Sports Dynamics: Discovery and Application, Edited by A.Subic, P. Trivailo & F.Alam, pp.57-62. RMIT Publishing, Melbourne. (2003)

SWEET SPOTS PREDICTION IN TERMS OF FEEL WITH THE EFFECT OF MASS AND MASS DISTRIBUTION OF A TENNIS RACKET

Y. KAWAZOE
Saitama Institute of Technology
1690, Okabe, Saitama, 369-0293, JAPAN
E-mail: ykawa@sit.ac.jp

R.TANAHASHI
Tanahashi Applied Research Laboratory
2-16-17 Handayama, Hamamatsu, Japan
E-mail: rtanahashi@dream.com

This paper derived the shock vibrations of the wrist joint caused by the impact when a player hits flat forehand drive. The predicted waveforms of the shock vibrations at the wrist joint agreed fairly well with the measured ones. The predicted shock vibrations with the light weight type is smaller at the top side and larger at the near side than those with the conventional weight balanced type of 100 in^2 face size racket.

1. Introduction

This paper investigated the feel or comfort of the arm or hand in an impact. It derived the shock vibrations of the wrist joint caused by the impact when a player hits flat forehand drive. It is based on the identification of the racket-arm system and the predicted coefficient of restitution between a racket and a ball.

2. Prediction of Shock Vibrations at the Racket Handle and the Wrist Joint

The acceleration of the shock vibrations at the player's wrist joint and at the racket handle was measured when a player hits flat forehand drive. The location of the accelerometer at the racket handle is 210 mm from the grip end. The waveforms of acceleration when struck at the off-center (top side) and those at the center were recorded during forehand stroke.

We consider the racket to be freely hinged to the forearm of the player, the forearm freely hinged to the arm and the arm freely hinged to the player's body. We can deduce that the inertia effect of the arm and the forearm can be attributed to a mass M_H concentrated in the hand. Therefore, the analysis of the impact between the ball and racket can be carried out by assuming that the racket is free in space, as long as the mass M_H is applied at the point that the hand grips the racket. If the impact force between a ball and the racket is given when the ball hits the racket, the shock force can be obtained [1][3][8]-[11]. The reduced mass M_r of a racket at the impact location on the string face can be derived from the principle of the conservation of angular momentum when the moment of inertia and the distance between an impact location and a center of gravity of the racket-arm system are given.

The vibration characteristics of a racket can be identified using experimental modal analysis [4][5] and the racket vibrations can be simulated by applying the approximate impact force-time curve to the hitting portion on the string face of the identified vibration model of the racket. When the impact force component of the *k*-th mode frequency f_k in the frequency region applies to the point *j* on the racket face, the amplitude $X_{ij\,k}$ of the *k*-th mode component at point *i* can be derived using the residue $r_{ij\,k}$ of the *k*-th mode between arbitrary point *i* and *j* [6][8].

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along a racket frame, when an impact location on the string face and the impact velocity are given.

The coefficient of restitution e_r (COR) between a ball and a racket can be derived by considering the energy loss due to the instantaneous large deformation of a ball and strings and that due to the racket vibrations [6][8]. Furthermore, the force-time curve of the impact between a ball and a racket considering the vibrations of a racket frame can be approximated. The contact time T_C during impact can be determined against the pre-impact velocity (V_{BO} - V_{RO}) between a ball and a racket assuming the contact time to be half the natural period of a whole system composed of the mass m_B of a ball, the equivalent stiffness K_{GB} of ball/strings, and the reduced mass M_r of the racket-arm system.

The shock acceleration $A_{nv}(t)$ at the hand grip considering the equivalent mass M_H of the arm system can be represented as:

$$A_{nv}(t) = S_0(t)\,[\,1/(M_R + M_H) - (a/I_G)X] \qquad (1)$$

where X denotes the distance between the center of mass of racket-arm system and the location of hand grip, a the distance between the center of mass of racket-arm system and the impact location of the racket, and I_G the moment of inertia around the center of mass of racket-arm system, respectively. The maximum shock force S_{1max} transmitted to a wrist joint corresponds to the maximum impact force $S_{0\,max}$. The vibration acceleration component $A_{i\,j,\,k}(t)$ of the *k*-th mode at the location *i* of the hand grip is represented as:

$$A_{ij,k}(t) = -(2\pi f_k)^2 r_{ijk} S_{0j}(2\pi f_k) exp(-2\pi f_k \zeta_k t)\, sin(2\pi f_k t) \qquad (2)$$

where j denotes the impact location between ball and racket on the string face, ζ_k the damping ratio of the k-th mode, and $S_{0j}(2\pi f_k)$ the Fourier spectrum of impact-force curve between a ball and strings. The summation of equation (1) and equation (2) represents the shock vibrations at the hand grip. The damping ratio of a hand-held racket during actual impact has been estimated as about 2.5 times that of the one identified by the experimental modal analysis with small vibrations amplitude. Furthermore, the damping of the waveform at the wrist joint has been 3 times that at the grip portion of the racket handle.

3. Estimation of the Sweet Spots in terms of Feel for Tennis Rackets having different weight and Weight Balance

Now we can predict the shock vibrations at the grip and the wrist joint during the impact and can estimate the sweet spots in terms of feel for the various rackets with different physical properties. Table 1 shows the specifications and the main physical properties of the light weight racket (EOS100: 290g) and the conventional weight and weight-balanced racket (PROTO-02: 370g), where the sign I_{GY} denotes the moment of inertia about the center of mass and the sign I_{GX} the moment of inertia about the longitudinal axis of racket head. Figure 8 shows the impact locations on the string face of the racket.

Figure 9 shows the predicted maximum shock accelerations at the racket grips (70 mm from the grip end) between the light weight racket (EOS100) and the conventional weight and weight-balanced racket (PROTO-02) when a ball strikes the freely suspended rackets. Figure 10 shows the predicted maximum shock

Table 1 physical property

Racket	EOS100	PROTO-02
Total length	680 mm	680 mm
Face area	606 cm^2	606 cm^2
Mass	290 g	370 g
Center of gravity from grip end	350 mm	317 mm
Moment of inertia I_{GY} about Y axis	34.1 gm^2	36.6 gm^2
Moment of inertia I_{GX} about X axis	1.121 gm^2	1.620 gm^2
1st frequency	171 Hz	215 Hz
Strings tension	55 lb	55 lb

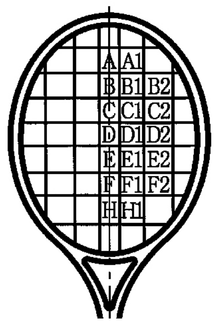


Fig.8 Hitting locations on the string face.

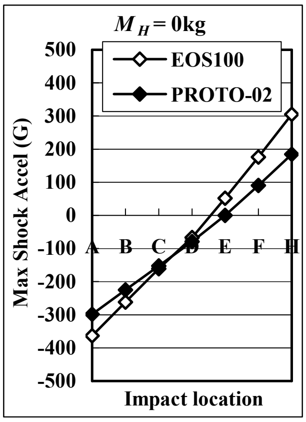


Fig.9 Predicted Maximum shock acceleration at the grip of freely suspended racket.

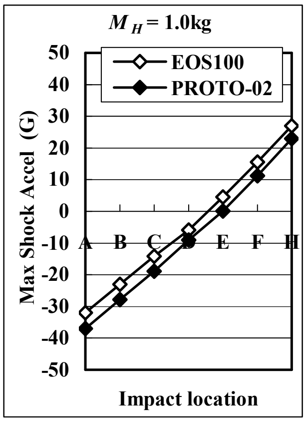
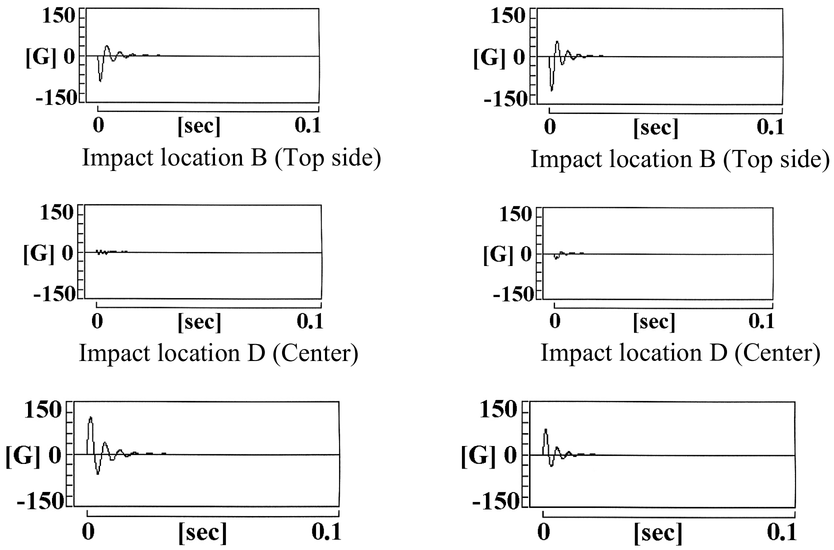


Fig.10 Predicted Maximum shock accelerations at the grip of hand-held racket



Impact location B (Top side) | Impact location B (Top side)

Impact location D (Center) | Impact location D (Center)

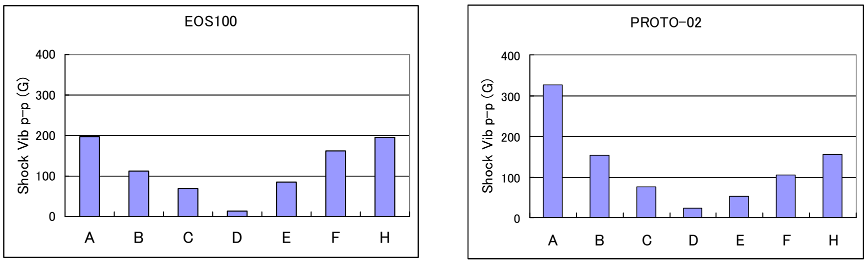
Impact location F (Near side) | Impact location F (Near side)

(a) Racket EOS100 (light weight) | (b) Racket PROTO-02 (Conventional)

Fig.11 Predicted shock vibrations of the player's wrist joint (impact velocity: 30 m/s).

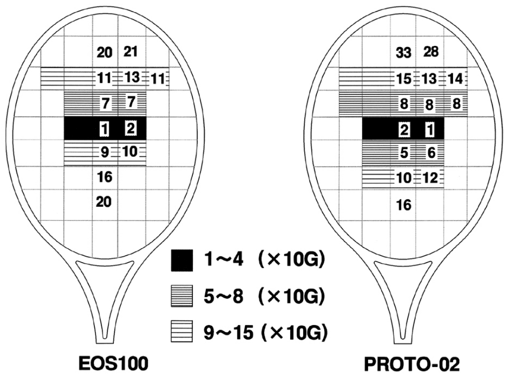
the grips of hand-held rackets. The equivalent mass of an arm is estimated as M_H= 1.0 kg [1]-[3]. The equivalent mass of an arm reduces remarkably the maximum shock acceleration of a racket grip on comparing with Fig.8.

Figure 11 shows the predicted shock vibrations at the wrist joint. Figure 12 shows the predicted shock vibrations estimated by the initial peak-peak value of wrist acceleration waveforms when the ball strikes the each hitting location along the longitudinal axis on the string face of hand-held rackets. Figure 13 shows the predicted sweet area in terms of feel or comfort of tennis rackets estimated by the initial peak-peak value of wrist acceleration waveforms, where the ball hits the string face. It is seen that the sweet area of both rackets



(a) Racket EOS100 (Light weight) **(b) Racket PROTO-02 (Conventional)**

Fig.12 The predicted shock vibrations estimated by the initial peak-peak value of wrist acceleration waveforms when the ball strikes the each hitting location along the longitudinal axis on the string face of hand-held rackets.



(a) EOS100 (Light weight) **(b) PROTO-02 (Conventional)**

Fig.13 The predicted sweet area in terms of feel or comfort estimated by the initial peak-peak value of wrist acceleration waveforms when the ball strikes the string face of hand-held rackets.

Sports Dynamics: Discovery and Application, Edited by A.Subic, P. Trivailo & F.Alam, pp.57-62. RMIT Publishing, Melbourne. (2003)

in terms of feel or comfort lies near the center of the string face, but the shock vibrations at the wrist joint when using a light weight racket is smaller at the top side and larger at the near side than that when using a conventional weight and weight-balanced racket.

4. Conclusions

The predicted waveform of the shock vibrations at the wrist joint agreed fairly well with the measured ones during actual forehand stroke by a player. The predicted results could explain the difference in sweet spots of a racket in terms of feel or comfort of 100 in^2 face size rackets with different weight and weight balance. The sweet area of both rackets in terms of feel or comfort lies near the center of the string face, but the shock vibrations at the wrist joint when using a light weight racket is smaller at the top side and larger at the near side than that when using a conventional weight and weight-balanced racket.

Acknowledgments

The authors would like to thank the International Tennis Federation (ITF) for funding the research. This work was supported by a Grant-in-Aid for Science Research of the Ministry of Education, Culture, Sports, Science and Technology of Japan, and also supported by the High-Tech Research Center of Saitama Institute of Technology.

References

1. Casolo,F. & Ruggieri,G., *Meccanica*, 24(1991), pp.501-504.
2. Y. Kawazoe, R. Tomosue& A. Miura, *Proc. of Int. Conf. on New Frontiersin Biomechanical Engineering* (1997), pp.285-288.
3. Y. Kawazoe, *Theoretical and Applied Mechanics*, 49, 11(2000).
4. Y. Kawazoe, *Proc. Int. Symposium on Advanced Computers for Dynamics and Design'89*, Japan Society of Mechanical Engineers, 243 (1989).
5. Y. Kawazoe, *Theoretical and Applied Mechanics*, 46, 165(1997).
6. Y. Kawazoe, *Theoretical and Applied Mechanics*, 42, 197(1993).
7. Y. Kawazoe, *Theoretical and Applied Mechanics*, 41, 3 (1992).
8. Y. Kawazoe, *Theoretical and Applied Mechanics*, 43, 223(1994).
9. Y. Kawazoe, *Science and Racket Sports*, E & FN SPON. 134(1994).
10. Y. Kawazoe, *5th Japan Int. SAMPE Symposium,* (1997), pp.1323-1328.
11. Y. Kawazoe & K. Yoshinari, *Tennis Science & Technology* (Edited by Haake S. & Coe A.), 91 (2000). Blackwell Science.