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Performance Prediction of Active Piezo Fiber Rackets in Terms of Tennis Power*

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Abstract

Several former top players sent a letter to the International Tennis Federation (ITF) encouraging the governing body to revisit the question of rackets. In the letter, the players wrote that racket technology has led to major changes in how the game is played at the top level. This paper investigated the physical properties of a new type of racket with active piezoelectric fibers appeared recently in the market, and predicted the various factors associated with the frontal impact, such as impact force, contact time, deformation of ball and strings, and also estimated the racket performance such as the coefficient of restitution, the rebound power coefficient, the post-impact ball velocity and the sweet areas relevant to the power in tennis. It is based on the experimental identification of the dynamics of the ball-racket-arm system and the approximate nonlinear impact analysis with a simple swing model. The predicted results with forehand stroke model can explain the difference in mechanism of performance between the new type racket with active piezoelectric fibers and the conventional passive representative rackets. It showed that this new type racket provides higher coefficient of restitution on the whole area of string face and also gives larger rebound power coefficients particularly at the topside and bigger powers on the whole area of string face but the difference was not so large. It seems that the racket-related improvements in play are relatively small and the players themselves continue to improve, accordingly there is a gap between a perception and reality.

Key words: Sports Engineering, Performance Prediction, Tennis Racket, Active Piezoelectric Fibers, Impact Analysis, Experimental Modal Analysis

1. Introduction

According to the recent news (August 2003), several former top players, including McEnroe, Boris Becker and Martina Navratilova, sent a letter to the International Tennis Federation (ITF) encouraging the governing body to revisit the question of rackets. In the letter, the players wrote that tennis has become "unbalanced and one-dimensional." "The sport has lost something, lost some subtlety, some strategy, some of the nuance. Rackets today allow players to launch the ball at previously unthinkable speeds, approaching 150 mph." "The reason for this change is clear to see," they wrote. "Over a period of years, modern racket technology has developed powerful, light, wide-bodied rackets that are easier to wield than wooden rackets were and have a much larger effective hitting area." "They're high-tech weapons made of graphite, Kevlar, titanium and exotic alloys. There's even a

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racket with a chip built into the handle that allows the racket to stiffen upon impact with the ball. All of this technology has led to major changes in how the game is played at the top level."

Since tennis should be learned from experience, it is a subjective thing. Thus, it is quite difficult to see how the physical properties of a tennis racket have an effect on the performance of a player(1)-(8).

The lightweight racket with handle-light configuration is recent tendency of high-tech rackets, increasing power with an increasing racket swing speed. However, the previous paper of the author showed that the lightest racket at present in the market has advantage for racket head speed, but disadvantage for coefficient of restitution, rebound power, and post-impact velocity for ground stroke, and it has also large shock vibrations at the racket handle compared to the ordinary lightweight racket. This means there is a limit to current lightweight design from the viewpoint of tennis racket performance(9). The engineers and racket designers at the racket companies seem to be under intense pressure to keep pumping out new and better technologies every year.

This paper investigates the physical properties of a new type of racket with active piezoelectric fibers appeared recently in the market (10), and predicts the various factors associated with the frontal impact, such as impact force, contact time, deformation of ball and strings, and also estimates the racket performance such as the coefficient of restitution, the rebound power coefficient, the post-impact ball velocity and the sweet areas relevant to the power in tennis. It is based on the experimental identification of the dynamics of the ball-racket-arm system and the approximate nonlinear impact analysis with a simple swing model. The vibration characteristics of a racket can be identified using the experimental modal analysis because the amplitudes are relatively small compared to the deformations of strings and a ball. The damping characteristics of the racket-arm system are identified at the wrist joint and the racket handle during actual impact in the forehand ground stroke. The racket vibrations can be simulated by applying the impact force-time curve derived from the nonlinear analysis based on the measured nonlinear restoring force characteristics of a ball and strings and the collision experiment between a ball and strings with the racket head clamped.

This system enables us to predict quantitatively the performance of various rackets with various specifications and physical properties, even with recent innovative complex structures like active piezoelectric fibers.

Figure 1 shows a racket with active piezoelectric fibers and a chip that allows the racket to stiffen upon impact with the ball according to the racket maker's catalogue.

Technical background was shown as follows (10). Piezoceramic materials have been applied for damping of vibrations. What makes the current technology radically different is that it is an active damping system. In the passive system, the electrical energy is dissipated across a shunt circuit, while in the active system the electrical energy is stored and released back into the materials such that it actively damps the vibration, without using any external energy. Head introduced their Intelligence rackets, incorporating newly developed piezo ceramics in the form of Intellifibers™, which were developed from piezoelectric fiber composites (PFCs) or Active fiber composites (AFCs). Intellifibers™ are manufactured from continuous thin PZT fibers, approximately 0.3 mm in diameter. These fibers are extruded ceramics, and polarized to exhibit piezoelectric properties. The individual fibers are very brittle and are sandwiched between two polyimide film layers to prevent mechanical failure. The electrodes that collect the voltage are etched in silver on the inside of one of the layers, resulting in thin flexible strips, containing about 50 fibers. The Intellifibers™ are connected via wires to a circuit board located in handle. During the impact, the vibrations constantly causes the Intellifibers™ to generate a very high potential at low current. this is stored in a coil on the circuit board in real time and released back to the Intellifibers™ in the optimal phase and wave form for the most efficient dampening. The stored energy is send back to the Intellifibers™ in a phase which causes an opposite mechanical force to the vibration, so reducing it. The circuit board is tuned to the first natural frequency of the racket and can only damp vibrations within a range of its design frequency. The frequency of the circuit board and placement of the Intellifibers^{TM} on the racket are determined through a vibration analysis of the final racket frame it is intended for. Four Intellifibers^{TM} are placed on the fork of the racket, on either side of each piller. A combined finite element and laboratory modal analysis established the optimal fiber positions, and frequencies to damp. Each is connected to the circuit board, which is located in the handle. The circuit board is tuned to the frequency considered as the most likely to have the largest effect at the handle. The Intellifibers^{TM} and the circuit board are assembled as a complete unit, the Chipsystem^{TM}, with the board being encased in Polyurethane form, to protect it from the heat and mechanical vibrations. The unit is added to the racket lay-up just before insertion into the mold, with the Intellifibers^{TM} forming the outside layer of the racket and the circuit board placed between the two carbon fiber tubes forming the racket handle. The unit is therefore molded as an integral part of the racket's structure. Test performed on rackets with the Chipsystem^{TM}, using an accelerometer, reveal up to a 50 % reduction in the damping ratio of the vibration of a freely suspended racket.

2. Method to Predict the Frontal Impact Between Ball and Racket

We introduce the reduced mass M_r of a racket at the impact location on the racket face in order to make the impact analysis simpler. It can be derived from the principle of the conservation of angular momentum if the moment of inertia and the distance between an impact location and a center of gravity are given. Consider a ball that impacts the front of a racket at a velocity of V_{B0} and also assume that the racket after impact rotates around the center of gravity, which moves along a straight line. The impulse S could be described as the following equation, where m_B is the mass of a ball, V_B the post-impact velocity of a ball, M_R the mass of a racket, V_G the post-impact velocity of the center of gravity (pre-impact velocity $V_{G0} = 0$).

$$S = m_B\,(\,V_{B0} - V_B\,) \tag{1}$$

$$S = M_R \cdot V_G \tag{2}$$

The following equation can be expressed if the law of angular momentum conservation is applied, where the distance b_0 between the center of gravity and the impact location, the inertial moment I_{G0X} around the center of gravity and the mass M_R of a racket, and the angular velocity ω immediately after impact (pre-impact angular velocity $\omega_0 = 0$) are given.

$$S \cdot b_o = I_{G0X} \cdot \omega \tag{3}$$

Based on the geometric relationships, the velocity V_R at the impact location of the racket after impact can be expressed as follows:

$$V_R = V_G + \omega b_o \tag{4}$$

When ω and V_G are eliminated, the following equation can be written:

$$S = \left(\frac{I_{G0X} M_R}{I_{G0X} + M_R bo^2} \right) V_R \quad = m_B\,(\,V_{B0} - V_B\,) \tag{5}$$

Thus, we can express the law of conservation of linear momentum as

$$m_B V_{B0} = m_B V_B + M_r V_R \tag{6}$$

where,

$$M_r = \frac{I_{G0X} M_R}{I_{G0X} + M_R b_0^{\ 2}} \qquad (7)$$

The symbol M_r refers to the reduced mass at the impact location for a racket freely suspended. Thus the motion of a racket as a rigid body could be analyzed as though the racket were a particle.

The inertial moment I_{G0X} is obtained using T_X; the measured pendulum vibration period, g; the gravity acceleration, a; the distance between the support location and the center of gravity of a racket as a physical pendulum.

$$I_{G0X} = \left(\frac{T_X}{2\pi}\right)^2 M_R g a - M_R a^2 \qquad (8)$$

The shock forces during impact are assumed to be one order of magnitude higher than those due to gravity and muscular action. Accordingly, we consider the racket to be freely hinged to the forearm of the player, the forearm being freely hinged to the arm and the arm freely hinged to the player's body. We can deduce that the inertia effect of the arm and the forearm can be attributed to a mass M_H concentrated in the hand; therefore the analysis of impact between ball and racket can be carried out by assuming that the racket is free in space, as long as the mass M_H is applied at the hand grip. The reduced mass M_r at the impact location with a racket-arm system can be derived as

$$M_r = \frac{I_{GMX}(M_R + M_H)}{I_{GMX} + (M_R + M_H) b^2} \qquad (9)$$

where

$$b = b_o + (L_{Go} - L_H) M_H / (M_R + M_H) \qquad (10)$$

$$I_{GMX} = I_{GoX} + M_R \triangle G^2 + M_H (L_{Go} - L_H - \triangle G)^2 \qquad (11)$$

$$\triangle G = (L_{Go} - L_H) M_H / (M_R + M_H) \qquad (12)$$

and L_{Go} denotes the distance between the center of mass and the grip end of the racket, I_{GoX} the moment of inertia with respect to the center of gravity of the racket, b_o the distance between the center of gravity and the impact location of the racket, and L_H the distance of the point of the hand grip from the grip end. The moment of inertia with respect to the center of gravity and the distance of the center of gravity from the impact location of the racket-arm system are indicated by I_{GMX} and b, respectively.

Figure 2 shows the single degree of freedom model of impact between a racket and a ball by introducing a reduced mass of a racket.

In case the vibration of the racket frame is neglected, the momentum equation and the measured coefficient restitution e_{GB} give the approximate post-impact velocity V_B of a ball and V_R of a racket at the impact location. The impulse could be described as the following equation using the law of conservation of linear momentum and the equation for the velocity difference with the coefficient of restitution, where m_B is the mass of a ball, M_r is the reduced mass of a racket-arm system at the hitting location, and V_{Bo} and V_{Ro} are the ball velocity and racket head velocity before impact, respectively.

$$\int F(t)\,dt = m_B V_{Bo} - m_B V_B = (V_{BO} - V_{Ro})(1+e_{GB})m_B/(1+m_B/M_r) \qquad (13)$$

Assuming the contact duration during impact to be half the natural period of a whole system composed of m_B, K_{GB} and M_r as shown in Fig.2, the contact time T_c could be obtained according to the vibration theory.

$$T_c = \pi\ m_B^{1/2}/(K_{GB}(1+m_B/M_r))^{1/2} \qquad (14)$$

In order to make the analysis simpler, the approximate equivalent force F_{mean} can be introduced during contact time T_c, which is described as

$$\int^{Tc} F(t)\,dt = F_{mean} \cdot T_c \qquad (15)$$

Thus, from Eq.(13), Eq.(14) and Eq.(15), the relationship between F_{mean} and corresponding K_{GB} against the pre-impact velocity (V_{BO} - V_{Ro}) is given by

$$F_{mean} = (V_{BO} - V_{Ro})(1+e_{GB})\, m_B^{1/2} K_{GB}^{1/2}/\pi(1+m_B/M_r)^{1/2} \qquad (16)$$

where e_{BG} is the measured coefficient of restitution (11) when a ball strikes the clamped string bed for estimating energy loss of the ball and the strings, being equivalent to the nonlinear damping coefficient C_{GB} in Fig.2.

On the other hand, the non-linear relationship between the measured restoring force F_{GB} (Fig.3) vs. stiffness K_{GB} (Fig.4) of the composed strings/ball system can be expressed in the form

$$F_{mean} = f(K_{GB}). \qquad (17)$$

From Eq.(16) and Eq.(17), K_{GB} and F_{mean} against the pre-impact velocity can be obtained, accordingly T_C can also be determined against the pre-impact velocity (11). A comparison between the measured contact times during actual forehand strokes and the calculated ones when a ball hits the center of the strings face of a conventional type racket, showing a good agreement (11). Since the force-time curve of impact has an influence on the magnitude of racket frame vibrations, it is approximated as a half-sine pulse, which is almost similar in shape to the actual impact force.

(a) Is-10(Head) (b) Piezoelectric fibers

Fig.1 Racket with active piezoelectric fibers.

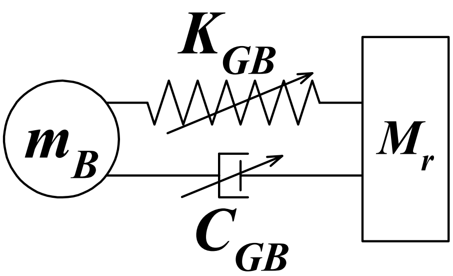


Fig.2 Non-linear impact model of a ball-string system.

The mathematical expression is

$$F(t) = F_{max}\sin(\pi t/T_c) \quad (0 \leqq t \leqq T_c) \tag{18}$$

where $F_{max} = \pi F_{mean}/2$. The Fourier spectrum of Eq.(18) is represented as

$$S(f) = 2F_{max}T_c \mid \cos(\pi f T_c) \mid / [\pi \mid 1 - (2fT_c)^2 \mid] \tag{19}$$

where f is the frequency.

The vibration characteristics of a racket can be identified using the experimental modal analysis [(9)-(14)] and the racket vibrations can be simulated by applying the impact force-time curve to the hitting portion on the racket face of the identified vibration model of a racket. When the impact force $S_j(2\pi f_k)$ applies to the point j on the racket face, the amplitude X_{ijk} of k -th mode component at point i is expressed as

$$X_{ijk} = r_{ijk} \; S_j(2\pi f_k) \tag{20}$$

where r_{ijk} denotes the residue of k-th mode between arbitrary point i and j, and $S_j(2\pi f_k)$ is the impact force component of k-th frequency f_k .

Figure 5 shows the string meshes for vibration model and impact locations for impact simulation, and Fig.6 shows an example of the calculated shock shape when a ball strikes the center on the string face.

The energy loss due to the racket vibration induced by impact can be derived from the amplitude distribution of the vibration velocity and the mass distribution along the racket frame. The coefficient of restitution e_r (COR) can be derived considering the energy loss E_1 due to racket vibrations and E_2 due to large deformations of a ball and strings corresponding to the coefficient e_{BG}. If a ball collides with a racket at rest ($V_{Ro} = 0$), the coefficient of restitution e_r corresponding to the total energy loss E $(= E_1 + E_2)$ can be obtained as

$$e_r = (V_R - V_B)/V_{BO} = [1 - 2E(m_B + M_r)/(m_B M_r V_{BO}^2)]^{1/2.} \tag{21}$$

The ratio of rebound velocity against the incident velocity of a ball when a ball strikes the freely suspended racket ($V_{Ro} = 0$) is defined as the rebound power coefficient e written as Eq.(22), because the coefficient e is often used to estimate the rebound power performance of a racket experimentally in the laboratory. A comparison between the measured e and the predicted e when a ball hit a freely-suspended racket (about 30 m/s) showed a good agreement between them [(11)].

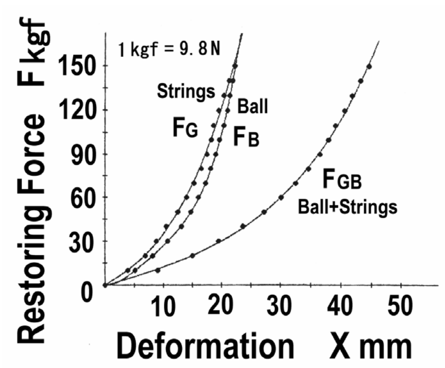


Fig.3 Restoring forces vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings [(11)].

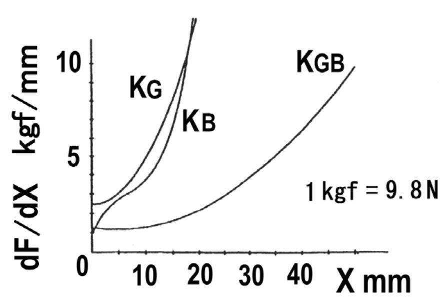
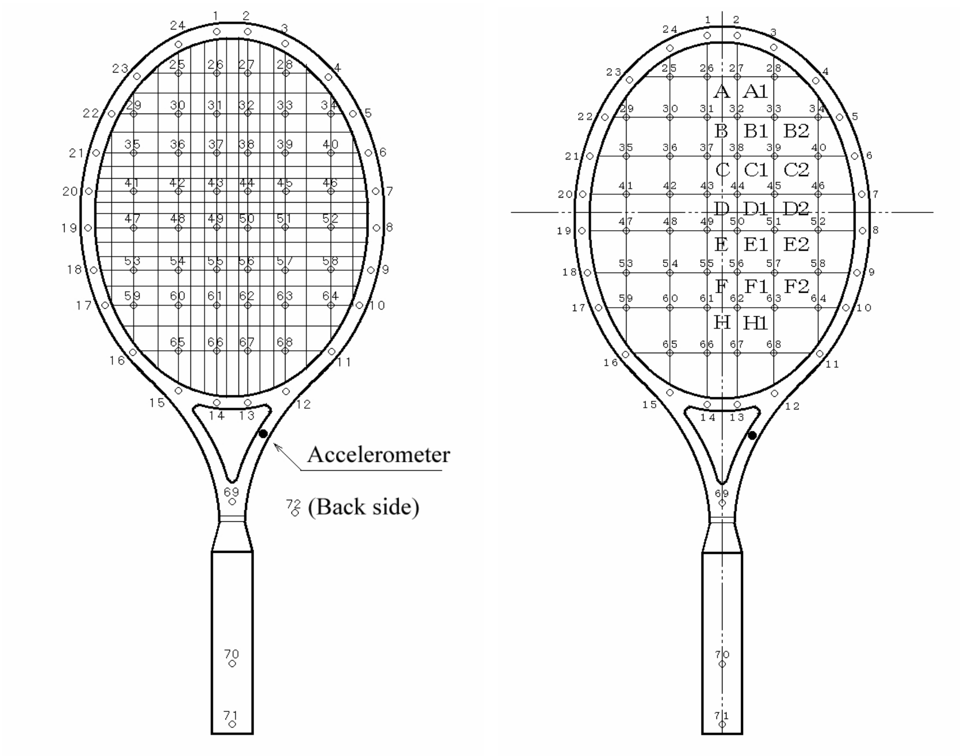


Fig.4 Stiffness vs. deformation of a ball, strings, and a composed ball/string system assuming that a ball deforms only at the side in contact with the strings [(11)].

$$e = -(V_B - V_{RO})\ /(V_{BO} - V_{RO}) = -V_B\ /\ V_{BO}\ =\ (\,e_r\ - m_B/M_r)\ /(1 + m_B/M_r\,) \qquad (22)$$

The power of the racket could be estimated by the post-impact ball velocity V_B when a player hits the ball. The V_B can be expressed as Eq.(23). The pre-impact racket head velocity V_{RO} is given by $L_X(\pi N_s / I_s)^{1/2}$, where L_X denotes the horizontal distance between the player's shoulder joint and the impact location on the racket face, N_s the constant torque about the shoulder joint, and I_s the moment of inertia of arm/racket system about the shoulder joint. Figure 7 shows a simple forehand ground stroke swing model (5),(6).

$$V_B\ =\ -V_{Bo}\ \ e\ \ + V_{Ro}\,(\,1 + e\ ) \qquad (23)$$



(a) (b)

Fig.5 String meshes for vibration model and impact locations for impact simulation.

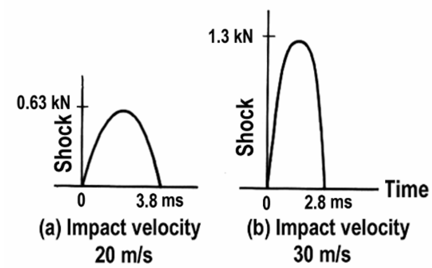


Fig.6 Calculated shock shape when a ball strikes the center on the String face.

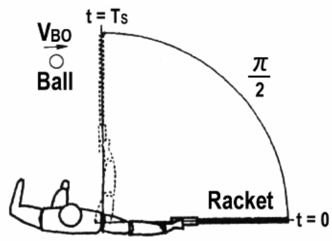


Fig.7 Forehand stroke model

3. Performance Estimation of Tennis Racket with Active Piezoelectric Fibers Compared to the Conventional Passive rackets in terms of Power

Now we can predict the various factors associated with the tennis impact when the impact velocity or swing model and the impact locations on the racket face are given. Furthermore we can estimate the performance of the various rackets with different physical properties. Figure 8 shows the geometry of Intelligent fiber racket Is-10. Table 1 shows the physical properties of three representative rackets (Intelligent fiber Is-10, Lightest racket TSL, Highest power racket EOS120A among available passive rackets), where I_{GY} denotes the moment of inertia about the center of mass, I_{GR} the moment of inertia about the grip 70 mm from grip end and I_{GX} the moment of inertia about the longitudinal axis of racket head. Table 2 shows the result of experimental vibration modal analysis and Fig.9 shows the mode shapes of rackets Is-10, TSL and EOS120A [(3),(12)-(14)]. The 1st mode frequency of racket Is-10 is higher than those of the other rackets considering the other frequencies. It is the reason that the piezo-electricity is embedded at the anti-node of 1st vibration mode of racket frame.

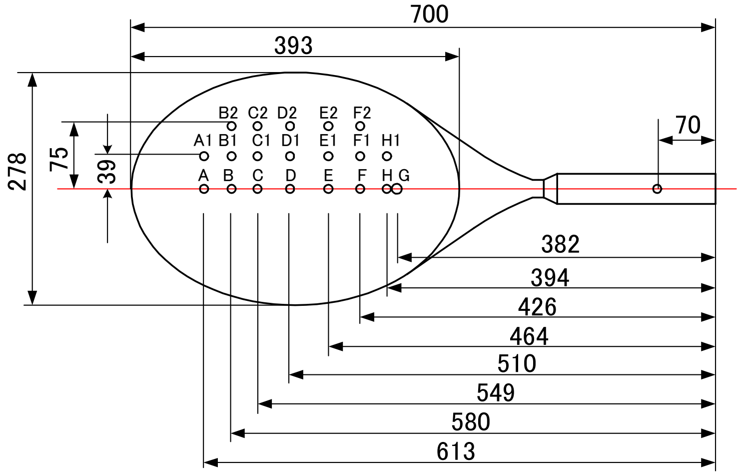


Fig.8 Geometry of of Intelligent fiber Racket Is-10.

Table 1 Physical properties

Racket	IS－10	TSL	EOS120A
Total length	700 mm	710 mm	690 mm
Face area	740 cm^2	742 cm^2	760 cm^2
Mass	241 g	224 g	292 g
Center of gravity from grip end	382 mm	379 mm	363 mm
Moment of intertia I_{GY} about Y axis	11.2 gm^2	11.0 gm^2	14.0 gm^2
Moment of intertia I_{GR} about grip	36.7 gm^2	32.4 gm^2	39.0 gm^2
Moment of intertia I_{GX} about X axis	1.51 gm^2	1.21 gm^2	1.78 gm^2
1st frequency	205 Hz	200 Hz	137 Hz
Strings tension	55 lb	55 Ib	79 Ib
Reduced mass (center)	179 g	152 g	206 g

Table 2 Frequencies of vibration modes of 3 rackets (Hz)

	Is-10	TSL	EOS120A
1st	205 Hz	200 Hz	137 Hz
2nd	400 Hz	474 Hz	322 Hz
3rd	493 Hz	557 Hz	391 Hz
4th	532 Hz	581 Hz	605 Hz

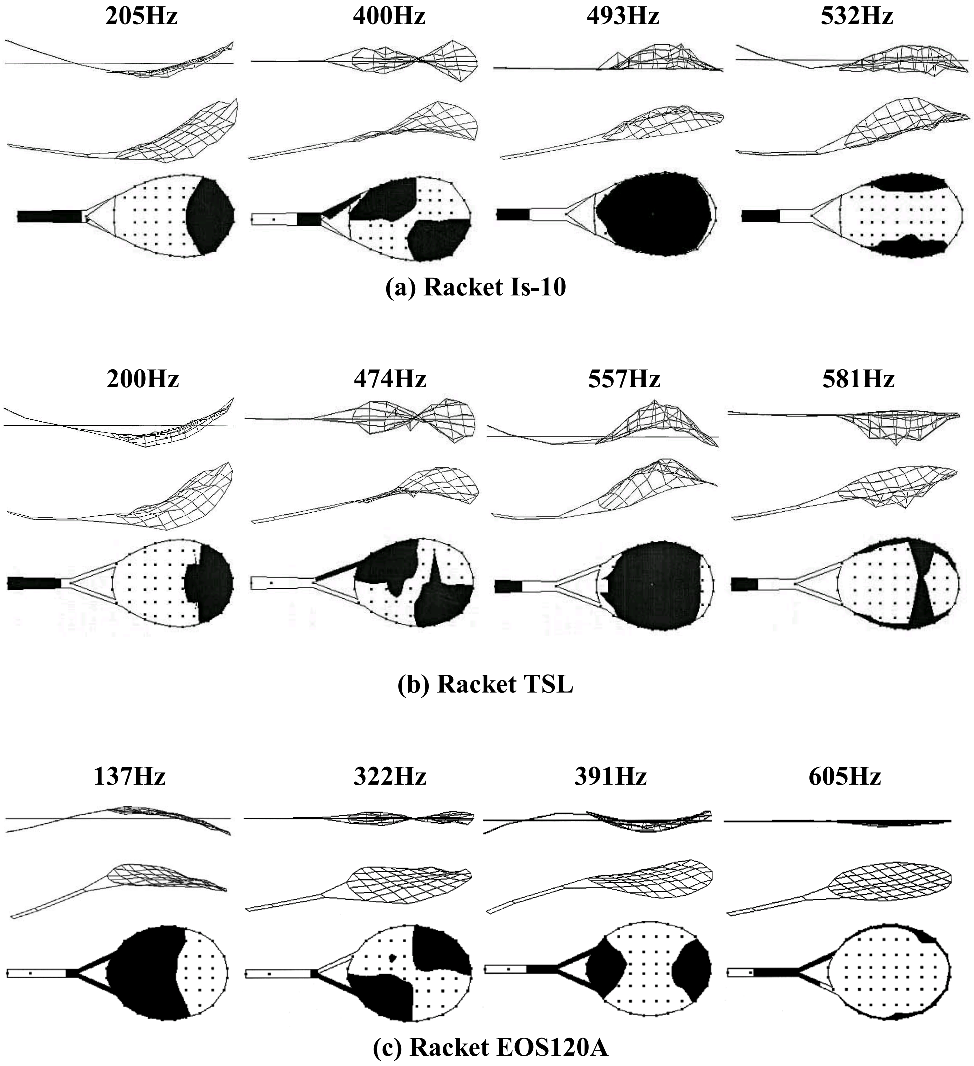


Fig.9 Experimentally identified vibration modes

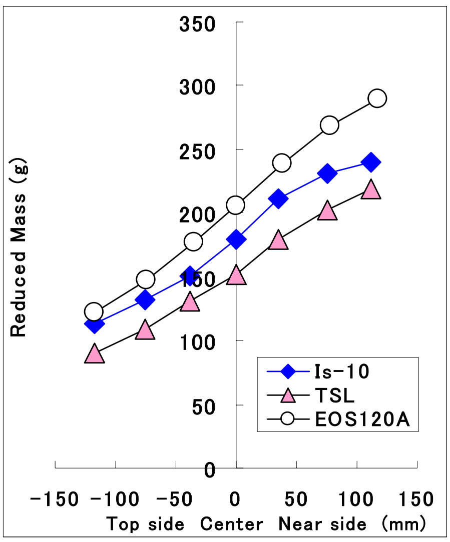


Fig.10 Reduced mass M_r of racket at the hitting locations.

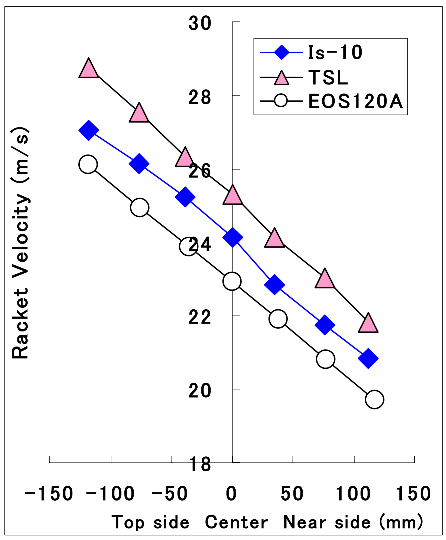


Fig.11 Predicted pre-impact racket head velocity V_{Ro} (Ns= 56.9 Nm, V_{B0} = 10m/s)

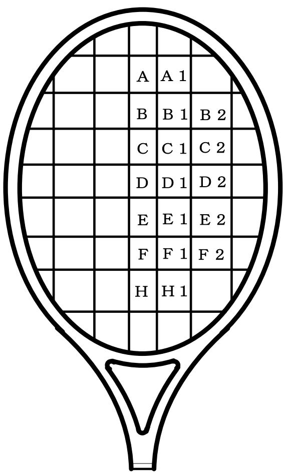
Figure 10 shows the reduced mass of racket at the hitting locations on the string face and Fig.11 shows the predicted pre-impact racket head velocity V_{Ro} (Ns= 56.9 Nm, V_{B0} =10m/s) at the hitting locations on the string face.

Figure 12 shows the comparison of the predicted coefficients of restitution e_r between three rackets during forehand stroke. It is seen that e_r of intelligent fiber racket is higher than that of a lightest racket TSL and quite high even at the top side off-center on the string face, because the energy loss due to frame vibrations are rather small.

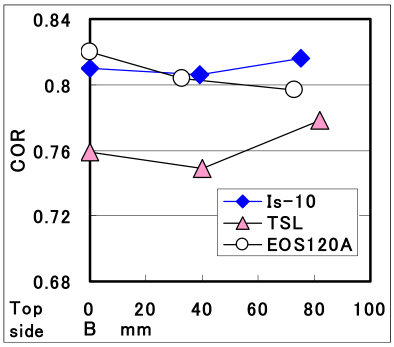
The intelligent fiber racket also gave larger rebound power coefficients particularly at the topside shown in Fig.13.

Figure 14 shows the predicted post-impact ball velocity V_B at each hitting location along the longitudinal centerline on the racket face.

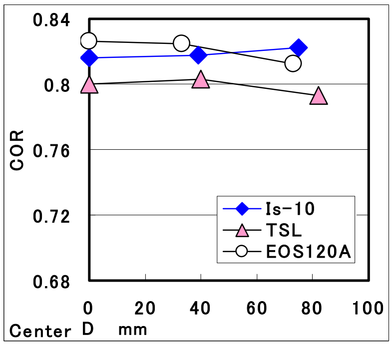
Figure 15 shows the difference in sweet area in terms of racket power or V_B between three rackets compared to a wooden racket (375 g). It is seen that V_B of intelligent fiber racket is higher than that of a lightest racket TSL and quite high even at the top off-center on the string face. The post-impact ball velocity V_B of racket is-10 is 5 % larger at the center



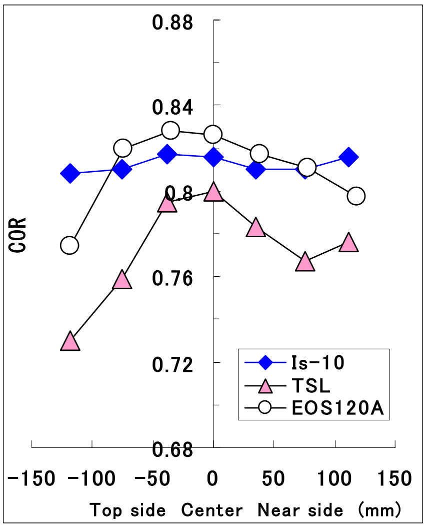
(a) Impact locations



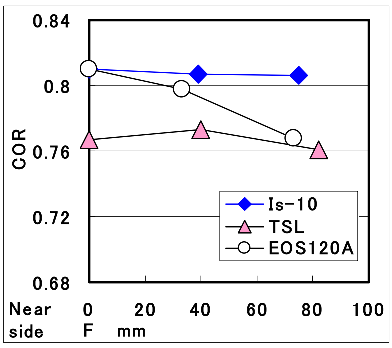
(c) Top side off-center



(d) Center off-center

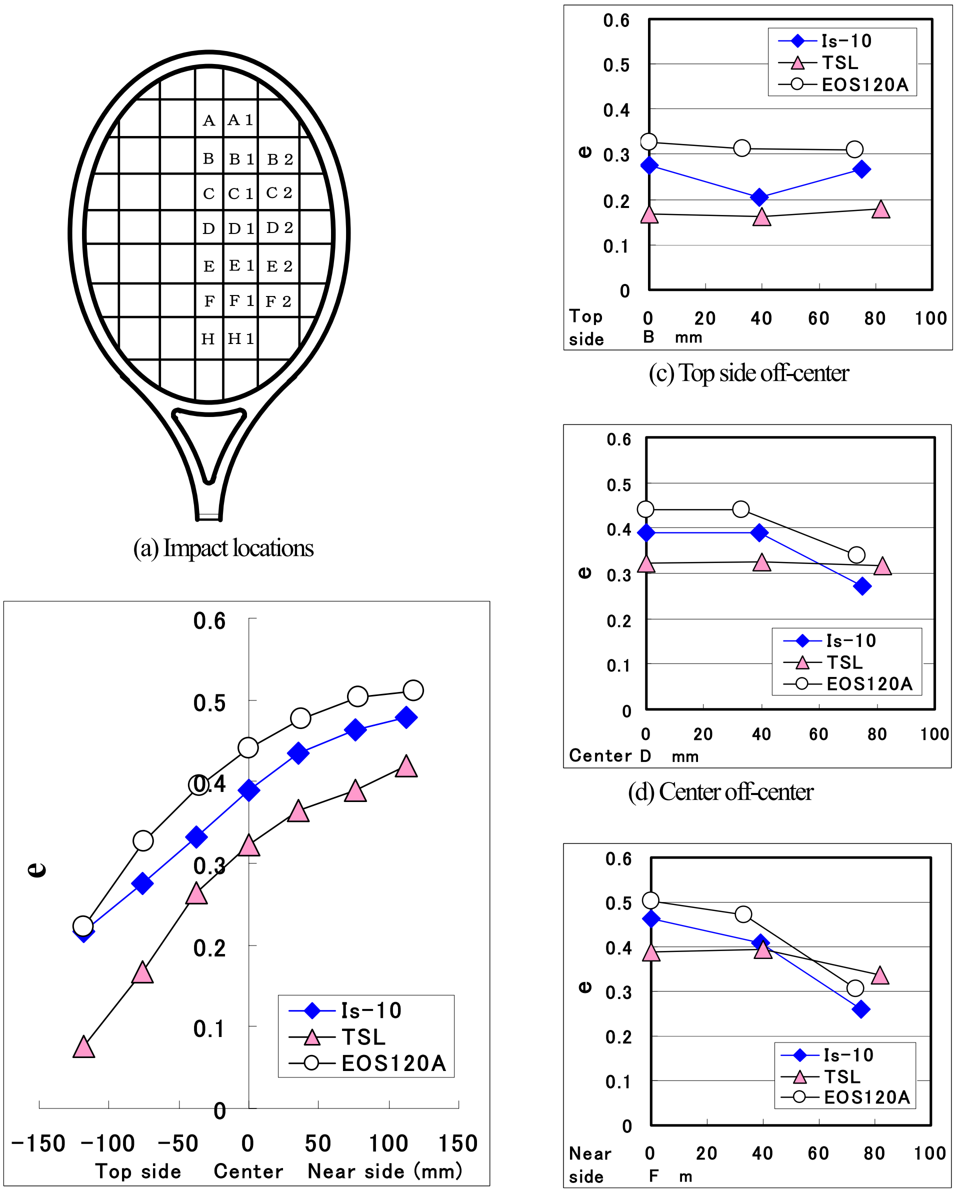


(b) Along the longitudinal center



(e) Near side off-center

Fig.12 Predicted Restitution coefficient e_r (Ns = 56.9Nm, V_{B0} = 10m/s) hitting and 14 % larger at the top off-center hitting compared to wooden racket. Although this new type racket surely provides higher coefficient of restitution on the whole area of string face and also gives larger rebound power coefficients at the topside and bigger powers on the whole area of string face but the difference was not so large. It seems that the racket-related improvements in play are relatively small and the players themselves continue to improve by making use of racket improvement, accordingly there is a gap between perception and reality.



(a) Impact locations

(b) Along the longitudinal center

(c) Top side off-center

(d) Center off-center

(e) Near side off-center

Fig.13 Predicted rebound power coefficient *e* (*Ns* = 56.9Nm, V_{BO} = 10m/s)

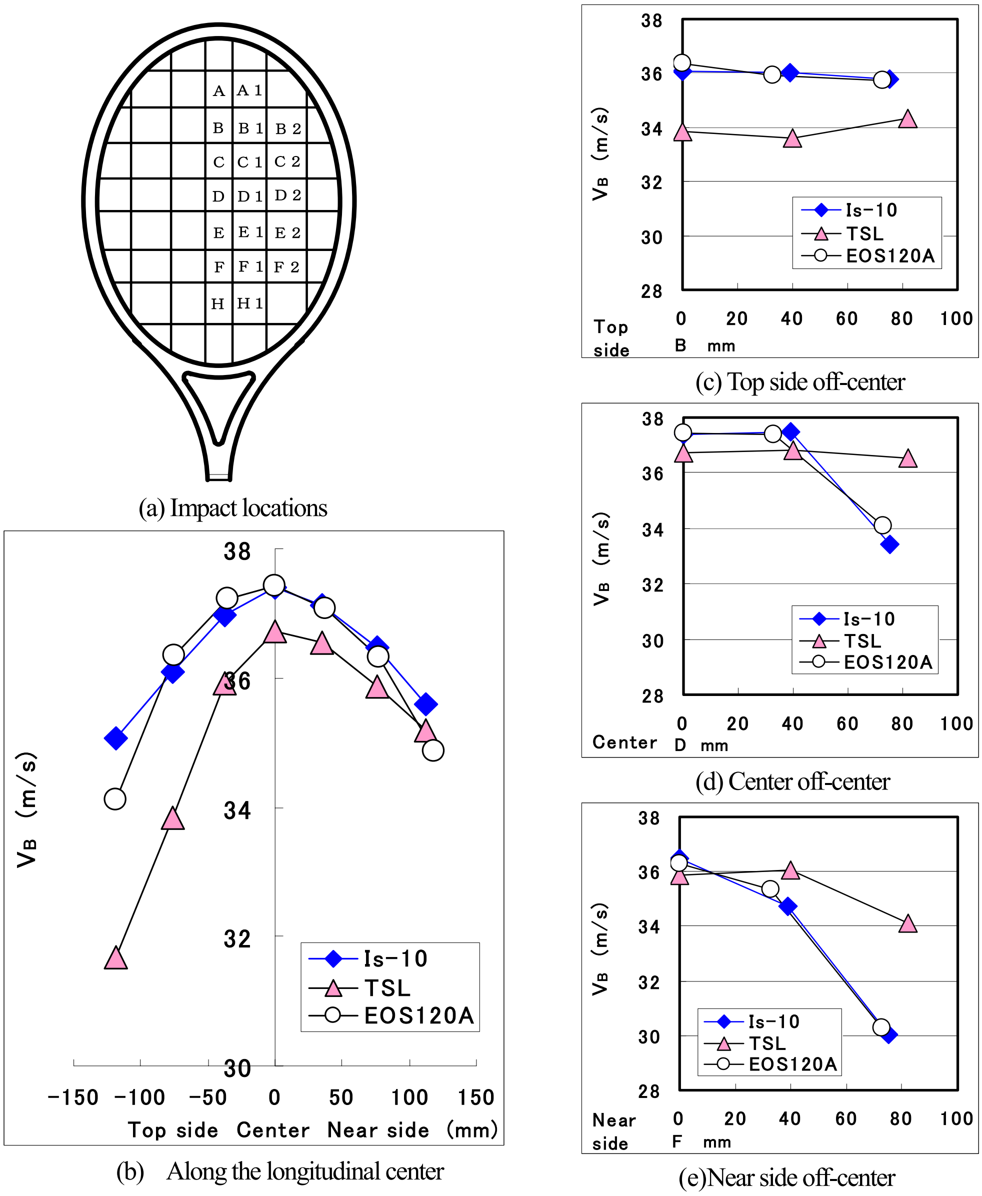


Fig.14 Predicted post-impact ball velocity V_B (Ns = 56.9Nm, V_{BO} = 10m/s)

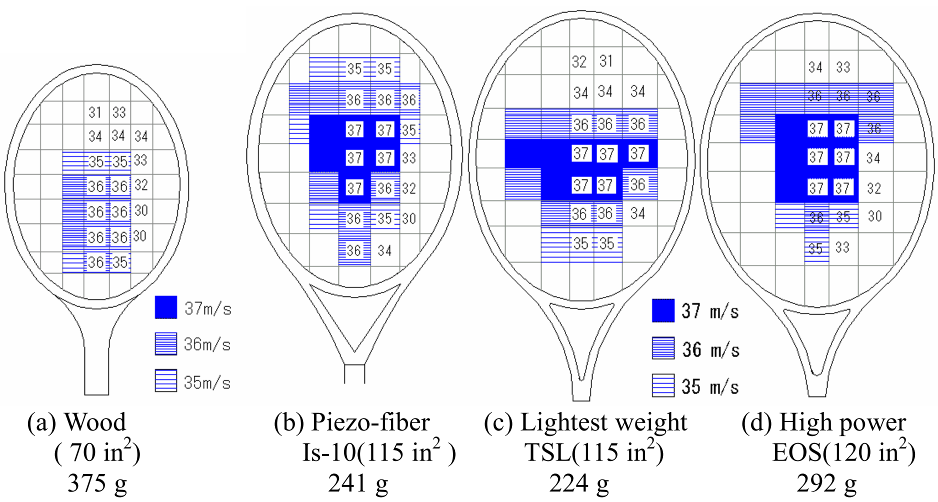


Fig.15 Predicted sweet area in terms of post-impact ball velocity V_B (Shoulder torque Ns = 56.9Nm, coming ball velocity V_{BO} = 10m/s)

4. Conclusions

This paper predicted the various factors associated with the frontal impact, such as impact force, contact time, deformation of ball and strings, and also estimated the racket performance such as the coefficient of restitution, the rebound power coefficient, the post-impact ball velocity and the sweet areas relevant to the power in tennis.

The predicted results with forehand stroke model could explain the difference in mechanism of performance between the new type racket with active piezoelectric fibers and the conventional passive representative rackets. It showed that this new type racket provides higher coefficient of restitution on the whole area of string face and also gives larger rebound power coefficients particularly at the topside and bigger powers on the whole area of string face but the difference was not so large. It seems that the racket-related improvements in play are relatively small and the players themselves continue to improve by making use of racket improvement, accordingly there is a gap between perception and reality.

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