719 Mechanism and Prediction of the Power of Tennis Racket

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Key Words: **Dynamics, Sport engineering, Tennis, Impact, Tennis racket, Coefficient of restitution, Power, Prediction of performance**

With sport equipment, engineering technology has advanced to enable manufacturers to discover and synthesize new materials and new design. There are rackets of all compositions, sizes, weights, shapes and string tension. At the current stage, very specific designs are targeted to match the physical and technical levels of each user. However, ball and racket impact in tennis is an instantaneous phenomenon creating vibrations and large deformations of ball/strings in the racket. The problem is further complicated by the involvement of humans in the actual strokes. Therefore, there are many unknown factors involved in the mechanisms explaining how the specifications and physical properties of the racket frame influence the racket capabilities.

This paper has investigated the physical properties of a racket, predicting racket performance in terms of the coefficient of restitution, the rebound power coefficient and the post-impact ball velocity. It is based on the experimental identification of the racket dynamics and the approximate nonlinear impact analysis with a simple forehand swing model.

The predicted results could explain the mechanism of tennis racket performance in terms of power and the difference in performance between the rackets with different physical properties.

Figure A1 is a comparison between the measured *e* and the calculated *e* when a ball hits a freely-suspended racket (about 30 m/s), showing a good agreement between them.

Figure A2 shows the predicted V_B of various types of tennis rackets available in the market shown in Table A1, when a player hits flat forehand drive. The velocity of the coming ball is 10 m/s and the shoulder joint torque is 56.9 N·m. The hitting locations on the racket face are the topside and the center. The most powerful racket in the ground stroke is the racket that weighs 292 grams and has hitting areas of 120 square inches.

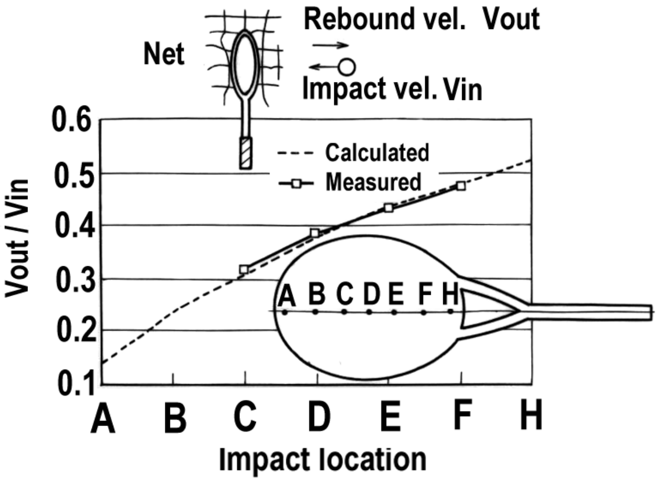

Fig.A1 Comparison between the measured *e* and the predicted *e*.

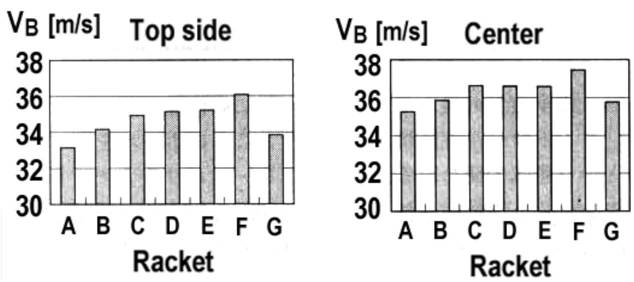

Fig.A2 Predicted post-impact ball velocity V_B of various types of tennis rackets.

Table A1 Physical properties of different type of rackets

Racket	A	B	C	D	E	F	G
Face area	100 in^2	100 in^2	100 in^2	110 in^2	120 in^2	120 in^2	68 in^2
Total length	27 in 680 mm	27 in 680 mm	27 in 680 mm	27 in 685 mm	27 in 685 mm	27 in 690 mm	27 in 685 mm
Mass (+Strings)	360 g	370 g	290 g	366 g	349 g	292 g	375 g
Center of Gravity	308 mm	317 mm	350 mm	325 mm	323 mm	363 mm	335 mm
I_{GY}	13.1 g·m^2	14.0 g·m^2	11.4 g·m^2	16.9 g·m^2	16.0 g·m^2	14.0 g·m^2	14.8 g·m^2
I_{GR}	33.5 g·m^2	36.6 g·m^2	34.1 g·m^2	40.7 g·m^2	38.0 g·m^2	39.0 g·m^2	41.2 g·m^2
I_{GX}	1.29 g·m^2	1.62 g·m^2	1.12 g·m^2	1.68 g·m^2	2.21 g·m^2	1.78 g·m^2	0.94 g·m^2
1st freq	122 Hz	215 Hz	171 Hz	132 Hz	142 Hz	137 Hz	103 Hz
Strings tension	55 lbs	55 lbs	55 lbs	63 lbs	79 lbs	79 lbs	50 lbs
Reduced mass	170 g	196 g	175 g	220 g	205 g	206 g	188 g

1. INTRODUCTION

With sport equipment, engineering technology has advanced to enable manufacturers to discover and synthesize new materials and new design. There are rackets of all compositions, sizes, weights, shapes and string tension. At the current stage, very specific designs are targeted to match the physical and technical levels of each user. However, ball and racket impact in tennis is an instantaneous phenomenon creating vibrations and large deformations of ball/strings in the racket. The problem is further complicated by the involvement of humans in the actual strokes. Therefore, there are many unknown factors involved in the mechanisms explaining how the specifications and physical properties of the racket frame influence the racket capabilities.

This paper investigates the physical properties of a racket, predicting racket performance in terms of the coefficient of restitution, the rebound power coefficient and the post-impact ball velocity. It is based on the experimental identification of the racket dynamics and the approximate nonlinear impact analysis with a simple forehand swing model.

2. PREDICTION OF THE COEFFICIENT OF RESTITUTION BETEEEN A BALL AND A RACKET

2.1 MAIN FACTORS ASSOCIATED WITH THE ENERGY LOSS AND COEFFICIENT OF RESTITUTION DURING IMPACT

2.1.1 Nonlinear restoring force characteristics of a ball and strings and a composed ball/strings system

Figure 1 shows the test for obtaining the applied force-deformation curves schematically, where the ball was deformed between two flat surfaces as shown in (a) and the ball plus strings were deformed with a racket head clamped as shown in (b). The results for the ball and racket strung at a tension of 246 N (55 lbs) are shown in Fig.2. According to the pictures of a racket being struck by a ball, it seems that the ball deforms only at the side, which contact to the strings.

Assuming that a ball with concentrated mass deforms only at the side in contact with the strings (Kawazoe, 1994), the curves of restoring force F_B vs. ball deformation, restoring force F_G vs. strings deformation, and the restoring force F_{GB} vs. deformation of the composed ball/strings system are obtained from Fig.2 as shown in Fig.3. These restoring characteristics are determined so as to satisfy a number of experimental data using the least square method. The curves of the corresponding stiffness K_B, K_G and K_{GB} are derived as shown in Fig.4 by differentiation of the equations of restoring force with respect to deformation, respectively.

The stiffness K_B of a ball, K_G of strings and K_{GB} of a composed ball/strings system exhibit the strong nonlinearity.

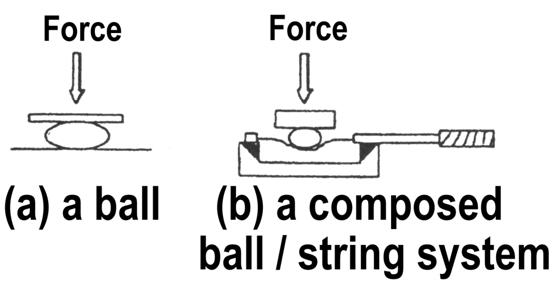

Fig.1 Illustrated applied force-Deformation test

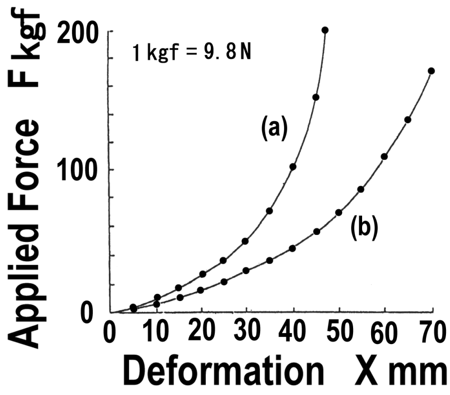

Fig.2 Results of a force-deformation test with pretension of strings 55 lbs(246 N)

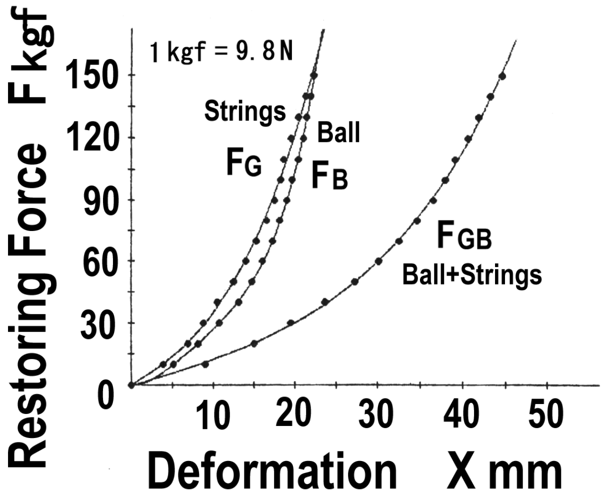

Fig.3 Restoring forces vs. deformation of a ball, strings, and a Composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

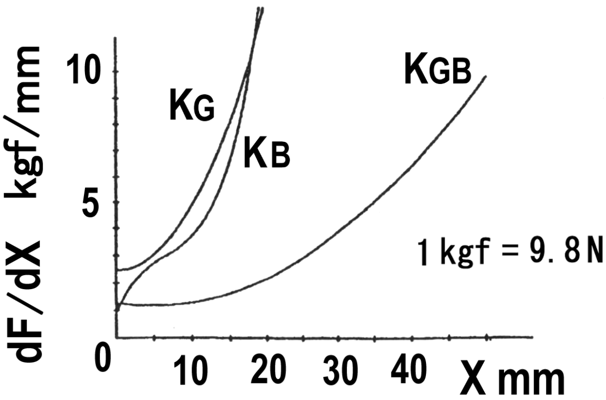

Fig.4 Stiffness vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

2.1.2 Energy loss in a collision between a ball and strings

The measured coefficient of restitution versus the incident velocity when a ball strikes the rigid wall is shown in Fig.5, while the measured coefficient of restitution e_{BG}, which is abbreviated as COR, when a ball strikes the strings with a racket head clamped is shown in Fig.6. Although the COR in Fig.5 decreases with increasing incident velocity, the coefficient e_{BG} with a racket head clamped is almost independent of ball velocity and strings tension. This value of COR can be regarded as being inherent to the materials of ball and strings, showing the important role of strings. This feature is due to the nonlinear restoring force characteristics of a composed ball/strings system(Kawazoe,1992). Accordingly, the energy loss of a ball and strings due to impact can be related to the coefficient e_{BG}.

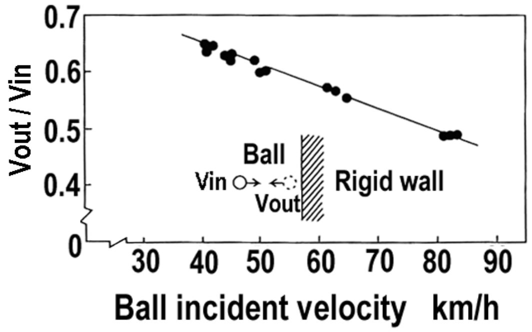

Fig.5. Measured coefficient of restitution (COR) between a ball and a rigid wall.

2.1.3 Remarks on the contact time between a racket and a ball during Impact

The result of measured contact time, which means how long the ball stays on the strings, with a normal racket and with a wide-body racket (stiffer) shows that the stiffness of the racket frame does not affect the contact time much (Kawazoe, 1992). Accordingly, the masses of a ball and a racket as well as the nonlinear stiffness of a ball and strings are the main factors in the deciding of a contact time. Therefore, the contact time can be calculated using a model assuming that a ball with a concentrated mass m_B and a nonlinear spring K_B, collides with the nonlinear spring K_G of strings supported by a frame without vibration, where the measured coefficient of restitution inherent to the materials of ball-strings impact is employed as one of the sources of energy loss.

2.1.4 Support condition of a racket handle

The result of the experimental modal analysis (Kawazoe, 1989,1997) showed that the fundamental vibration mode of a conventional type racket supported by a hand has two nodes being similar to the mode of a freely supported racket. Since this study deals with the racket performance in terms of power, it is assumed that the racket is freely suspended.

2.2 DERIVATION OF THE APPROXIMATE IMPACT FORCE AND THE CONTACT TIME

The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location are given.

In case the vibration of the racket frame is neglected, the momentum equation and the coefficient restitution e_{BG} give the post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse could be described as the following equation, where m_B is the mass of a ball, M_r is the reduced mass of a racket at the hitting location, and (V_{BO} - V_{Ro}) is the pre-impact velocity.

$$\int F\ (\ t)\ dt = m_B V_{Bo} - m_B V_B = (V_{BO} - V_{Ro})(1+ e_{BG}) m_B/(1+ m_B/M_r). \quad (1)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of m_B , K_{GB}, and M_r , it could be obtained as

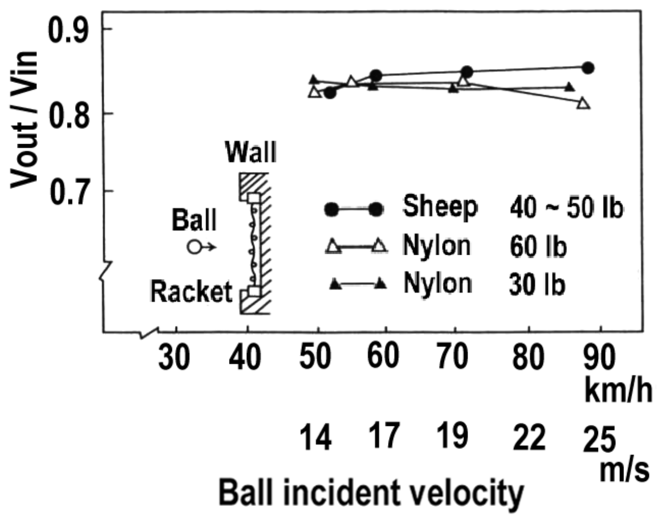

Fig.6. Measured COR between a ball and strings with frame clamped.

$$T_c = \pi\ m_B^{1/2} / (\ K_{GB} (1+ m_B/M_r\ \ )^{1/2} \quad (2)$$

In order to make the analysis simpler, the equivalent force F_{mean} can be introduced during contact time T_c, which is described as

$$\int^{Tc} F\ (\ t\ )\ dt = F_{mean} \cdot T_c \quad (3)$$

Thus, from Eq.(1), Eq.(2) and Eq.(3), the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity (V_{BO} - V_{Ro}) is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1+ e_{BG})\, m_B^{1/2}\, K_{GB}^{1/2} / \pi (\ 1+ m_B/M_r\ \ )^{1/2} \quad (4)$$

On the other hand, from the approximated curves shown in Fig.3 and Fig.4, F_{GB} can be expressed as the function of K_{GB} in the form

$$F_{GB} = func.\ (\ K_{GB}\ \ ). \quad (5)$$

From Eq.(4) and Eq.(5), K_{GB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be determined against the pre-impact velocity by using Eq.(2). Figure 7 is a comparison between the measured contact times during actual forehand strokes(Nagarta,1983) and the calculated ones when a ball hits the center of the strings face of a conventional type racket(360 g), showing a good agreement between them.

Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force. The mathematical expression is

$$F(t) = F_{max} \sin(\ \pi\, t /\ T_c) \quad (0 \leqq t \leqq\ \ T_c\ \ ) \quad (6)$$

where F_{max} $= \pi\ F_{mean}/2$. The fourier spectrum of Eq.(6) is represented as

$$S(f) = 2F_{max} T_c\ |\ \cos(\ \pi f T_c)\ |\ /\ [\ \pi\ \ |\ 1 - (2 f T_c\ \ )^2\ |\ ] \quad (7)$$

where f is the frequency. Figure 8 shows the examples of the calculated shock shape during impact, where the ball strikes the center on the string face at a velocity of (a) 20 m/s and (b) 30 m/s with the racket strung at 55 lb, respectively.

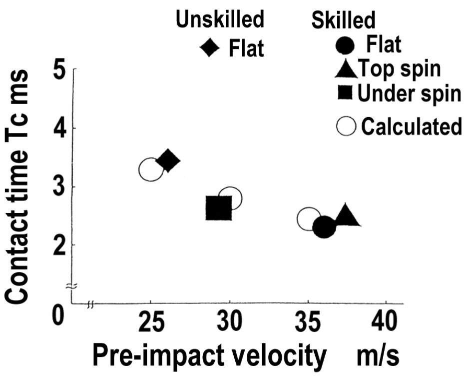

Fig.7. Comparison between the measured contact times during Strokes and the calculated results.

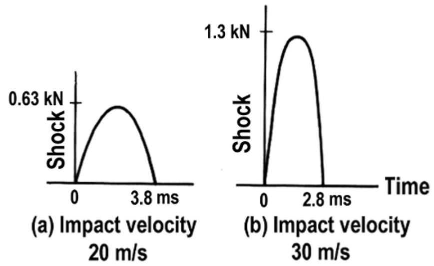

Fig.8 Calculated shock shape when a ball strikes the center on the String face of the racket at velocities of 20 m/s and 30 m/s.

2.3 PREDICTION OF THE RACKET VIBRATIONS

The vibration characteristics of a racket can be identified using the experimental modal analysis (Kawazoe, 1989,1990) and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of a racket. When the impact force $S_j(2\pi f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i is expressed as

$$X_{ijk} = r_{ijk}\ S_j(2\pi f_k) \qquad (8)$$

where r_{ijk} denotes the residue of k-th mode between arbitrary point i and j, and $S_j(2\pi f_k)$ is the impact force component of k-th frequency f_k (Kawazoe,1993). Figure 9 shows the predicted vibration amplitude of a racket struck by a ball at a velocity of 30 m/s.

2.4 ENERGY LOSS DUE TO RACKET VIBRATIONS INDUCED BY IMPACT

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along a racket frame. If the longitudinal mass distribution of racket frame is assumed to be uniform, the energy loss E_1 due to racket vibrations can be easily derived.

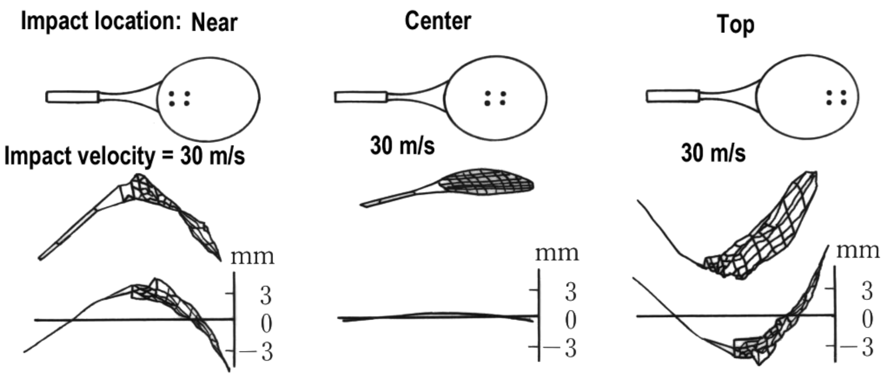

Fig.9 Predicted initial amplitude of 1st mode component of racket frame vibrations.

2.5 DERIVATION OF THE COEFFICIENT OF RESTITUTION

The coefficient of restitution (COR) can be derived considering the energy loss during impact. The main sources of energy loss is E_1 as well as. E_2 due to the instantaneous large deformation of a ball and strings which is calculated by using the coefficient e_{BG}. If a ball collides with a racket at rest (V_{Ro} = 0), the energy loss E_2 could be easily obtained. The coefficient of restitution e_r corresponds to the total energy loss E (= $E_1 + E_2$) obtained as

$$e_r = (V_R - V_B)/V_{BO} = [1 - 2E\,(m_B + M_r)/(m_B M_r V_{BO}^2)]^{1/2.} \qquad (9)$$

Figure 10 shows an example of predicted e_r at the longitudinal axis on the racket face when a player hits a coming ball with a velocity V_{BO} of 10 m/s, where a simple forehand ground stroke swing model

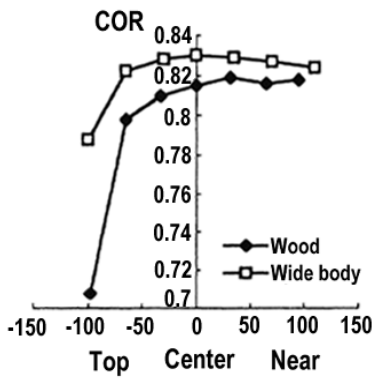

Fig.10 Examples of predicted e_r on the racket face when a player hits a ball.

(Kawazoe et al.,. 1993) is used as shown in Fig.11.
It is seen that e_r of a composite racket is higher than that of a wooden

racket, particularly at the top of the string face

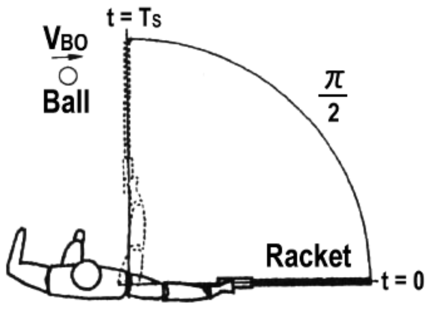

Fig.11 Simple forehand ground stroke swing model.

3. PREDICTION OF THE REBOUND POWER COEFFICIENT

The post-impact ball velocity V_B is represented as

$$V_B = -V_{Bo}(e_r - m_B/M_r)/(1+m_B/M_r) + V_{Ro}(1+e_r)/(1+m_B/M_r) \qquad (10)$$

Accordingly, if the ratio of rebound velocity against the incident velocity of a ball when a ball strikes the freely suspended racket (V_{Ro} = 0) is defined as the rebound power coefficient, it is written as Eq.(11) . The rebound power coefficient is often used to estimate the rebound power performance of a racket experimentally in the laboratory.

$$e = -V_B / V_{BO} = (e_r - m_B/M_r)/(1+m_B/M_r) \qquad (11)$$

When a player hits a coming ball with a pre-impact racket head velocity V_{Ro}, the coefficient e can be expressed as

$$e = -(V_B - V_{Ro}) / (V_{BO} - V_{Ro}) \qquad (12)$$

Figure 12 is a comparison between the measured e and the calculated e when a ball hits a freely-suspended racket (about 30 m/s), showing a good agreement between them.

4. PREDICTION OF THE POST-IMPACT BALL VELOCITY

The power of the racket could be estimated by the post-impact ball velocity V_B when a player hits a ball. The V_B can be expressed as Eq.(13). The V_{Ro} is given by $L_X(\pi N_s/I_s)^{1/2}$, where L_X denotes the holizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque around the shoulder joint, and I_s
the moment of inertia of arm/racket system around the shoulder joint. Figure 13 shows the examples of the predicted V_B at each hitting location on the racket face (V_{Ro}= 10 m/s, N_s =56.9 Nm).

$$V_B = -V_{Bo}\, e + V_{Ro}(1+e) \qquad (13)$$

Figure 14 shows the predicted V_B of various types of tennis rackets available in the market shown in Table 1, where the sign I_{GY} denotes the moment of inertia about the center of mass, the I_{GR} the moment of inertia about the grip portion 70 mm from the grip end, the I_{GX} the moment of inertia about the longitudinal axis of racket head.

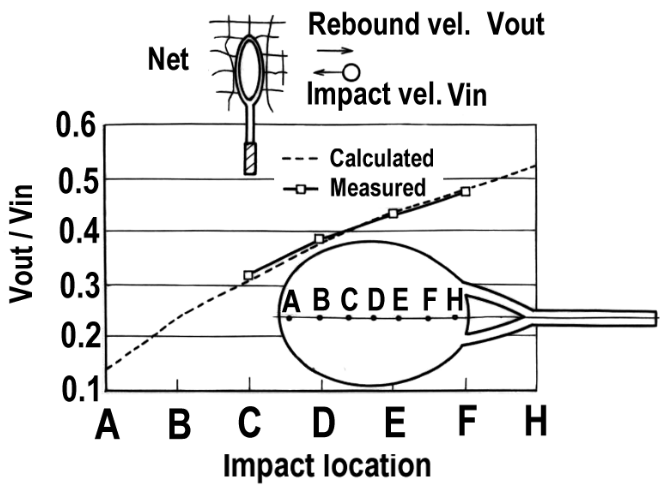

Fig.12 Comparison between the measured *e* and the predicted *e*.

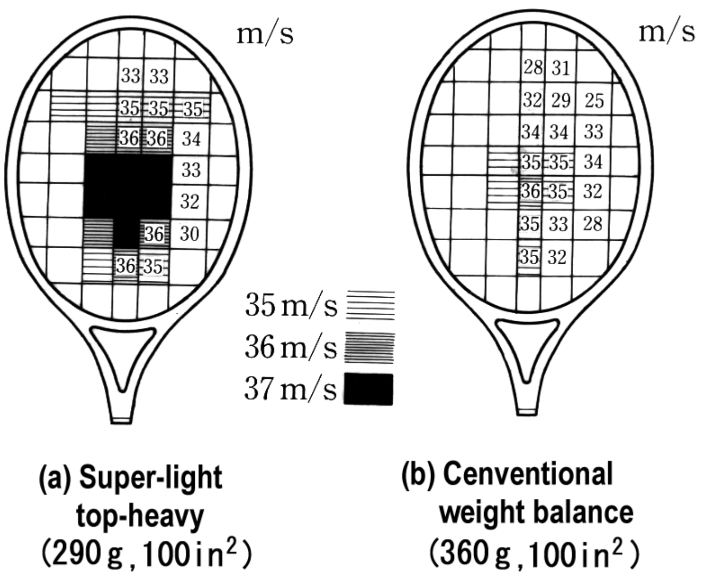

Fig.13 Examples of predicted V_B (V_{Ro}= 10 m/s, N_s =56.9 Nm).

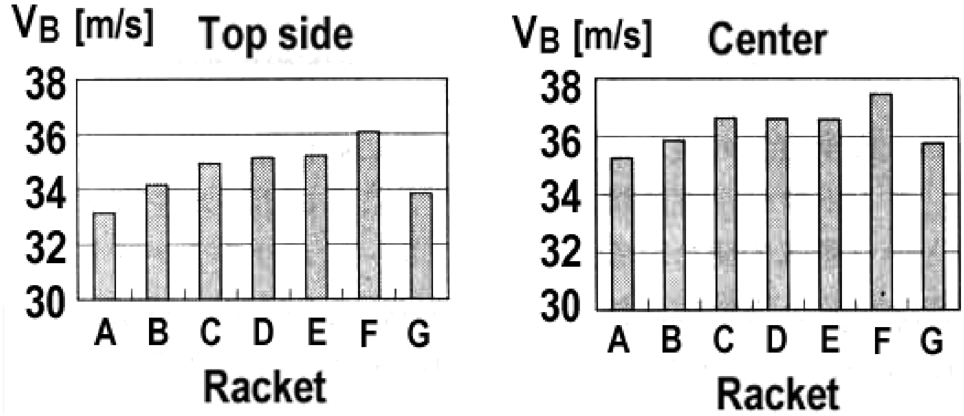

Fig.14 Predicted post-impact ball velocity V_B of various types of tennis rackets.

Table 1 Physical properties of different type of tennis rackets

Racket	A	B	C	D	E	F	G
Face area	100 in^2	100 in^2	100 in^2	110 in^2	120 in^2	120 in^2	68 in^2
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5. CONCLUSIONS

This paper investigated the physical properties of a tennis racket, predicting the racket performance in terms of power being related to the coefficient of restitution, rebound power coefficient, post-impact ball velocity when a ball is struck by a player by using a simple ground stroke swing model. The predicted results could explain the mechanism of tennis racket performance in terms of power and the difference in performance between the rackets with different physical properties. This work was supported by a Grant-in-Aid for Science Research(B) of the Ministry of Education, and Culture of Japan, and a part of this work was also supported by the High-Tech Research Center of Saitama Institute of Technology.

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