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Dynamics of the Ball-Racket Impact in Tennis: Contact Force, Contact Time, Coefficient of Restitution, and Deformation

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Abstract

This paper has investigated the physical properties of a racket and has derived the contact forces, contact time, coefficient of restitution, and deformations during impact between a ball and racket. Furthermore, it has predicted the power or post- impact ball velocity with a forehand groundstroke. It is based on the experimental identification of the dynamics of racket-arm system and the approximate nonlinear impact analysis with a simple forehand stroke swing model. The predicted results could explain the mechanism of impact between a ball and a racket with different physical properties. It enables us to predict the various factors associated with impact and performance of the various racket.

1. Introduction

Traditional wooden frame tennis rackets were used for a century, but they have evolved through several technology advancements over the past 30 years. Advanced engineering technology has enabled manufacturers to discover and synthesize new materials and new designs of sport equipment. There are rackets of all compositions, sizes, weights, shapes and strings tension, and very specific designs are targeted to match the physical and technical levels of each user [1][2].
However, at the current stage, the perception of performance of the tennis racket is actually based on the feel of an experienced tester or player. Since the optimum racket depends on the physical and technical levels of each user, there are many unknowns regarding the relationship between the performance estimated by a player and the physical properties of a tennis racket.
The ball-racket impact in tennis is an instantaneous phenomenon creating large deformations of ball/strings and vibrations in the racket. The problem is further complicated by the involvement of humans in the actual strokes.
This paper derives the contact forces, contact time, coefficient of restitution, and deformations during impact between a ball and racket. Furthermore, it predicts the power or post- impact ball velocity with a forehand groundstroke. It is based on the experimental identification of the dynamics of racket-arm system and the approximate nonlinear impact analysis with a simple forehand stroke swing model. The predicted results could explain the mechanism of impact between a ball and a racket with different physical properties.

2. Prediction of Impact Forces, Contact Time, Energy Loss, and Coefficient of Restitution between Ball and Racket [3]-[7]

2.1 Main Factors Associated with Impact Analysis

Figure 1 shows schematically the test for obtaining the applied force- deformation curves, where the ball is deformed between two flat surfaces as shown in (a) and the ball plus strings is deformed with a racket head clamped as shown in (b). The results for the ball and racket are shown in Fig.2. According to the pictures of a racket being struck by a ball, it seems that the ball deforms only at the side, which contact to the strings.
Assuming that a ball with concentrated mass deforms only at the side in contact with the strings [7], the curves of restoring force F_B vs. ball deformation, restoring force F_G vs. strings deformation, and the restoring force F_{GB} vs. deformation of the composed ball/strings system are obtained from Fig.2 as shown in Fig.3. These restoring characteristics are determined in order to satisfy a number of experimental data using the least square method. The curves of the corresponding stiffness K_B, K_G and K_{GB} are derived as shown in Fig.4 by differentiation of the equations of restoring force with respect to deformation. The stiffness K_B of a ball, K_G of strings and K_{GB} of a composed

ball/strings system exhibit strong nonlinearity.
The measured coefficient of restitution versus the incident velocity when a ball strikes the rigid wall is shown in Fig.5, while the measured coefficient of restitution e_{BG}, which is abbreviated as COR, when a ball strikes the strings with a racket head clamped is shown in Fig.6. Although the COR in Fig.5 decreases with increasing incident velocity, the coefficient e_{BG}

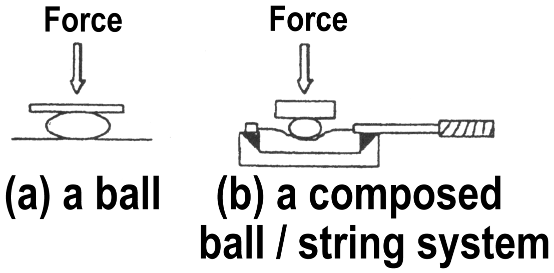


Fig.1 Illustrated applied force- deformation test

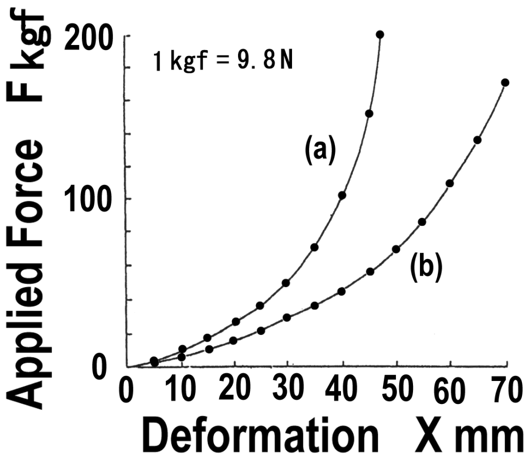


Fig.2 Results of a force-deformation test.

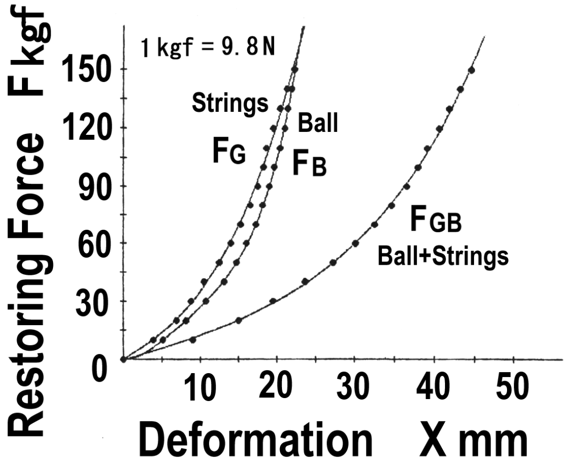


Fig.3 Restoring forces vs. deformation of a ball, strings, and a Composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

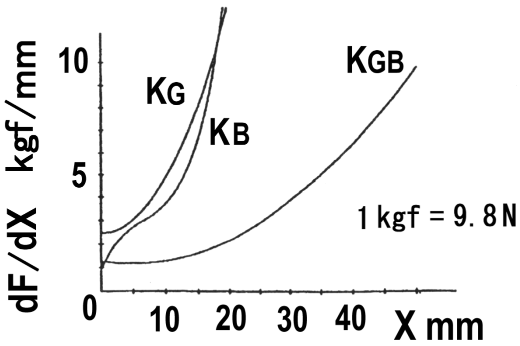


Fig.4 Stiffness vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings.

with a racket head clamped is almost independent of ball velocity and strings tension. This value of COR can be regarded as being inherent to the materials of ball and strings, showing the important role of strings. This feature is due to the nonlinear restoring force characteristics of a composed ball/strings system [4].
Since equivalent spring stiffness K_{GB} of the compound system increases as the impact velocity increases, the independence of the damping coefficient ratio with respect to impact velocity means that damping coefficient C_{GB} is proportional to $K_{GB}^{1/2}$ and increases with increases in impact velocity. The energy loss of a ball and strings due to impact can be related to the coefficient e_{BG}.

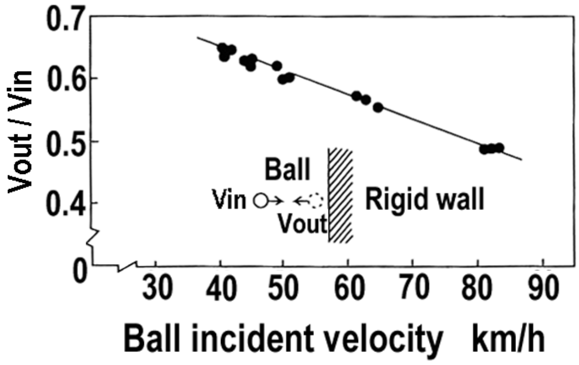


Fig.5 Measured coefficient of restitution (COR) between a ball and a rigid wall.

The result of measured contact time, which means how long the ball stays on the strings, with a normal racket and with a wide-body racket (stiffer) shows that the stiffness of the racket frame does not affect the contact time much [4]. Accordingly, the masses of a ball and a racket as well as the nonlinear stiffness of a ball and strings are the main factors in the deciding of a contact time. Therefore, the contact time can be calculated using a model assuming that a ball with a concentrated mass m_B and a nonlinear spring K_B, collides with the nonlinear spring K_G of strings supported by a frame without vibration, where the measured coefficient of restitution inherent to the materials of

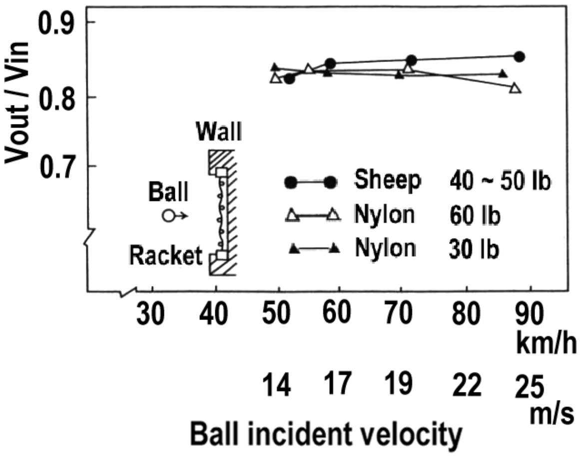


Fig.6. Measured COR between a ball and strings with frame clamped.

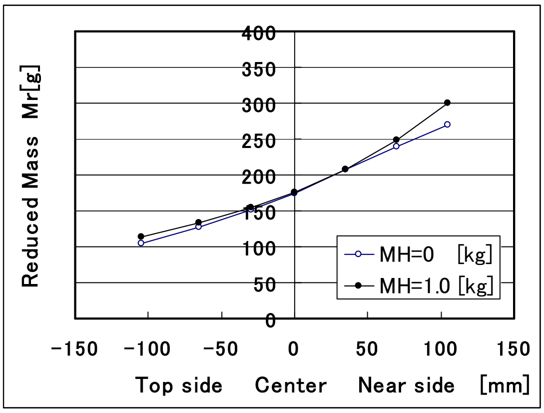


Fig7. The effect of the arm on the reduced mass of a racket at the impact locations (super-light weighted racket 290 g, M_H=1.0 kg: with arm, M_H=0 kg: without arm)

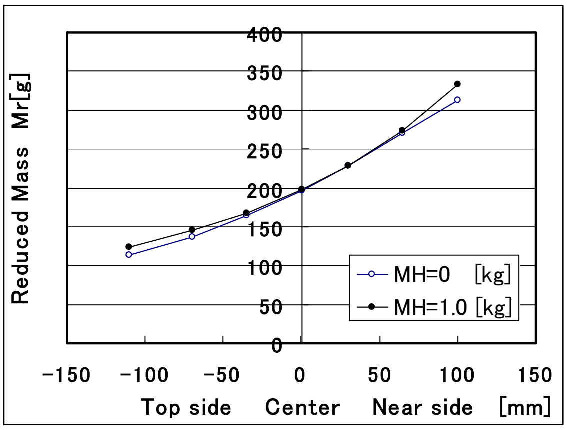


Fig.8 The effect of the arm on the reduced mass of a racket at the impact locations (conventional balanced racket 370 g, M_H=1.0 kg: with arm, M_H=0 kg: without arm)

ball-strings impact is employed as one of the sources of energy loss.

The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location and a center of gravity are given. Figure 7 (super-light weighted racket 290 g) and Fig.8 (conventional weight balanced racket 370 g) show the effect of an arm on the reduced mass on the longitudinal axis on the racket face. There is no big difference between the reduced masses with the arm and without it, particularly around the center of the racket face.

The result of the experimental modal analysis [3][9] showed that the fundamental vibration mode of a conventional type racket supported by a hand has two nodes being similar to the mode of a freely supported racket. The racket is assumed to be freely suspended in terms of the performance of power.

2.2 Derivation of Approximate Impact Force and Contact Time

In case the vibration of the racket frame is neglected, the momentum equation and the coefficient of restitution e_{BG} give the post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse could be described using the following equation, where m_B is the mass of a ball, M_r is the reduced mass of a racket at the hitting location, and (V_{BO} - V_{Ro}) is the pre-impact velocity.

$$\int F(t)\,dt = m_B V_{Bo} - m_B V_B = (V_{BO} - V_{Ro})(1+e_{BG})m_B/(1+m_B/M_r) \qquad (1)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of m_B , K_{GB} , and M_r , it could be obtained as

$$T_c = \pi m_B^{1/2}/[K_{GB}(1+m_B/M_r)]^{1/2} \qquad (2)$$

In order to make the analysis simpler, the equivalent force F_{mean} can be introduced during contact time T_c , which is described as

$$\int^{T_c} F(t)\,dt = F_{mean} \cdot T_c \qquad (3)$$

Thus, from Eq.(1), Eq.(2) and Eq.(3), the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity (V_{BO} - V_{Ro}) is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1+e_{BG})\, m_B^{1/2} K_{GB}^{1/2} / \pi (1+m_B/M_r)^{1/2} \qquad (4)$$

On the other hand, from the approximated curves shown in Fig.3 and Fig.4, F_{GB} can be expressed as the function of K_{GB} in the form

$$F_{GB} = f(K_{GB}). \qquad (5)$$

From Eq.(4) and Eq.(5), K_{GB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be determined against the pre-impact velocity by using Eq.(2). Figure 9 is a comparison between the measured contact times during actual forehand strokes [11] and the calculated ones when a ball hits the center of the strings face of a conventional type racket, showing a good agreement between them.

Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force. The mathematical expression is

$$F(t) = F_{max} \sin(\pi t / T_c) \quad (0 \leqq t \leqq T_c) \qquad (6)$$

where $F_{max} = \pi F_{mean}/2$. The Fourier spectrum of Eq.(6) is represented as

$$S(f) = 2F_{max} T_c \mid \cos(\pi f T_c) \mid / [\pi \mid 1 - (2fT_c)^2 \mid] \qquad (7)$$

where f is the frequency.

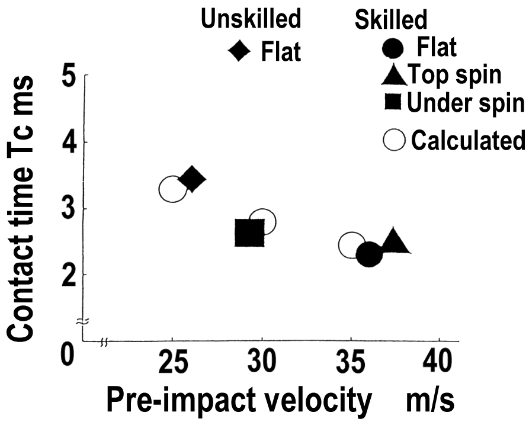


Fig.9 Comparison between the measured contact times during strokes and the calculated results.

Figure 10 shows the examples of the calculated shock shape during impact, where the ball strikes the center on the string face at a velocity of (a) 20 m/s and (b) 30 m/s with the racket strung at 55 lb, respectively.

2.3 Prediction of Racket Vibrations

The vibration characteristics of a racket can be identified using experimental modal analysis [3][9] and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of the racket. When the impact force $S_j(2\pi f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i is expressed as

$$X_{ijk} = r_{ijk} \ S_j(2\pi f_k) \qquad (8)$$

where r_{ijk} denotes the residue of k-th mode between arbitrary point i and j, and $S_j(2\pi f_k)$ is the impact force component of k-th frequency f_k [5].

Figure 11 shows the string mesh and impact location on the racket face, and Fig.12 shows the example of predicted vibration amplitude of the racket struck by a ball at 30 m/s.

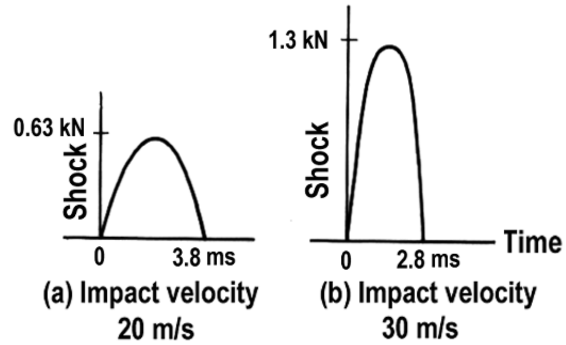


Fig.10 Calculated shock shape when a ball strikes the center on the String face at velocities of 20 m/s and 30 m/s.

2.4 Energy Loss Due to Racket Vibrations Induced by Impact

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along the racket frame. If the longitudinal mass distribution of racket frame is assumed to be uniform, the energy loss E_1 due to racket vibrations can be easily derived.

2.5 Derivation of Coefficient of Restitution

The coefficient of restitution (COR) can be derived considering the energy loss during impact. The main sources of energy loss are E_1 and E_2 due to the instantaneous large deformation of a ball and strings which is calculated by using the coefficient e_{BG}. If a ball collides with a racket at rest ($V_{Ro} = 0$), the energy loss E_2 could be easily obtained. The coefficient of restitution e_r corresponds to the total energy loss E (= E_1 + E_2) obtained as

$$e_r = (V_R - V_B)/V_{BO} = [1 - 2E(m_B + M_r)/(m_B M_r V_{BO})]^{1/2} \qquad (9)$$

3. Prediction of Rebound Power Coefficient

The post-impact ball velocity V_B is represented as

$$V_B = -V_{Bo}(e_r - m_B/M_r)/(1+m_B/M_r) + V_{Ro}(1+e_r)/(1+m_B/M_r) \qquad (10)$$

Accordingly, if the ratio of rebound velocity against the incident velocity of a ball when a ball strikes the freely suspended racket ($V_{Ro} = 0$) is defined as the rebound power coefficient e, it is written as Eq.(11) . The rebound power coefficient is often used to estimate the rebound power performance of a racket experimentally in the laboratory.

$$e = -V_B / V_{BO} = (e_r - m_B/M_r)/(1 + m_B/M_r) \qquad (11)$$

When a player hits a coming ball with a pre-impact racket head velocity V_{Ro}, the coefficient e can be expressed as

$$e = -(V_B - V_{Ro}) / (V_{BO} - V_{Ro}) \qquad (12)$$

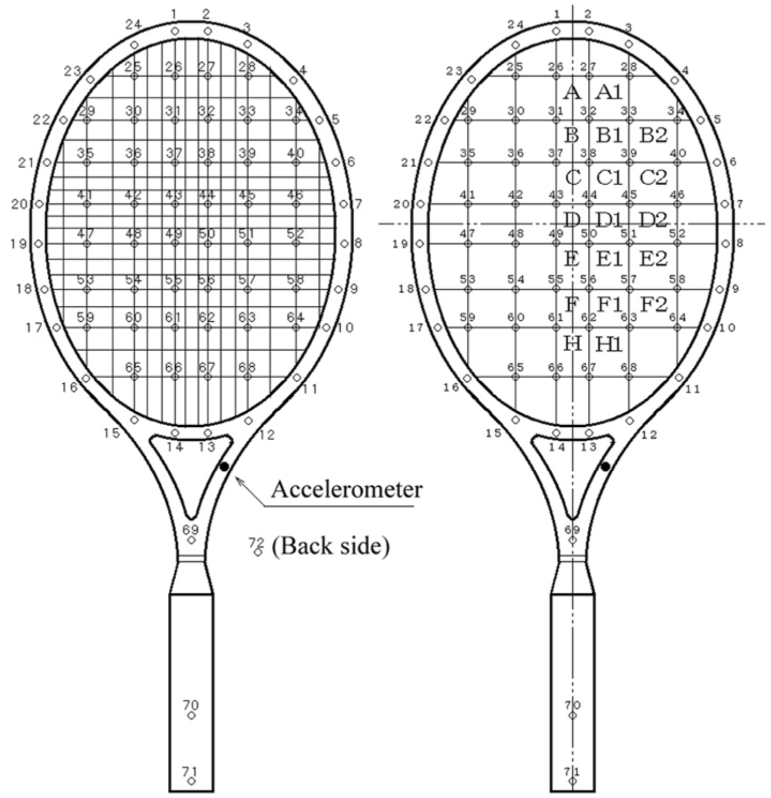


Fig.11 String mesh (Left side) and impact location on the racket face (Right side).

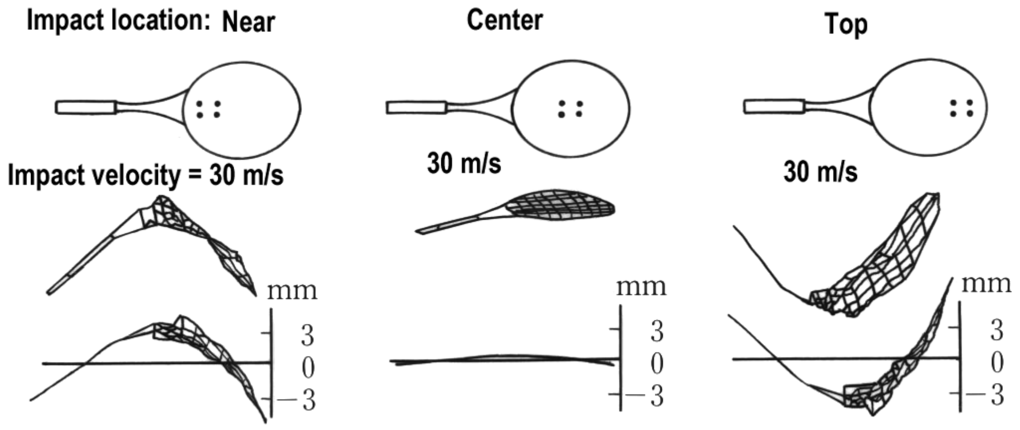


Fig.12 Predicted initial amplitude of 1st mode component of racket frame vibrations.

Figure 13 is a comparison between the measured *e and* the predicted *e* when a ball hits a freely-suspended racket (about 30 m/s), showing a good agreement between them [5][6].

4. Prediction of Post-impact Ball Velocity

The power of the racket could be estimated by the post-impact ball velocity V_B when a player hits the ball [13][14]. The V_B can be expressed as Eq.(13). The V_{Ro} is given by $L_X\,(\pi N_s/I_s)^{1/2}$, where L_X denotes the horizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque about the shoulder joint, and I_s the moment of inertia of arm/racket system about the shoulder joint.

$$V_B = -V_{Bo}\ e + V_{Ro}\,(1+e) \tag{13}$$

5. Ball Control and Racket Stability

Control is simply being able to put the ball where desired, but it

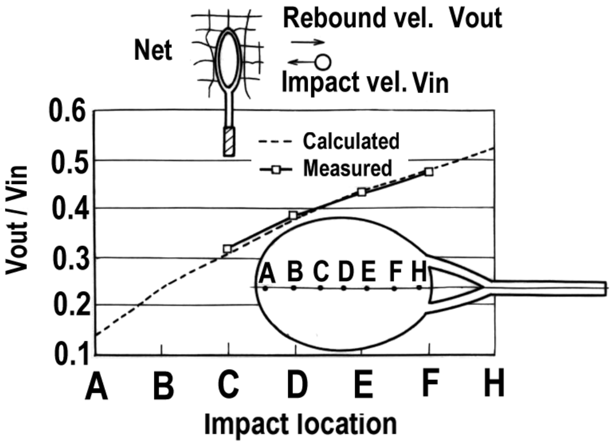


Fig.13 Comparison between the measured rebound power coefficient *e* and the predicted one ($V_B = V_{out}$, $V_{BO} = V_{in}$, $V_{Ro} = 0$).

is the most difficult to analyze. Designing equipment to optimize control for all types of players is nearly impossible. Tennis players have a wide variety of styles from smooth stroking to whippy and wristy. These style differences require different equipment to optimize control. However, one characteristic required for control is stability. Stability refers to the ability at impact to maintain its swing path without deviation. Stability is also defined as the ability to resist off center hits. It is desirable to maximize stability [1].

We can estimate the racket stability by the amount of twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket.

6. Estimation of the Performance of Tennis Rackets having Different Weight and Weight Balance

Now we can predict the various factors associated with the tennis impact when the impact velocity or swing model and the impact locations on the racket face are given. Furthermore we can estimate the performance of the various rackets with

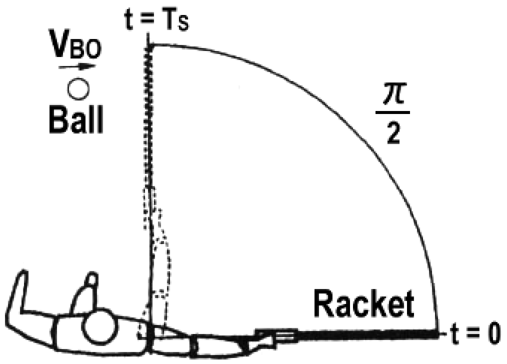


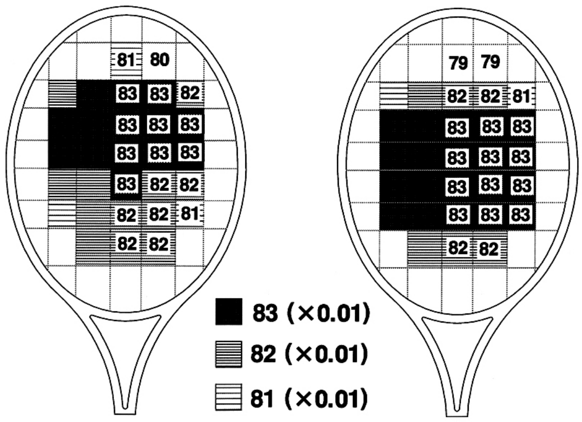
Fig.14 Simple forehand groundstroke swing model.

different physical properties. Figure 14 shows a simple forehand ground stroke swing model [12].

Figure 15 shows the comparison of the predicted coefficients of restitution e_r between the super-light weight racket (290 g) and conventional weight and weight balanced racket (370 g). It is when a player hits a coming ball with a velocity V_{BO} of 10 m/s. Figure 16 is at the longitudinal axis on the racket face. It is seen that e_r of a super-light weight racket is higher than that of a conventional weight and weight balanced racket at the top of the string face.

Figure 17 shows the predicted rebound power coefficient *e* (N_S=56.9Nm,V_{Bo}=10m/s). It is seen that *e* of a conventional weight and weight balanced racket is higher than that of a super-light weight racket anywhere on the string face.

Figure 18 and Fig.19 show the comparison of the predicted V_B at each hitting location on the racket face. We can see the difference in sweet area in terms of racket power between a super-light weighted racket (EOS100, 290 g) and conventional heavier weighted racket (PROTO-02, 370 g).



(a) EOS100 (290 g) (b) PROTO-02 (370 g)

Fig.15 Predicted Restitution coefficient e_r on the racket face when a player hits a ball (N_S=56.9Nm,V_{Bo}=10m/s).

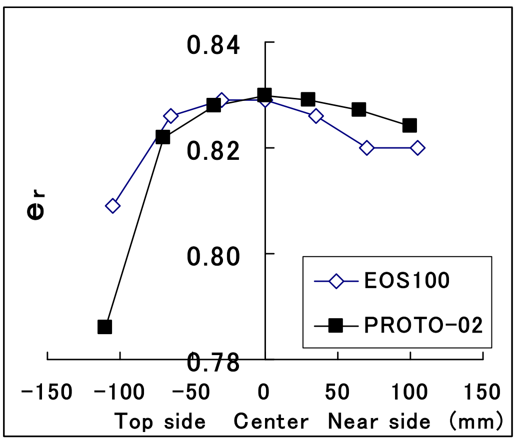


Fig.16 Predicted Restitution coefficient e_r on the longitudinal axis of racket face when a player hits a ball (N_S=56.9 Nm,V_{Bo}=10 m/s).

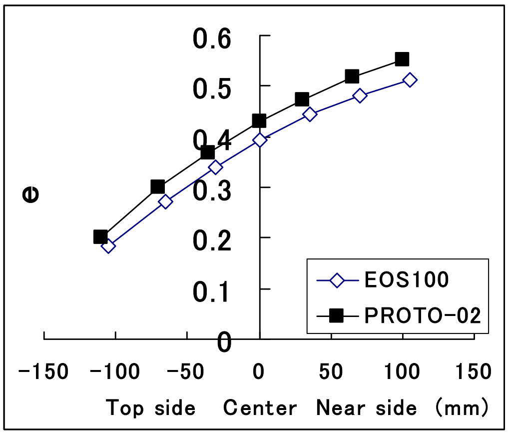
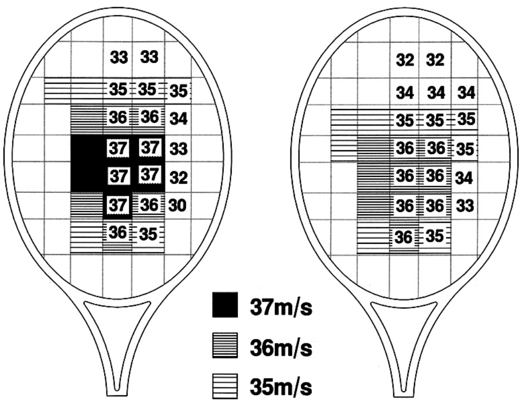


FIg.17 Predicted rebound power coefficient e (N_S= 56.9 Nm, V_{Bo}=10m/s)

Figure 20 shows the twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket. Figure 21 shows the predicted amount of the racket twist vs. distance of the impact location from the long axis, assuming that there is no friction between the hand and the racket grip. It is the comparison between a Super light weighted racket: (EOS100, 290 g) and conventional heavier weighted racket (PROTO-02, 370 g) at the topside, the center and the near side on the racket face away from the long axis. There is no twist about long axis at the topside away from the long axis, because the racket turns about the location near the grip. There is big difference in twist angles at the near side on the racket face but there is no big difference at the topside and the center away from long axis between the lighter racket and the heavier racket. The conventional heavier racket seems to be desirable in stability. However, since the hitting area for the groundstroker is usually at the topside from the center, there is no big disadvantage for the super-lighted weighted racket.



(a) EOS100 (290g) (b) PROTO-02 (370g)

Fig.18 Predicted post-impact ball velocity V_B on the racket face representing sweet area in terms of power.

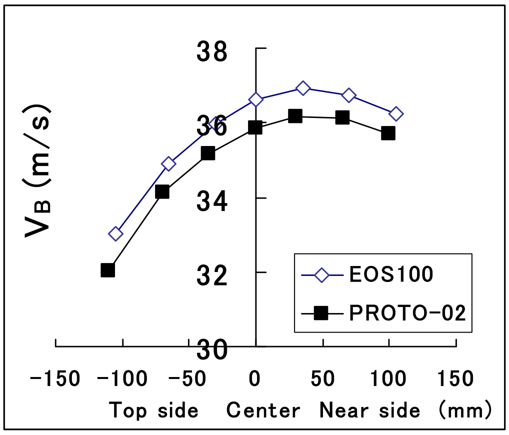


Fig.19 Predicted post-impact ball velocity on the longitudinal axis of a racket face (VBo= 10 m/s, Ns =56.9 Nm).

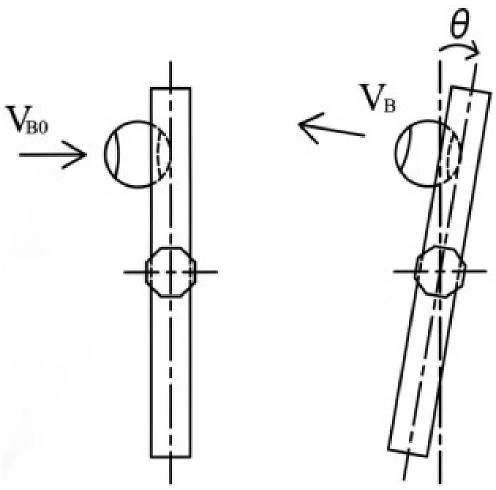


Fig.20 Twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket.

7. CONCLUSIONS

This paper has investigated the physical properties of a racket and has derived the contact forces, contact time, coefficient of restitution, and deformations during impact between a ball and racket. Furthermore, it has predicted the power or post- impact ball velocity with a forehand groundstroke. It is based on the experimental identification of the dynamics of racket-arm system and the approximate nonlinear impact analysis with a simple forehand stroke swing model. It enables us to predict the various factors associated with impact and performance of the various racket.

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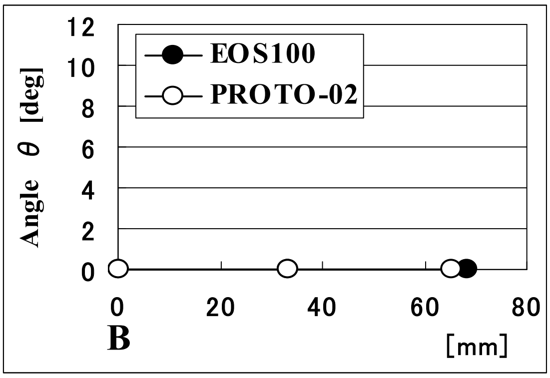
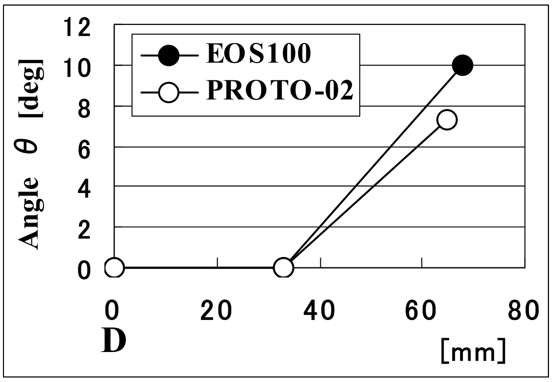
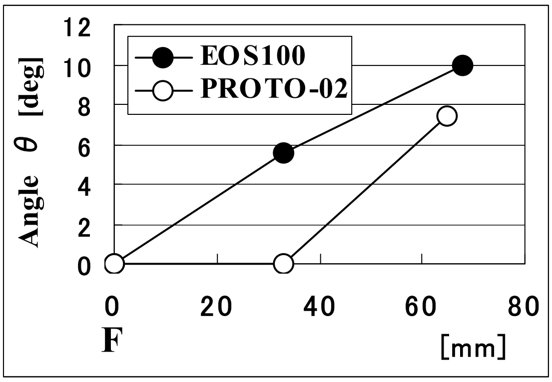


(a) Top side on the racket face



(b) Center on the racket face



(c) near side on the racket face

Fig.21 Calculated amount of the racket twist vs. distance of the impact location from the long axis, assuming that there is no friction between the hand and the racket grip. (Super light weighted racket: EOS100: 290 g, Conventional weighted racket: PROTO-02: 370 g)

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