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PREDICTION OF CONTACT FORCES, CONTACT TIMES, RESTITUTION COEFFICIENTS AND RACKET STABILITIES DURING TENNIS IMPACT WITH THE EFFECT OF RACKET MASS AND MASS DISTRIBUTION

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This paper has derived the contact forces, contact time, restitution coefficients and racket stabilities during impact between a ball and racket. It is based on the experimental identification of the dynamics of racket-arm system and the approximate nonlinear impact analysis with a simple forehand stroke swing model. It predicted the impacts of 100 in^2 face size rackets with different weight and weight balance. The predicted results could explain the mechanism of difference in power and stability between a light weighted racket and a conventional weight balanced racket.

1. Introduction

There are rackets of all compositions, sizes, weights, shapes and strings tension, and very specific designs are targeted to match the physical and technical levels of each user [1][2]. This paper derives the contact forces, contact times, coefficients of restitution, deformations and racket stabilities during impact between a ball and racket. It predicts the impacts of 100 in^2 face size rackets with different weight and weight balance.

2. Prediction of Impact Forces, Contact Time, Energy Loss, and Coefficient of Restitution between Ball and Racket

Figure 1 shows the non-linear impact model of a ball-string system. The approximate impulse could be obtained using the mass m of a ball, the reduced mass M_r of a racket-arm system at the hitting location, and the pre-impact velocity (V_{Bo} - V_{Ro}) between a ball and a racket. The contact time could be obtained using m, K_{GB} of the stiffness of ball/strings system and M_r. The relationship between the equivalent force F_{mean} and corresponding K_{GB} against the pre-impact velocity (V_{BO} - V_{RO}) is given by

$$F_{mean}= (V_{BO} - V_{RO})(1+ e_{BG})\, m_B^{1/2}\, K_{GB}^{\,1/2} / \pi(1+ m/M_r)^{1/2} \qquad (1)$$

On the other hand, from the approximated restoring force F_{GB} can be expressed as the function of the stiffness K_{GB} in the form

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$$F_{GB} = f(K_{GB}). \tag{2}$$

From Eq.(1) and Eq.(2), K_{GB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be determined against the pre-impact velocity. A comparison between the measured contact times during actual forehand strokes and the calculated ones when a ball hits the center of the strings face of a conventional type racket, showing a good agreement [5]. Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force. The mathematical expression is

$$F(t) = F_{max} \sin(\pi t / T_c) \quad (0 \leqq t \leqq T_c) \tag{3}$$

where $F_{max} = \pi F_{mean}/2$. Figure 2 shows the examples of the calculated shock shape during impact, where the ball strikes the center on the string face at a velocity of (a) 20 m/s and (b) 30 m/s, respectively.

The vibration characteristics of a racket can be identified using experimental modal analysis [3][9] and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the string face of the identified vibration model of the racket. When the Fourier spectrum $S_j(2\pi f_k)$ of the impact force component of k-th frequency f_k applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i is obtained using the residue r_{ijk} of k-th mode between arbitrary point i and j [5].

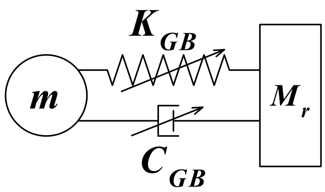

Fig.1 Non-linear Impact model of a ball-string system.

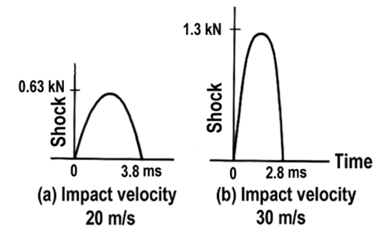

Fig.2 Calculated shock shape when a ball strikes the center on the String face at velocities of 20 m/s and 30 m/s.

Figure 3 shows the example of predicted vibration amplitude of the racket struck by a ball at 30 m/s.

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along the racket frame. The coefficient of restitution e_r (COR) can be derived considering the energy loss E_1 due to racket vibrations and E_2 due to large deformations of a ball and strings corresponding to the coefficient e_{BG}. If a ball collides with a racket at rest ($V_{Ro} = 0$), the coefficient of restitution e_r corresponding to the total energy loss E (= $E_1 + E_2$) can be obtained. The ratio of rebound velocity against the incident velocity of a ball when a ball strikes the freely suspended racket ($V_{Ro} = 0$) is defined as the rebound power coefficient e written as Eq.(4), because the coefficient e is often used to estimate the rebound power performance of a racket experimentally in the laboratory. A comparison between the

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measured *e* and the predicted *e* when a ball hits a freely-suspended racket (about 30 m/s) showed a good agreement between them [5].

$$e = -V_B \;/\; V_{BO} \;=\; (\,e_r\; - m_B/M_r\,)\,/(1+ m_B/M_r\,) \qquad (4)$$

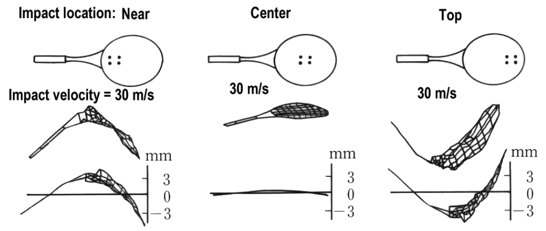

Fig.3 Predicted initial amplitude of 1st mode component of racket frame vibrations.

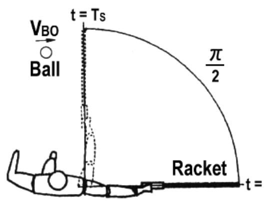

Fig.4 Simple forehand groundstroke swing model.

4. Prediction of Post-impact Ball Velocity

The power of the racket could be estimated by the post-impact ball velocity V_B when a player hits the ball. The V_B can be expressed as Eq.(5). The pre-impact racket head velocity V_{RO} is given by $L_X\,(\,\pi N_s\,/\,I_s\,)^{1/2}$, where L_X denotes the horizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque about the shoulder joint, and I_s the moment of inertia of arm/racket system about the shoulder joint. Figure 4 shows a simple forehand ground stroke swing model [10].

$$V_B \;=\; -\,V_{Bo}\;\; e \;\; +V_{Ro}\,(\,1+e\,\;) \qquad (5)$$

5. Ball Control and Racket Stability

Control is simply being able to put the ball where desired, but it is the most difficult to analyze. Designing equipment to optimize control for all types of players is nearly impossible. Tennis players have a wide variety of styles from smooth stroking to whippy and wristy. These style differences require different equipment to optimize control. However, one characteristic required for control is stability. Stability refers to the ability at impact to maintain its swing path without deviation. Stability is also defined as the ability to resist off center hits. It is desirable to maximize stability [1].

We can estimate the racket stability by the amount of twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket.

6. Estimation of the Performance of Tennis Rackets having Different Weight and Weight Balance

Now we can predict the various factors associated with the tennis impact when the impact velocity or swing model and the impact locations on the racket face are given. Furthermore

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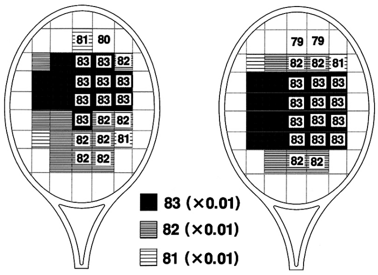
we can estimate the performance of the various rackets with different physical properties.

Figure 5 shows the comparison of the predicted coefficients of restitution e_r between



(a) EOS100 (290g) (b) PROTO-02 (370g)

Fig.5 Predicted Restitution coefficient e_r on the racket face when a player hits a ball (N_S= 56.9Nm, V_{BO}= 10m/s).

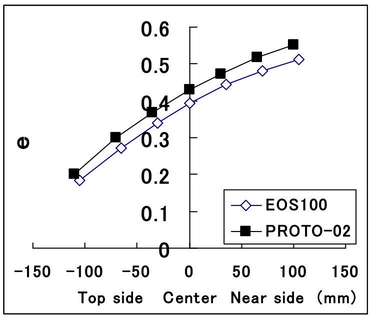


FIg.6 Predicted rebound power coefficient e (N_S= 56.9 Nm, V_{BO}= 10m/s)

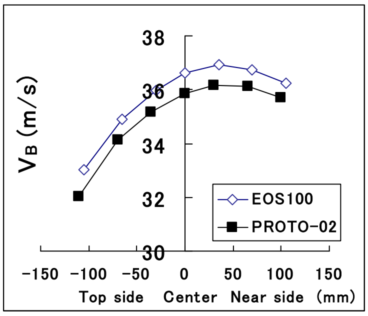
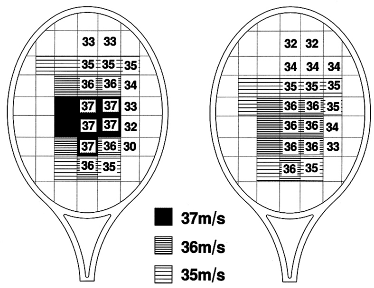


Fig.7 Predicted post-impact ball velocity on the longitudinal axis of a racket face (V_{BO}= 10 m/s, N_s = 56.9 Nm).



(a) EOS100 (290g) (b) PROTO-02 (370g)

Fig.8 Predicted post-impact ball velocity V_B representing sweet area in terms of power.

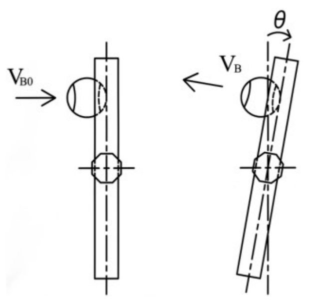


Fig.9 Twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket.

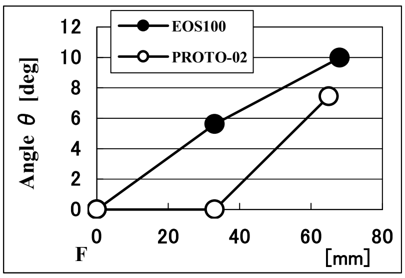


Fig.10 Calculated amount of the racket twist vs. distance of the impact location from the long axis (at the near side on the racket face). Light weighted racket: EOS100: 290 g, Conventional weighted racket: PROTO-02: 370 g)

the light weight racket (290 g) and conventional weight and weight balanced racket (370 g). It is when a player hits a coming ball with a velocity V_{BO} of 10 m/s. It is seen that e_r of a light weight racket is higher than that of a conventional weight and weight balanced racket at the top of the string face. Figure 6 shows the predicted rebound power coefficient e of a light weight racket is higher than that of a conventional weight and weight balanced racket anywhere on the string face. Figure 7 shows the predicted V_B at each hitting location along the longitudinal centerline on the racket face. Figure 8 shows the difference in sweet area in terms of racket power between a light weight racket and conventional heavier weight racket. Although the power of light weighted racket is larger along the centerline on the racket face, it is lower at the near side on the racket face away from the long axis than that of conventional heavier weighted racket. Figure 9 shows the twist or turn about the long axis when the ball hits the strings at the location away from the long axis of a racket face. Figure 10 shows the predicted amount of the racket twist vs. distance of the impact location from the long axis, assuming that there is no friction between the hand and the racket grip. It is found that the twist angle of light weighted racket (EOS100, 290 g) is larger than that of conventional heavier weighted racket (PROTO-02, 370 g) at the near side on the racket face away from the long axis. However, there is no big difference at the topside and the center away from long axis between them. Since the hitting areas for the groundstroker and the service are usually at the topside from the center, there is no big disadvantage for the super-lighted weighted racket for the groundstroker and the service.

7. Conclusions

This paper has derived the contact forces, contact times, coefficient of restitutions, deformations and racket stabilities during impact between a ball and racket. It predicted the impacts of 100 in^2 face size rackets with different weight and weight balance. The predicted results could explain the mechanism of difference in power and stability between a light weighted racket and a conventional weight balanced racket.

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