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Impact prediction between a ball and racket in terms of contact forces, contact times, restitution coefficients and sweet spot in table tennis

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1 Introduction

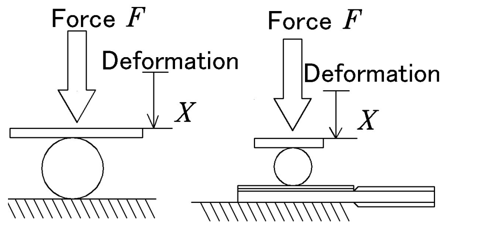
Currently, very specific designs for racket sport equipment are targeted to match the physical and technical levels of each player. However, the ball/racket impact is an instantaneous phenomenon, complicated by the involvement of a human. Many unknown factors are involved in the mechanisms that explain how the specifications and physical properties of the racket and the ball influence the racket capabilities (Kawazoe *et al.* 1998, 2000, 2002 a, 2002 b).

This paper investigates the physical properties of the table tennis racket and ball, and predicts the impact force, the contact time, the deformation of ball and rubber, the coefficient of restitution and the racket rebound power associated with the frontal impact when the impact velocity and the impact location on the racket face are given. It is based on the experimental identification of the dynamic characteristics of the ball-racket- arm system and an approximate nonlinear impact analysis. Also considered is the vibrations at the grip portion of the racket handle. The diameter of the ball is 38 mm. The comparison of the 40 mm ball with the 38 mm ball is reported in a separate paper.

2 Main factors associated with frontal impact between a ball and a racket in table tennis

Figure 1 shows the test for obtaining the applied force-deformation curves schematically, where the ball was deformed between two flat surfaces as shown in (a) and the ball plus rubbers were deformed with a racket head clamped as shown in (b). Figure 2 shows the results of the force-deformation tests (38 mm ball, 2.5 g).

By assuming that a ball deforms only at the side in contact with the rubbers, we could obtain the curves of restoring force vs. ball deformation X_B, restoring force vs. rubber deformation X_R, and the restoring force vs. deformation X_{RB} of the composed ball/rubber system from the results of deformation tests. These restoring characteristics are determined in order to satisfy a number of experimental data using the least squares method. The curve of the corresponding stiffness is derived by differentiation of the equations of restoring force with respect to deformation. The stiffness K_{RB} of a composed ball/rubbers system exhibits strong non-linearity.



(a) Ball (b) Composed rubber & ball system

Fig.1 Illustrated applied force - Deformation test

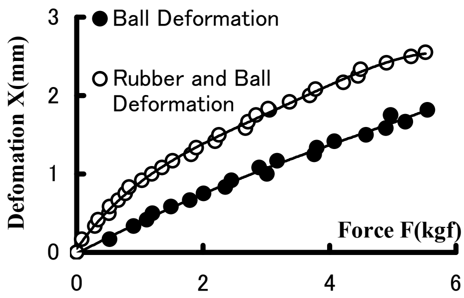
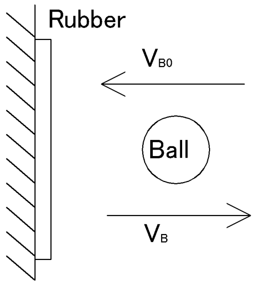
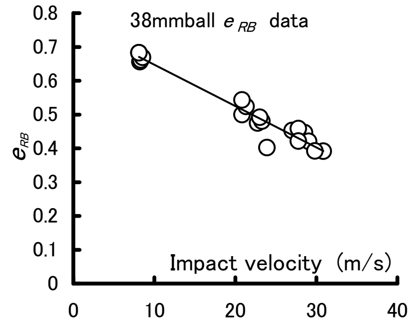


Fig.2 Results of force-deformation tests of a ball and a composed rubber & ball system (38 mm ball, 1 kgf = 9.8 N)

Figure 3 shows the measured coefficient of restitution $e_{RB} = V_B/V_{Bo}$ versus the incident velocity V_{Bo} when a ball strikes the clamped rubbers for estimating energy loss of the ball and the rubber.



(a)



(b) Measured $e_{RB} = V_B/V_{Bo}$

Fig.3 Measured coefficient of restitution between a ball and the clamped rubber (38 mm ball).

The reduced mass M_r of a racket at the impact location on the racket face can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location and a center of gravity are given. Figure 4 shows the single degree of freedom model of impact between a racket and a ball by introducing a reduced mass M_r of a racket, where m is the mass of a ball. Figure 5 shows the impact locations and the center of gravity of the tested racket made by Tamasu Co. Ltd. The mass of the racket (BISIDE) is 171 g including 79.5 g of two sheets rubbers (SRIVER). Figure 6 shows the comparison of the reduced mass at the locations along the longitudinal centreline on the racket face between the freely-suspended racket and the handled racket (Kawazoe 2000 a). The player's arm gives a remarkable effect on the reduced mass of racket.

3 Derivation of the impact force, contact time, coefficient of restitution and the vibration

If we neglect the vibration of the racket frame as a first approximation, the momentum equation and the measured coefficient e_{RB} give the approximate post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse $\int F(t)\,dt$ could be described. Assuming the contact time T_c to be half the natural period of a whole system composed of m, K_{RB} and M_r, it could be obtained according to the vibration theory.

In order to make the analysis simpler, the approximate equivalent force F_{mean} can be introduced during contact time T_c . Thus, the relationship between F_{mean} and corresponding K_{RB} against the pre-impact velocity (V_{BO} - V_{Ro}) is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1 + e_{RB})\, m_B^{1/2}\, K_{RB}^{\ 1/2} / \pi(\ 1 + m_B/M_r\ )^{1/2} \qquad (1)$$

On the other hand, the measured force curve can be expressed as the function of K_{RB}

$$F \ = f(\ K_{RB}\ ). \qquad (2)$$

From Eq.(1) and Eq.(2), K_{RB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be calculated against the pre-impact velocity. The force-time curve of impact is approximated as a half-sine pulse $F(t) = F_{max}\sin(\pi t/ T_c)$ $(0 \leqq t \leqq T_c)$, where $F_{max} = \pi F_{mean}/2$. The Fourier spectrum of half-sine pulse is represented as $S(f)$ where f is the frequency (Kawazoe 1992 a).

The vibration characteristics of a racket can be identified using experimental modal analysis (Kawazoe 1989, 1992 a, 1993, 1994, 1997) and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the racket face of the identified vibration model of a racket. When the impact force $S_j(f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k-th mode component at point i is derived using the residue r_{ijk} of k-th mode between arbitrary point i and j.

The coefficient of restitution e_r between a ball and a racket can be derived considering the energy loss due to rubber/ball deformation and the racket vibrations during impact. The coefficient of restitution e_r corresponding to the total energy loss E is

$$e_r = (\ V_R\ - V_B)/\ V_{BO}\ = [1 - 2E\,(\ m_B\ + M_r)/\,(m_B M_r V_{BO}^{\ 2})]^{1/2.} \qquad (3)$$

The maximum shock acceleration A_{grip} at the handgrip considering the equivalent mass

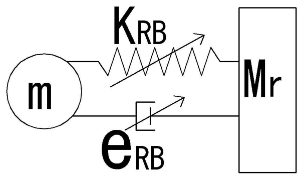


Fig.4 Single degree of freedom model of impact between a racket and a ball by introducing a reduced mass of a racket-arm system.

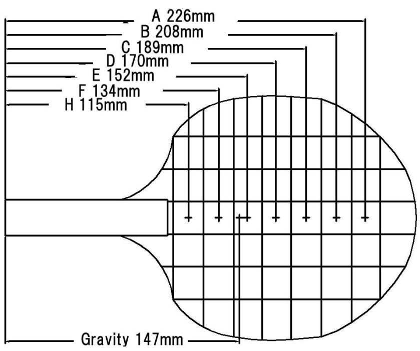


Fig.5 Impact locations and the center of gravity on the racket face of the tested racket BISIDE with rubber SRIVER (1.9 mm sponge) made by Tamasu Co. Ltd

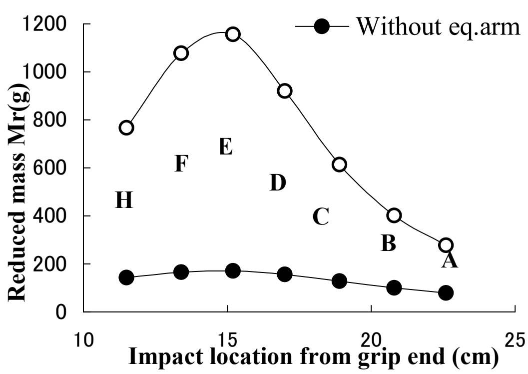


Fig.6 Reduced mass at the locations along the longitudinal centerline on the racket face.

M_H of the arm can be derived using the distance a between the center of mass of racket-arm system and the impact location of the racket, the distance between the center of mass of racket-arm system and the location of hand grip, the moment of inertia I_G with respect to the center of mass of racket-arm system, the total mass m_{racket} of the racket.

The initial vibration amplitude $A_{grip\ j\ k}$ of k-th mode acceleration component at the racket handle 50 mm from the grip end with the impact point j is derived using the residue of k-th mode (Kawazoe 1989, 1992 a, 1993, 1994, 1997).

4 Results and discussion

Figures 7-10 show the calculated impact force, contact time, deformation of the ball and deformation of the rubber against impact velocities respectively.

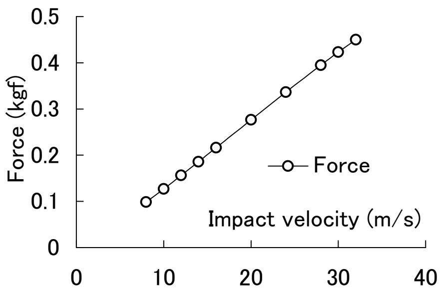


Fig.7 Calculated impact force vs. impact velocity. (1 kgf = 9.8 N)

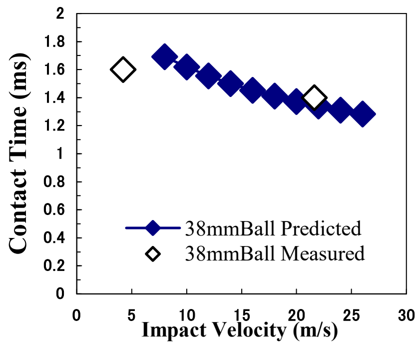


Fig.8 Predicted contact time vs. impact velocity compared to the measured.

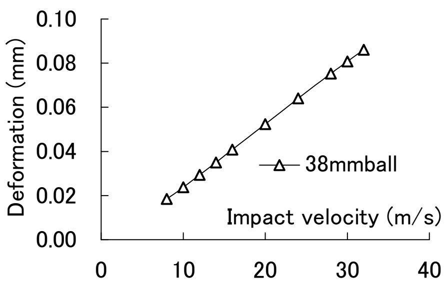


Fig.9 Calculated deformation of the ball vs. impact velocity.

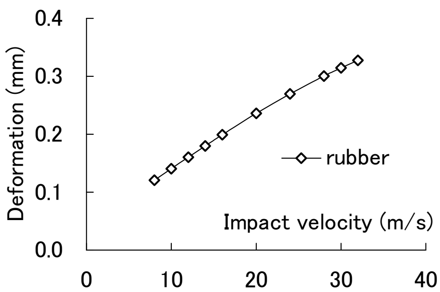


Fig.10 Calculated deformation of the rubber vs. impact velocity.

The ratio e of rebound velocity V_B against the incident velocity V_{BO} of a ball when a ball strikes the standstill racket (V_{Ro}= 0) is

$$e = -V_B\ /\ V_{BO} = (\ e_r\ - m_B/M_r)\ /(1+ m_B/M_r\ ) \qquad (4)$$

We define this coefficient e the rebound power coefficient. The coefficient e is often used to estimate the rebound power performance of a racket experimentally in the laboratory.

Figure 11 shows the effect of reduced mass on the rebound power coefficient. The player's arm has a remarkable effect on the reduced mass of racket but it does not have an effect on the rebound ball velocity because the mass of a ball is too small compared to the mass of a racket. Figure 12 shows the predicted rebound power coefficient e of a racket when a ball strikes at the location of A (top side of racket face), comparing with racket board vibrations and without. There is no big effect of board vibrations on the rebound power coefficient. Figure 13 shows the predicted rebound power coefficient *e* vs. impact locations of longitudinal centreline on the racket face. Figure14 shows the predicted rebound ball velocity vs. impact velocities when a ball strikes the location of D (center) and A (top side).

Figure 15 shows the main vibration modes of table tennis racket with rubbers (1st mode and 2nd mode). Figure 16 shows the predicted shock component and initial vibration amplitude component at the racket handle 50 mm from the grip end when a ball hits a racket at various impact locations with a velocity of 20 m/s. The vibration component is much larger than the shock component at the racket handle.

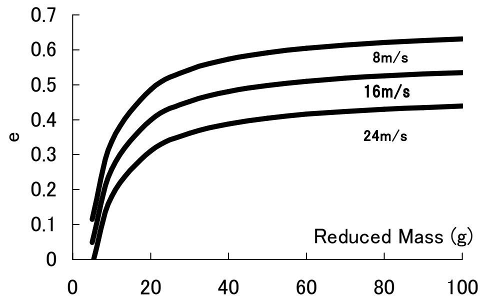


Fig.11 Effect of reduced mass on the rebound power coefficient

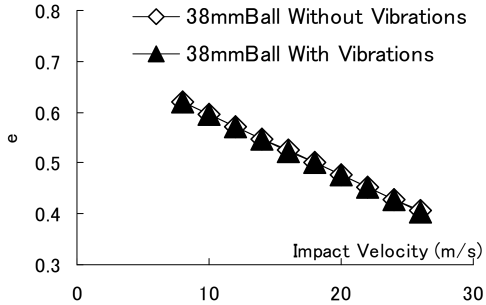


Fig.12 Predicted rebound power coefficient *e* of a racket when a ball strikes at the location of A (top side).

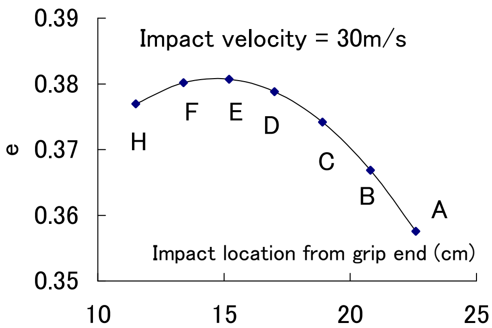


Fig.13 Predicted rebound power coefficient *e* when a ball strikes the longitudinal centerline on the racket.

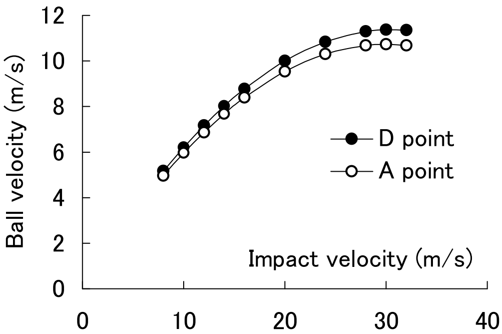
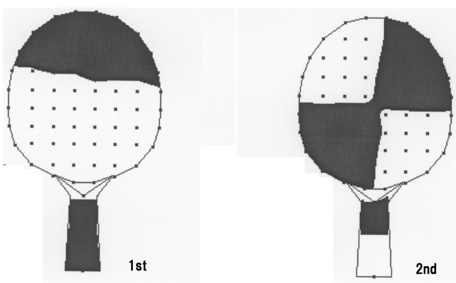


Fig.14 Predicted rebound ball velocity when a ball strikes the location of D(center) and A(top side).



(a) 1st mode (b) 2nd mode

Fig.15 Vibration modes of table tennis racket with rubbers.

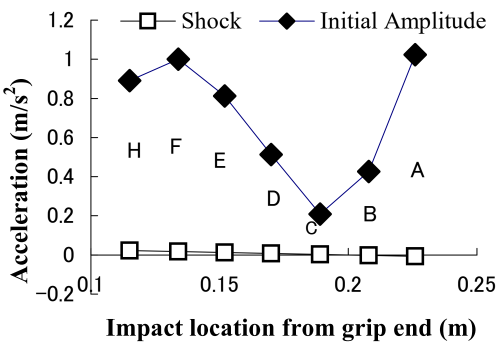


Fig.16 Shock and vibrations at the racket handle 50 mm from grip end with impact velocity 20 m/s.

Furthermore, the sweet spot in terms of the vibration is very remarkable. There seems to be a possibility that players might sense impact location on the racket face through the magnitude of vibrations at the grip portion. This seems to play an important role for a performance in table tennis.

5 Conclusions

The racket rebound power decreases remarkably with increasing impact velocity. Although the player's arm has a remarkable effect on the reduced mass of racket, it does not have an effect on the rebound ball velocity because the mass of ball is too small compared to the mass of racket.

The vibration component is much larger than the shock component at the racket handle. Furthermore, the sweet spot in terms of the vibration is very remarkable. There seems to be a possibility that players might sense impact location on the racket face through the magnitude of vibrations at the grip portion. This seems to play an important role for a performance in table tennis.

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