

Prediction of Table Tennis Racket Restitution Performance Based on the Impact Analysis

Yoshihiko KAWAZOE* and Daisuke SUZUKI**

* *Department of Mechanical Engineering, Saitama Institute of Technology, Saitama*

** *Hitachi Kodaira Semicon Ltd., Tokyo*

This paper investigated the physical properties of the racket and the ball, and predicted the impact force, the contact time, the deformation of ball and rubber, the coefficient of restitution and the racket rebound power associated with the frontal impact when the impact velocity and the impact location on the racket face are given. This study is based on the experimental identification of the dynamic characteristics of the ball-racket- arm system and an approximate nonlinear impact analysis, where the contact time is determined by the natural period of the whole system composed of the mass of the ball, the nonlinear stiffness of the ball and rubber, and the reduced mass of the handled racket at the impact location on the rubber face. Also considered is the energy loss during the impact. It was found that the racket rebound power peaks when the hitting point is 16 cm from the grip end of the racket and then decreases because of the mass distribution of the racket. The racket rebound power decreases remarkably with increasing impact velocity. Although the player's arm gives a remarkable effect on the reduced mass of racket, it does not give an effect on the rebound ball velocity because the mass of ball is too small compared to the mass of racket. This study enables us to predict quantitatively the various factors associated with frontal impact between a racket and a ball in table tennis.

1. INTRODUCTION

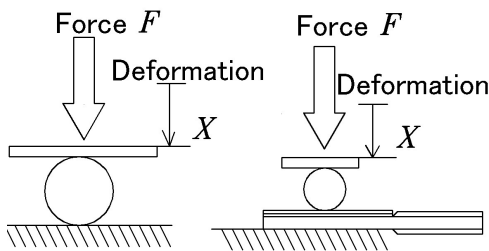
Advanced engineering technology enables manufacturers to discover and synthesize new materials and new designs for sport equipment. There are rackets of various compositions, sizes, weights, shapes and rubbers. Currently, very specific designs are targeted to match the physical and technical levels of each player.¹⁾⁻⁸⁾ However, the ball-and-racket impact is an instantaneous phenomenon, and is complicated by the involvement of a human. Many unknown factors are involved in the mechanisms that explain how the specifications and physical properties of the racket and the ball influence the racket capabilities.⁹⁾⁻¹⁶⁾

This study investigated the physical properties of the racket and the ball, and predicts the impact force, the contact time, the deformation of ball and rubber, the coefficient of restitution and the racket rebound power associated with the frontal impact when the impact velocity and the impact location on the racket face is given. It clarifies the origin of ball speed. It is based on the experimental identification of the dynamic characteristics of the ball-racket system and an approximate nonlinear impact analysis, where the contact time is determined by the natural period of the whole system composed of the mass of the ball, the nonlinear stiffness of the ball and rubber, and the reduced mass of the racket at the impact location on the rubber face. Also considered is the energy loss during the impact.

2. METHOD TO PREDICT THE FRONTAL IMPACT BETWEEN A BALL AND A RACKET IN TABLE TENNIS

2.1 Nonlinear Restoring Force Characteristics of a Ball and Rubbers

Figure 1 shows the test method for obtaining the applied force-deformation curves schematically, where the ball was deformed between two flat rigid surfaces as shown in (a) and the ball plus rubbers were deformed with a racket head clamped as shown in (b). Figure 2 shows the results of force-deformation tests of a ball and a composed rubber and ball system (38 mm ball). The solid curves were computed using a least squares program. The force-deformation curve of the rubber can be obtained from the difference between the two curves.



(a) Ball (b) Composed rubber & ball system
Fig.1 Illustrated applied force - Deformation test

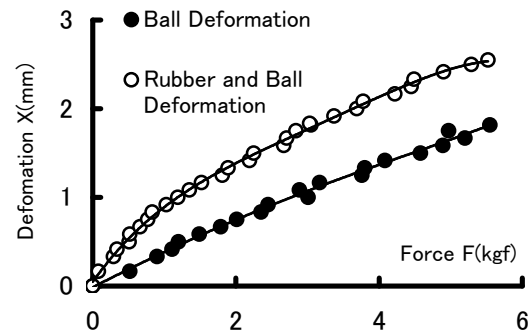


Fig.2 Results of force-deformation tests of a ball and a composed rubber & ball system (38 mm ball, 1 kgf = 9.8 N)

According to the pictures of a racket being struck by a ball, it seems that the ball deforms only at the side, which contact to the rubbers. By assuming that a ball deforms only at the side in contact with the rubbers, we could obtain the curves of $X_R - F$ the restoring force vs. ball deformation, $X_B - F$ the restoring force vs. rubbers deformation, and $X_{RB} - F$ the restoring force vs. deformation of the composed ball/rubbers system from the results of deformation tests. The curve $X_R - F$ in Fig.3 was obtained from difference between two curves in Fig.2. The curve $X_B - F$ was obtained from halving the deformation of the ball in Fig.2. The curve $X_{RB} - F$ was obtained from summation of the curve $X_R - F$ and the curve $X_B - F$ (38 mm ball). The restoring characteristics $X_B = f(F)$, $X_R = f(F)$, and $X_{RB} = f(F)$ were determined using a least squares program.

Figure 4 shows $F_{RB} = f(X)$ the restoring force vs. deformations of the composed rubber/ball system (38

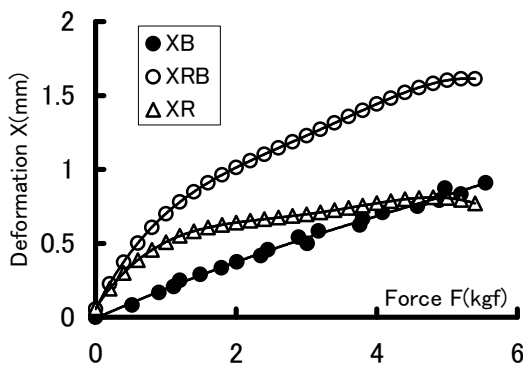


Fig.3. Deformation of a ball X_B , rubber X_R , and a composed rubber/ball system X_{RB} against applied forces when it is assumed that a ball deforms only at the side in contact with the rubber (38 mm ball).

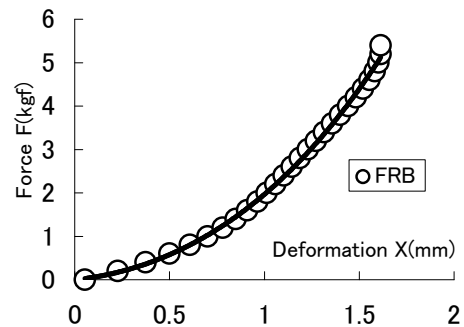


Fig.4 Restoring force vs. deformations of the composed rubber/ball system (38 mm ball).

mm ball) obtained from $X_{RB} = f(F)$ using a least squares program again. The curve $K_{RB} = f(X)$ of the corresponding stiffness vs. deformation of the composed rubber/ball system is derived by differentiation of restoring force $F_{RB} = f(X)$ with respect to deformation as shown in Fig.5. The stiffness of a composed ball/rubbers system exhibits the strong non-linearity.

Figure 6 shows the characteristics $F = f(K_{RB})$ of the restoring force vs. stiffness of the composed rubber/ball system obtained from $F_{RB} = f(X)$ (Fig.4) and $K_{RB} = f(X)$ (Fig.5) by eliminating X . This will be used later as the nonlinear characteristics of stiffness depending on the restoring force in the impact analysis.

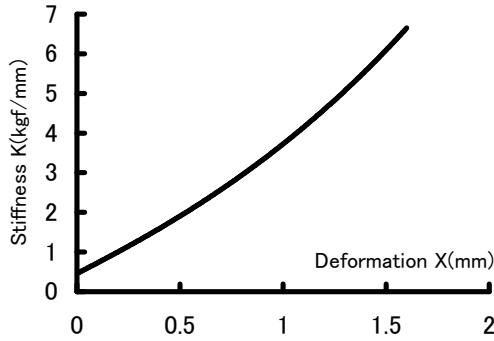


Fig.5 Stiffness vs. deformation of the composed rubber/ball system (38 mm ball).

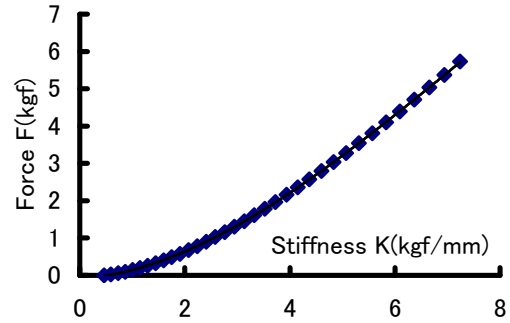


Fig.6 Restoring force vs. stiffness of the Composed rubber/ball system (38 mm ball).

2.2 Energy Loss in a Collision between a Ball and Rubbers

Figure 7(c) shows the measured coefficient of restitution $e_{RB} = V_B/V_{B0}$ versus the incident velocity V_{B0} when a ball strikes the clamped rubbers for estimating energy loss of the ball and the rubber as shown in Fig.7(a),(b).

2.3 Remarks on the Contact Time between a Racket and a Ball during Impact

By analogy from the experimental and theoretical analysis with tennis racket ^{10), 11)}, it is assumed that the masses of a ball and a racket as well as the nonlinear stiffness of a ball and rubbers are the main factors in the deciding of a contact time.

We introduced the reduced mass M_r of a racket at the impact location on the racket face in order to make the impact analysis simpler. It can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location and a center of gravity are given.

Figure 8 shows schematically the impact model for deriving the reduced mass M_r at the impact locations on the racket face.

Consider a ball that impacts the front of a racket at a velocity of V_{B0} and also assume that the racket after impact rotates around the center of gravity, which moves along a straight line.

The impulse S could be described as the following equation, where m is the mass of a ball, V_B the post-impact velocity of a ball, M_R the mass of a racket, V_G the post-impact velocity of the center of gravity (pre-impact velocity $V_{G0} = 0$).

$$S = m (V_{B0} - V_B) \quad (1)$$

$$S = M_R \cdot V_G \quad (2)$$

The following equation can be expressed if the law of angular momentum conservation is applied, where the distance b between the center of gravity and the impact location, the inertial moment I_{G0X}

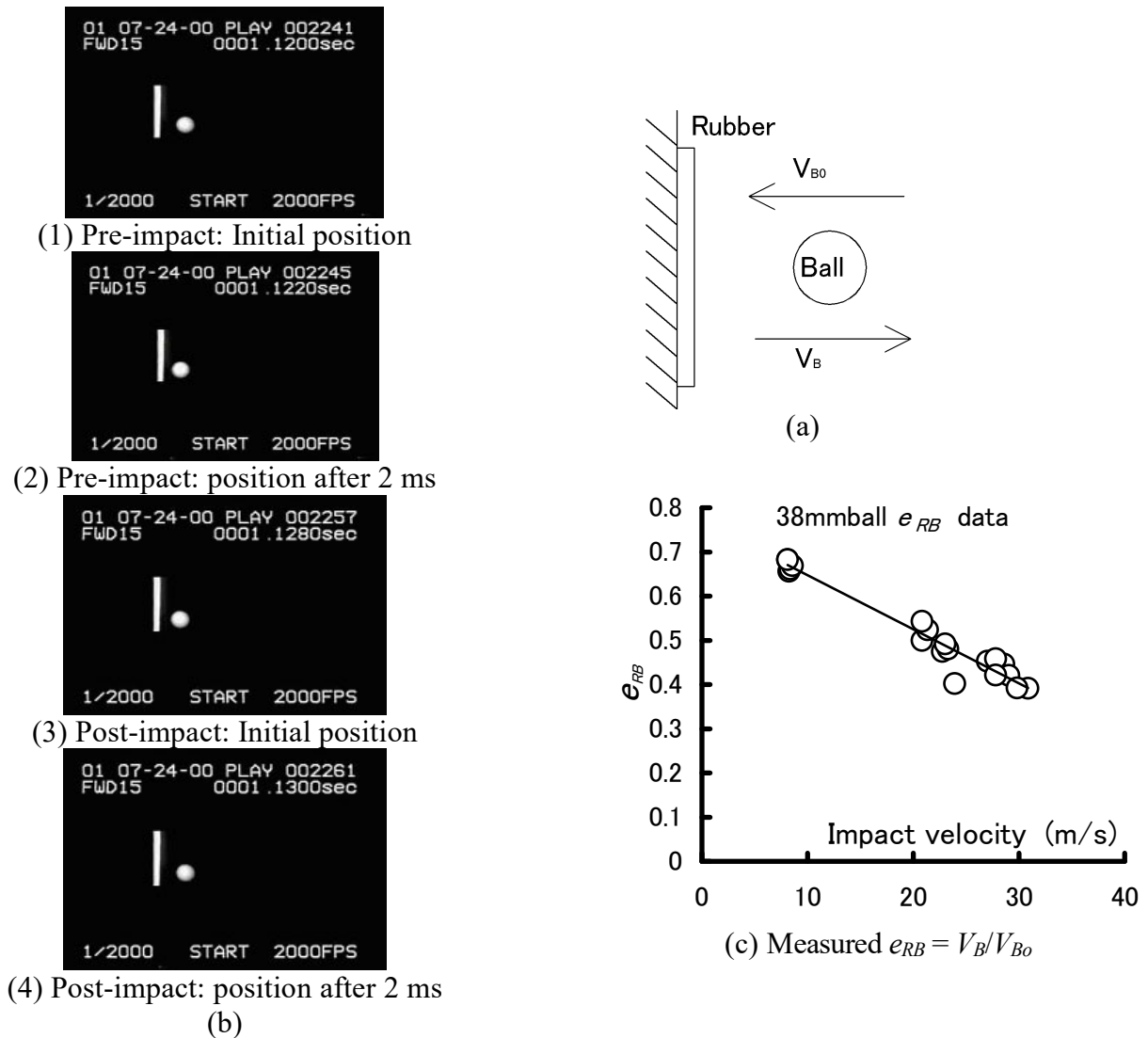


Fig.7 Measured coefficient of restitution between a ball and the clamped rubber (38 mm ball).

around the gravity and the mass M_R of a racket, and the angular velocity ω immediately after impact (pre-impact angular velocity $\omega_0 = 0$) are given.

$$S \cdot b_o = I_{G0X} \cdot \omega \quad (3)$$

Based on the geometric relationships, the velocity V_R at the impact location of the racket after impact can be expressed as follows:

$$V_R = V_G + \omega b_o \quad (4)$$

When ω and V_G are eliminated, the following equation can be written:

$$S = \left(\frac{I_{G0X} M_R}{I_{G0X} + M_R b_o^2} \right) V_R = m (V_{B0} - V_B) \quad (5)$$

Thus, we can express the law of conservation of linear momentum as

$$mV_{B0} = mV_B + M_r V_R \quad (6)$$

where,

$$M_r = \frac{I_{G0X} M_R}{I_{G0X} + M_R b_0^2} \quad (7)$$

The symbol M_r refers to the reduced mass at the impact location for a racket freely suspended. Thus the motion of a racket as a rigid body could be analyzed as though the racket were a particle.

Figure 9 shows the measurement of physical pendulum frequency for obtaining the moment of inertia of a racket. The inertial moment I_{G0X} is obtained using T_X the measured pendulum vibration period, g the gravity acceleration, a the distance between the support location and the center of gravity of a racket as a physical pendulum.

$$I_{G0X} = \left(\frac{T_X}{2\pi} \right)^2 M_R g a - M_R a^2 \quad (8)$$

The shock forces during impact are assumed to be one order of magnitude higher than those due to gravity and muscular action. Accordingly, we consider the racket to be freely hinged to the forearm of the player, the forearm being freely hinged to the arm and the arm freely hinged to the player's body. We can deduce that the inertia effect of the arm and the forearm can be attributed to a mass M_H concentrated in the hand; therefore the analysis of impact between ball and racket can be carried out by assuming that the racket is free in space, as long as the mass M_H is applied at the hand grip as shown in Fig.8(b). The reduced mass M_r at the impact location with a racket-arm system can be derived as

$$M_r = \frac{I_{GMX} (M_R + M_H)}{I_{GMX} + (M_R + M_H) b^2} \quad (9)$$

where

$$b = b_0 + (L_{G0} - L_H) M_H / (M_R + M_H) \quad (10)$$

$$I_{GMX} = I_{G0X} + M_R \Delta G^2 + M_H (L_{G0} - L_H - \Delta G)^2 \quad (11)$$

$$\Delta G = (L_{G0} - L_H) M_H / (M_R + M_H) \quad (12)$$

and L_{G0} denotes the distance between the center of mass and the grip end of the racket, I_{G0X} the moment of inertia with respect to the center of gravity of the racket, b_0 the distance between the center of gravity and the impact location of the racket, and L_H the distance of the point the hand grip from the grip end. The moment of inertia with respect to the center of gravity and the distance of the center of gravity from the impact location of the racket-arm system are indicated by I_{GMX} and b , respectively⁴⁾.

Figure 10 shows the single degree of freedom model of impact between a racket and a ball by introducing a reduced mass of a racket.

Figure 11 shows the impact locations and the center of gravity on the racket face of the tested racket *BISIDE* with rubber *SRIVER* (1.9 mm sponge) made by Tamasu Co. Ltd. Table 1 shows the physical properties of table tennis racket used in the study. The diameter and the mass of the ball are 38 mm and 2.5 g respectively and the mass of the racket (named *BISIDE*) is 171 g including 79.5 g of two sheets rubbers (named *SRIVER*).

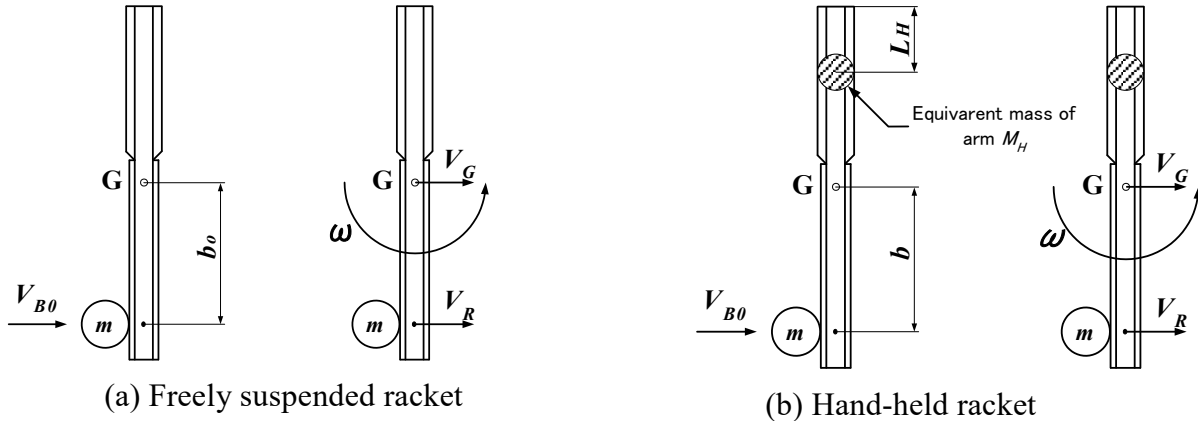


Fig.8 Impact model for deriving the reduced mass at the impact locations on the racket face.

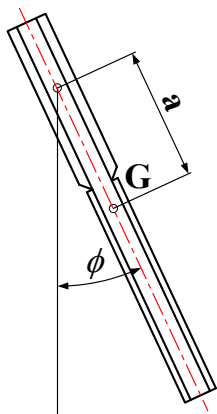


Fig.9 Measurement of physical pendulum frequency for moment of inertia of a racket.

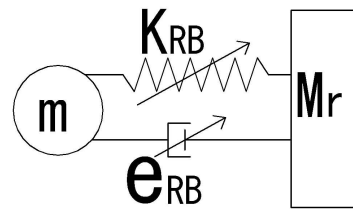


Fig.10 Single degree of freedom model of impact between a racket and a ball by introducing a reduced mass of a racket-arm system.

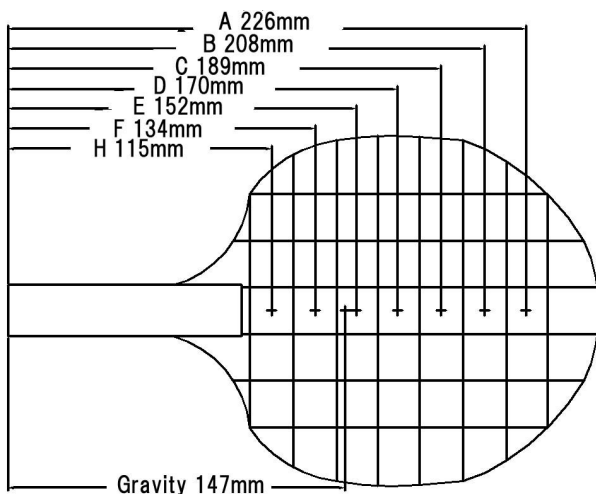


Fig.11 Impact locations and the center of gravity on the racket face of the tested racket *BISIDE* with rubber *SRIVER* (1.9 mm sponge) made by Tamasu Co. Ltd

Table 1 Physical properties of table tennis racket

used in the study.

Racket	BISIDE with rubber	BISIDE without rubber
Face area	185 cm ²	185 cm ²
Mass	171g	91.5g
Center of gravity from grip end	147 mm	130 mm
Moment of inertia I_{GY} about Y axis	2.51 gm ²	1.10 gm ²
Moment of inertia I_{GX} about X axis	0.26 gm ²	0.155 gm ²
1st frequency	253 Hz	351 Hz

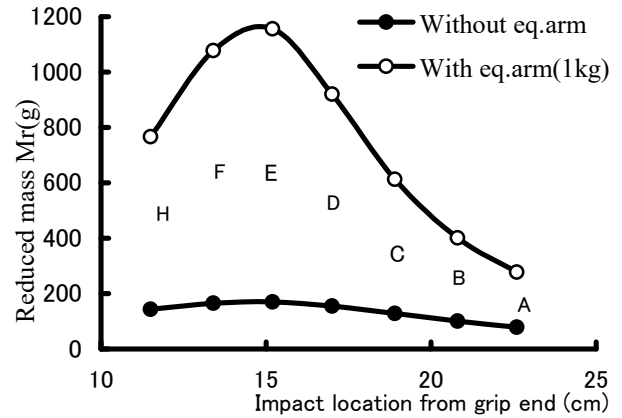


Fig.12 Reduced mass at the locations along the longitudinal centerline on the racket face.

Figure 12 shows the comparison of the reduced mass at the locations along the longitudinal centerline on the racket face between the freely- suspended racket and the handled racket.⁴⁾ It is seen that the player's arm gives a remarkable effect on the reduced mass of racket. It would be shown later whether the increased reduced mass has the effect on the rebounded power of the racket or not.

2.4 Derivation of the Impact Force, Contact Time and Coefficient of Restitution

In case the vibration of the racket frame is neglected, the momentum equation and the coefficient restitution e_{RB} give the approximate post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse could be described as the following equation using the law of conservation of linear momentum and the equation for the velocity difference with the coefficient of restitution, where m_B is the mass of a ball, M_r is the reduced mass of a racket-arm system at the hitting location, and V_{Bo} and V_{Ro} are the ball velocity and racket head velocity before impact, respectively.

$$\int F(t) dt = m_B V_{Bo} - m_B V_B = (V_{Bo} - V_{Ro})(1 + e_{RB})m_B / (1 + m_B/M_r) \quad (13)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of m_B , K_{RB} and M_r as shown in Fig.13, the contact time T_c could be obtained according to the vibration theory

$$T_c = \pi m_B^{1/2} / (K_{RB}(1 + m_B/M_r))^{1/2} \quad (14)$$

In order to make the analysis simpler, the approximate equivalent force F_{mean} can be introduced during contact time T_c , which is described as

$$\int^{T_c} F(t) dt = F_{mean} \cdot T_c \quad (15)$$

Thus, from Eq.(13), Eq.(14) and Eq.(15), the relationship between F_{mean} and corresponding K_{RB} against the pre-impact velocity $(V_{Bo} - V_{Ro})$ is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1 + e_{RB}) m_B^{1/2} K_{RB}^{1/2} / \pi (1 + m_B/M_r)^{1/2} \quad (16)$$

On the other hand, the non-linear relationship between the restoring force vs. stiffness of the composed rubber/ball system shown in Fig.6 can be expressed in the form

$$F_{mean} = f(K_{RB}). \quad (17)$$

From Eq.(16) and Eq.(17), K_{RB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be calculated against the pre-impact velocity by using Eq.(14). Figure 14 shows the graphical description of the derivation of approximate equivalent impact force F_{mean} and the equivalent stiffness K_{RB} of the composed rubber/ball system against the impact velocity ($V_{BO} - V_{Ro}$) during impact between a ball and a racket.

Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force. The mathematical expression is

$$F(t) = F_{max} \sin(\pi t / T_c) \quad (0 \leq t \leq T_c) \quad (18)$$

where $F_{max} = \pi F_{mean}/2$. The Fourier spectrum of Eq.(6) is represented as

$$S(f) = 2F_{max} T_c | \cos(\pi f T_c) | / [\pi | 1 - (2f T_c)^2 |] \quad (19)$$

where f is the frequency.

The vibration characteristics of a racket can be identified using the experimental modal analysis⁹⁻¹⁴ and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the racket face of the identified vibration model of a racket. When the impact force $S_j(2\pi f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k -th mode component at point i is expressed as

$$X_{ijk} = r_{ijk} S_j(2\pi f_k) \quad (20)$$

where r_{ijk} denotes the residue of k -th mode between arbitrary point i and j , and $S_j(2\pi f_k)$ is the impact force component of k -th frequency f_k .

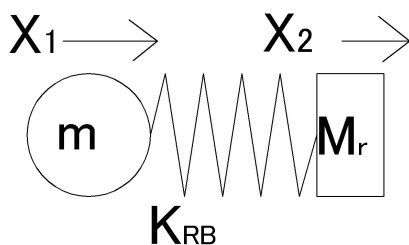


Fig.13 Model for deriving the contact time during impact between a racket and a ball.

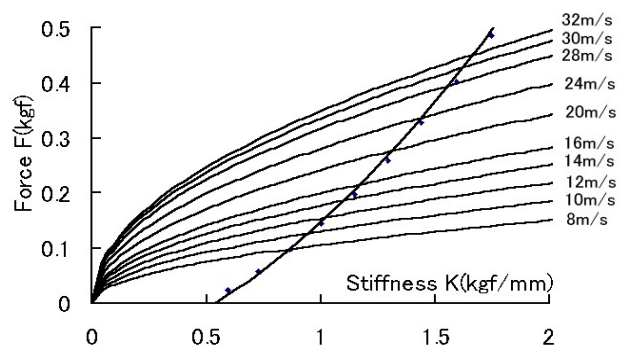


Fig.14 Graphical description of the derivation of equivalent impact force and the equivalent stiffness of the composed rubber/ball system against the impact velocity during impact between a ball and a racket. 1 kgf= 9.8 N.

Figure15 shows an example of predicted maximum amplitude of table tennis racket vibrations

immediately after a ball hits a racket at impact location A (top side on the racket face) with a velocity of 20 m/s using a performance prediction system developed in this study.

The coefficient of restitution e_r can be derived considering the energy loss due to rubber/ball deformation and the racket board vibrations during impact. The coefficient of restitution e_r corresponding to the total energy loss E is obtained as

$$e_r = (V_R - V_B) / V_{BO} = [1 - 2E (m_B + M_r) / (m_B M_r V_{BO}^2)]^{1/2}. \quad (21)$$

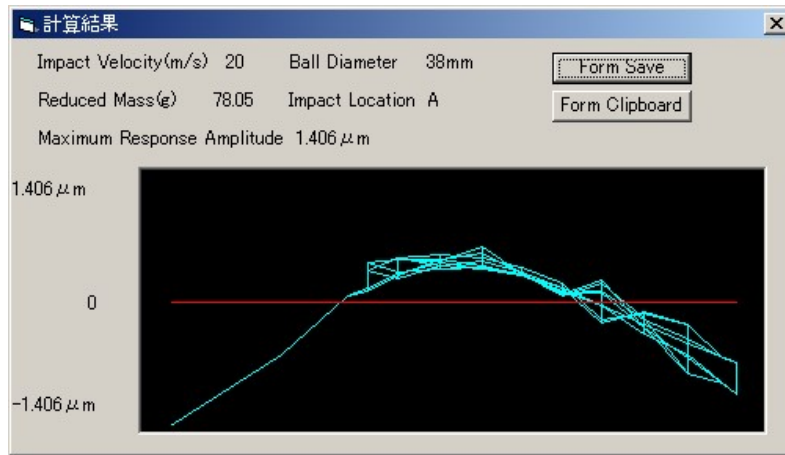


Fig.15 Predicted initial amplitude of table tennis racket vibration when a ball hits a racket at impact location A (top side on the racket face) with a velocity of 20 m/s using a performance prediction system developed in this study.

3. RESULTS AND DISCUSSION

3.1 Predicted Factors Associated with Frontal Impact between a Ball and a Racket

Figure 16 -19 show the calculated impact force F_{mean} , contact time, deformation of the ball and deformation of the rubber against impact velocities ($V_{BO} - V_{Ro}$) respectively.

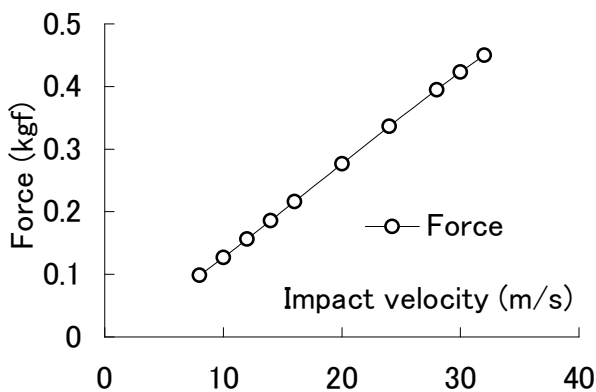


Fig.16 Calculated impact force vs. impact velocity.

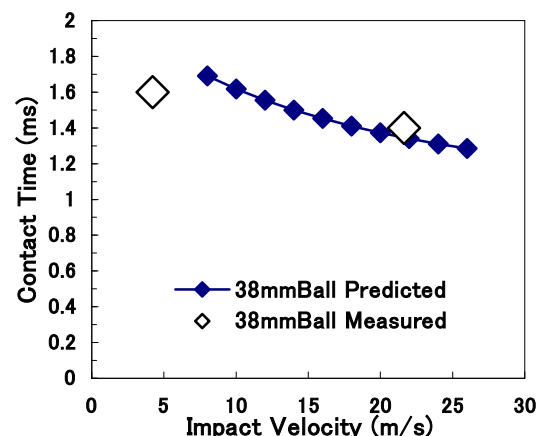


Fig.17 Predicted contact time vs. impact velocity compared to the measured.

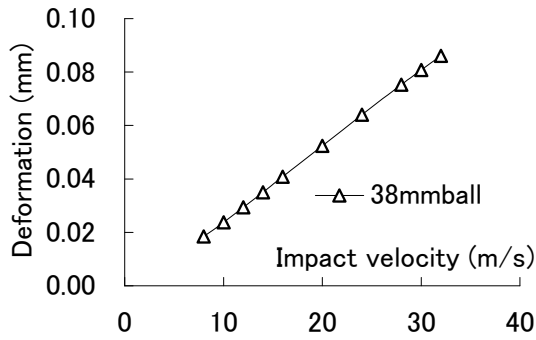


Fig.18 Calculated deformation of the ball vs. impact velocity.

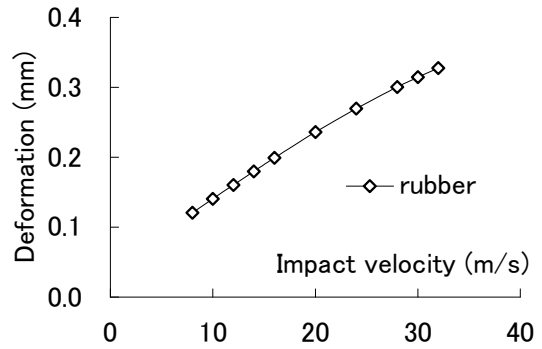


Fig.19 Calculated deformation of the rubber vs. impact velocity.

3.2 Predicted Rebound Power Coefficient

The post-impact ball velocity V_B is represented as

$$V_B = -V_{Bo}(e_r - m_B/M_r)/(1+m_B/M_r) + V_{Ro}(1+e_r)/(1+m_B/M_r) \quad (22)$$

Accordingly, the ratio e of rebound velocity against the incident velocity of a ball when a ball strikes the freely suspended racket ($V_{Ro} = 0$) is written as Eq.(11). We define this coefficient e the rebound power coefficient. The rebound power coefficient e is often used to estimate the rebound power performance of a racket experimentally in the laboratory.

$$e = -V_B / V_{Bo} = (e_r - m_B/M_r)/(1+m_B/M_r) \quad (23)$$

Figure 20 shows the effect of reduced mass on the rebound power coefficient. It shows the interesting fact that the player's arm gives a remarkable effect on the reduced mass of racket but it does not give an effect on the rebound ball velocity because the mass of ball is too small compared to the mass of racket.

Figure 21 shows the predicted rebound power coefficient e of a racket, (a) when a ball strikes at the location of D (center of racket face) and (b) when a ball strikes at the location of A (top side of racket face), comparing with board vibrations and without. There is no big effect of board vibrations on the rebound power coefficient. Figure 22 shows the predicted rebound power coefficient e when a ball strikes a suspended racket at the locations of longitudinal centerline on the racket face. Fig.23 shows the predicted rebound ball velocity when a ball strikes a suspended racket at the location of D (center) and A (top side).

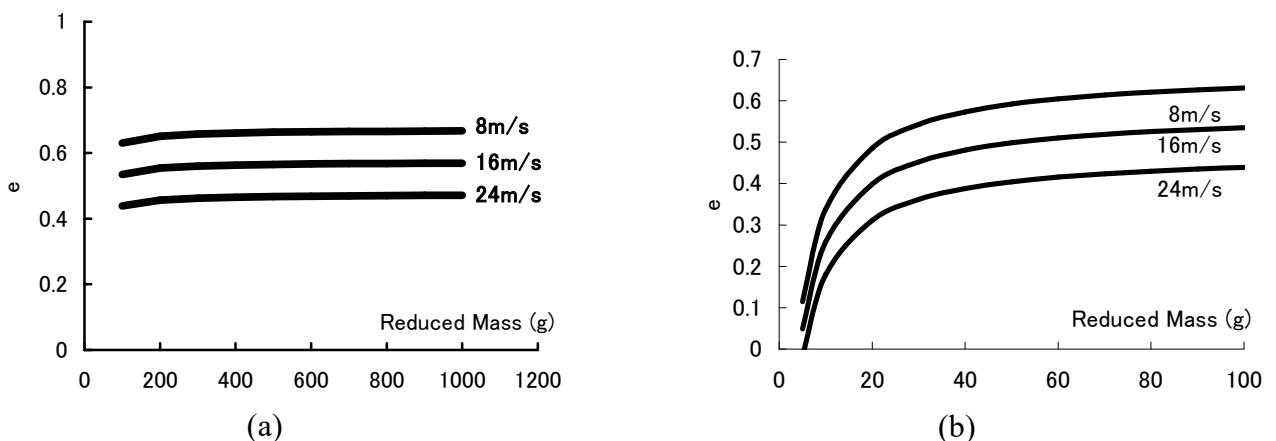
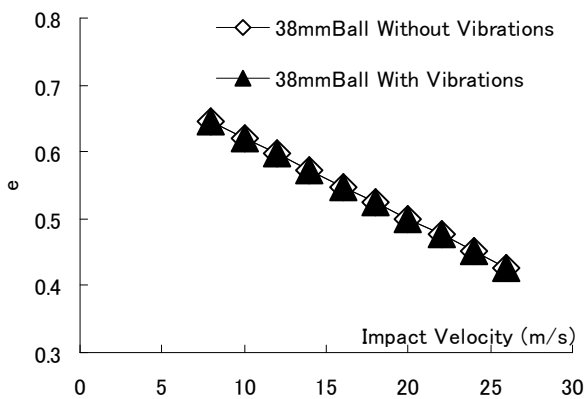
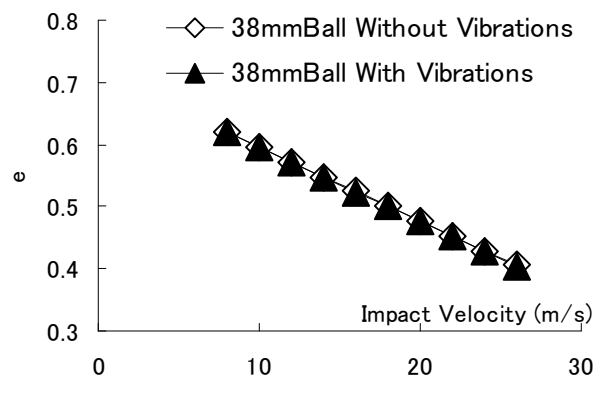


Fig.20 Effect of reduced mass on the rebound power coefficient



(a) when a ball strikes at the location of D (center).



(b) when a ball strikes at the location of A (top side).

Fig.21 Predicted rebound power coefficient e of a racket.

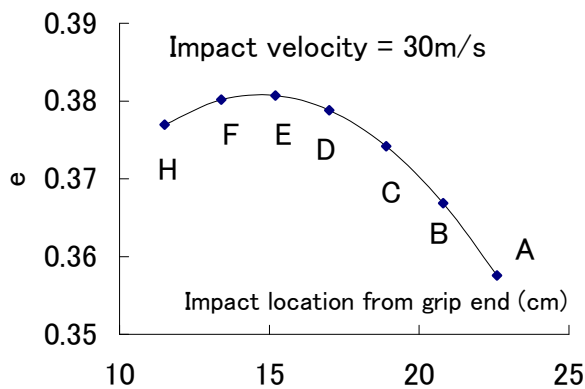


Fig.22 Predicted rebound power coefficient e when a ball strikes a suspended racket at the locations of longitudinal centerline on the racket face.

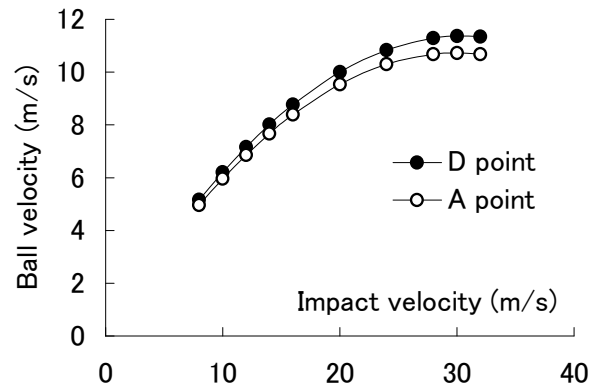


Fig.23 Predicted rebound ball velocity when a ball strikes a suspended racket at the location of D(center) and A(top side).

4. CONCLUSIONS

This study investigated the physical properties of the racket and the ball, and predicted the impact force, the contact time, the deformation of ball and rubber, the coefficient of restitution and the racket rebound power associated with the frontal impact when the impact velocity and the impact location on the racket face are given. It is based on the experimental identification of the dynamic characteristics of the ball- racket-arm system and an approximate nonlinear impact analysis. It enables us to predict quantitatively the various factors associated with frontal impact between a racket and a ball in table tennis. The result showed that the rebound power coefficient peaks at 16 cm from the grip end of the racket and then decreases because of the mass distribution of the racket. The rebound power coefficient decreases remarkably with increasing impact velocity. It also found that the player's arm gives a remarkable effect on the reduced mass of racket but it does not give an effect on the rebound ball velocity because the ball is too small compared to the racket in mass. The mechanism of the feel and the role of vibrations of the racket might be separately reported in the near future.

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