ball-racket system (Effects of frame vibrations and optimum racket design), JSME International Journal, Series C, Vol.40, No.1, pp.9-16.